Towards energy efficient operation of Heating, Ventilation and Air Conditioning systems via advanced supervisory control design

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Abstract. This paper presents conceptual control solution for reliable and energy efficient operation of heating, ventilation and air conditioning (HVAC) systems used in large volume building applications, e.g. warehouse facilities or exhibition centres. Advanced two-level scalable control solution, designed to extend capabilities of the existing low-level control strategies via remote internet connection, is presented. The high-level, supervisory controller is based on Model Predictive Control (MPC) architecture, which is the *state-of-the-art* for indoor climate control systems. The innovative approach benefits from using passive heating and cooling control strategies for reducing the HVAC system operational costs, while ensuring that required environmental conditions are met.

1. Introduction

Thermal systems are one of the main application domains of modern control engineering with the main focus on thermal comfort and efficient energy utilisation [1]. The research presented in this paper focuses on design of energy efficient control strategies for heating, ventilation and air conditioning (HVAC) systems used in large volume buildings. In this regard, the correct design and operation of the control system is essential to ensure that the HVAC system is achieving good thermal comfort at minimum energy use, operating cost and initial cost, see [2] and [3]. That is, at the design stage the HVAC system equipment should be accommodated to specific control needs together with the consideration for energy efficiency of chosen equipment and overall cost of the solution.

Since the use of HVAC systems is energy intensive and large amounts of energy amplify the peak demand for gas and electrical energy, the effective utilization of HVAC systems is essential. At present, the temperature control for large volume open plan industrial and commercial building structures such as warehouses, hangars and retail park stores, is niche research area, where considerable energy savings could be achieved. The thermal control of these structures is demanding because of their large space, small rate of change in indoor air temperature (compared to e.g. office buildings), and thermal mass within the building may change over the time. Some commercial sectors are imposed with legislative requirements creating a demand for accurate temperature regulation within the building, e.g. storage of pharmaceutical products and food.

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In recognition of the fact that the computing capabilities of the existing local, low-level, controllers are limited as well as the fact that these controllers are already designed and installed it is proposed to deploy a supervisory control solution. This control solution is designed to operate remotely via internet connection, while eliminating the need to purchase additional HVAC control equipment. Such a solution can be offered on top of the existing, low-level, industrial standard control solution. The proposed control system makes use of adaptive and predictive control methods to provide optimal energy management without compromising HVAC system performance or indoor air quality. The overall control strategy has been implemented with use of MATLAB computational environment.

1.1. Motivation

Starting from simple heating optimizers, the development gradually progressed resulting in intelligent control systems, which are able to adjust themselves to upcoming changes in indoor conditions and control environment, e.g. outdoor climate, occupation scheme of the building, and control parameters [3]. Advantages resulting from predictive character of control have inspired the researchers to seek adoption of model predictive control (MPC) methods to improve energy performance and minimize operation costs of HVAC systems, see [4,5] for further discussion on MPC methods. Privara et al. [6] emphasize the contrast of MPC to conventional control strategies and its importance in building climate control, while Afram and Janabi-Sharifi [7] scrutinize theory and applications of MPC to HVAC systems and reflect on factors affecting performance of this technique.

The idea of optimal control was captured, among others, by Morris, Braun and Treado [8] which demonstrated use of thermal storage potential of building mass to reduce cooling costs. This study considered two optimal control strategies: minimum total energy cost and minimum peak electrical demand, showing significant energy reduction achieved by both control methods. The obtained results have been validated on experimental test facility showing that up to 51% of total cooling load can be shifted to electricity off-peak time. The peak cooling load was reduced by 15% using energy minimization control, and by 40% using peak minimization control, when compared to conventional night setback control.

2. Plant details

A typical setup of large volume building and HVAC system with heating and cooling capabilities is depicted in figure 1. In this case, the setup is based on an existing pharmaceutical warehouse in Midlands, UK, which resembles the facility in size and thermal response. The overall volume of conditioned space is 185,472m^3. In figure 1, the overall HVAC system comprises exterior air handling unit (AHU) with inlet air mixing damper, control unit and necessary ductwork.

To describe the functionality of the HVAC system, consider the air mixing damper shown in figure 1. The position of damper blades is regulated by the control unit, taking one of two possible states: closed, which is full recirculation of the return air from the controller area, and open, where only fresh air intake is assumed. The air leaving the mixing damper section passes through the AHU comprising a cooling unit, heating unit and main supply fan. The operation of the components within the AHU is regulated by the control unit, which continuously monitors the indoor and outdoor air temperatures. Subsequently, the controlled area is supplied with conditioned air, from the AHU, which is distributed evenly throughout the area via ductwork. By default, the air mixing damper is closed, i.e. full recirculation is assumed. Note that if the air mixing damper is closed, i.e. full recirculation is assumed, there is enough fresh air within the building due to air leakage (infiltration).

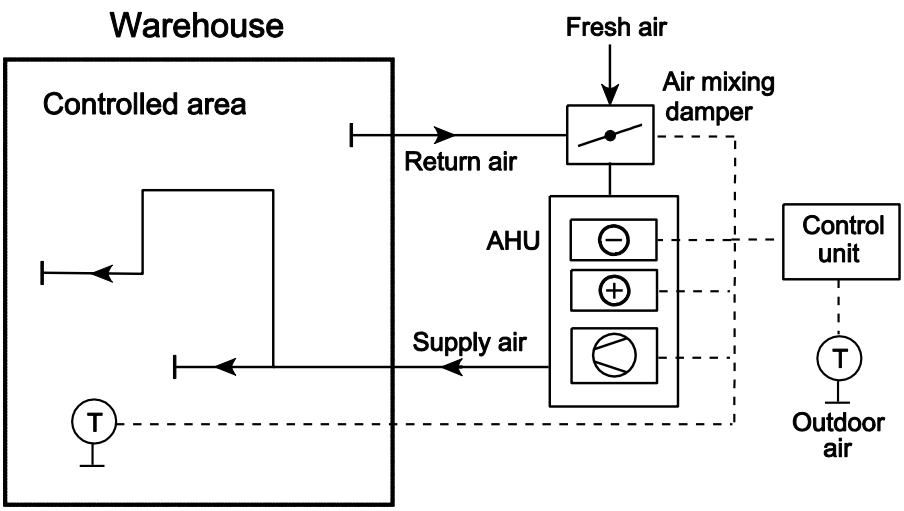

Figure 1. Air conditioning system of the pharmaceutical warehouse.

2.1. Demanded environmental conditions

The required indoor air temperature conditions are determined based on personnel comfort requirements, product storage requirements, and legislative requirements. Figure 2 shows an example of prescribed indoor air temperature set-point values of the warehouse facility. The space temperature is actively regulated to be within the control dead-band, denoted *db*. The control dead-band is defined by a cooling set-point from above, denoted r_c, and by a heating set-point from below, denoted r_h. In the case the indoor air temperature rises above the cooling set-point, i.e. is within the cooling band, the cooling unit is enabled. Similarly, if the air temperature decreases below the heating set-point, i.e. is within the heating band, the heating unit is enabled. Note, that no cooling or heating takes place within the control dead-band.

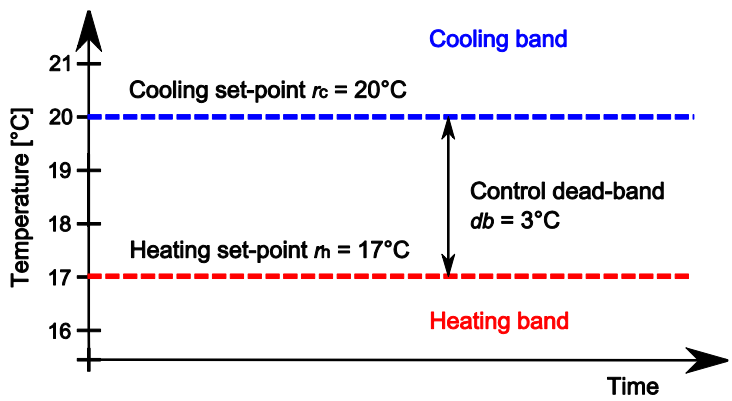

Figure 2. Demanded indoor air temperature conditions.

3. Control system structure

The proposed advanced control system has a two-level topology comprising low-level and high-level control unit. The concept of two-level control topology and its operation is depicted in figure 3. The low-level (or slave) controller represents the traditional standalone solution for control of HVAC systems, which is commonly located within the air conditioned building. The controller uses the local measurements of indoor and outdoor air temperatures, denoted *temperature* in figure 3, to compute the *control input* for actuators of the AHU. The control strategy of the low-level controller is designed to meet the demanded indoor air temperature conditions and contains basic, sub-optimal, strategy for energy efficient operation of the HVAC system.

Recognizing the fact that the computing capabilities of the local, low-level, controllers are limited as well as the fact that these controllers are already designed and installed on site it is proposed to deploy a supervisory control solution. This solution is designed to expand and build on the capabilities of the existing low-level control units via remote internet connection, while not compromising their safe operation. The supervisory controller is located in a remote command and control centre hosting the server room. The control solution, or service, is deployed via internet connection, where the controllers establish a communication link between each other in order to share information and achieve optimal, energy efficient, HVAC system operation. It should be emphasised, that the local control units work as a standalone HVAC system control solution, and hence if the remote connection is broken, the local controller continues to work as per industrial standard.

The high-level (or supervisory) controller is based on model predictive control architecture, which is in greater detail described in subsequent section 4. In general, the supervisory controller constantly monitors the performance of the existing low-level control units together with local environmental data, depicted as *warehouse env. data* in figure 3. Moreover, the controller is supplied with weather forecast data, current and anticipated energy prices, and anticipated heating and cooling loads. The supervisory controller uses a mathematical thermal model of the building together with the energy optimization algorithm to compute *optimal control settings* for the low-level controller. This process is continuously repeated in an adaptive, self-learning manner, to ensure continuous energy efficient HVAC system operation throughout its lifespan.

3.1. Low-level control design methodology

The control strategy for the low-level controller is designed and implemented using robust software design methods. The designed control strategy is validated by model-in-the-loop simulation techniques to achieve a rapid, low cost and, more importantly, reliable control software solution. For this purpose, the high-level control requirements are cascaded down to functional requirements, which then formulate the designed control strategy. Such control strategy is implemented in Matlab/Simulink programming environment and is tested, via simulation, against the functional requirements – this constitutes the model-in-the-loop testing and development methodology. Currently a prototype solution is under development and real-time trials are planned. The designed and validated low-level control strategy is implemented on the industrial programmable logic controller (PLC) of Trend Control Systems Ltd. The PLC belongs to a so called IQ line of products with enabled Ethernet connectivity, which enables the use of the proposed supervisory controller.

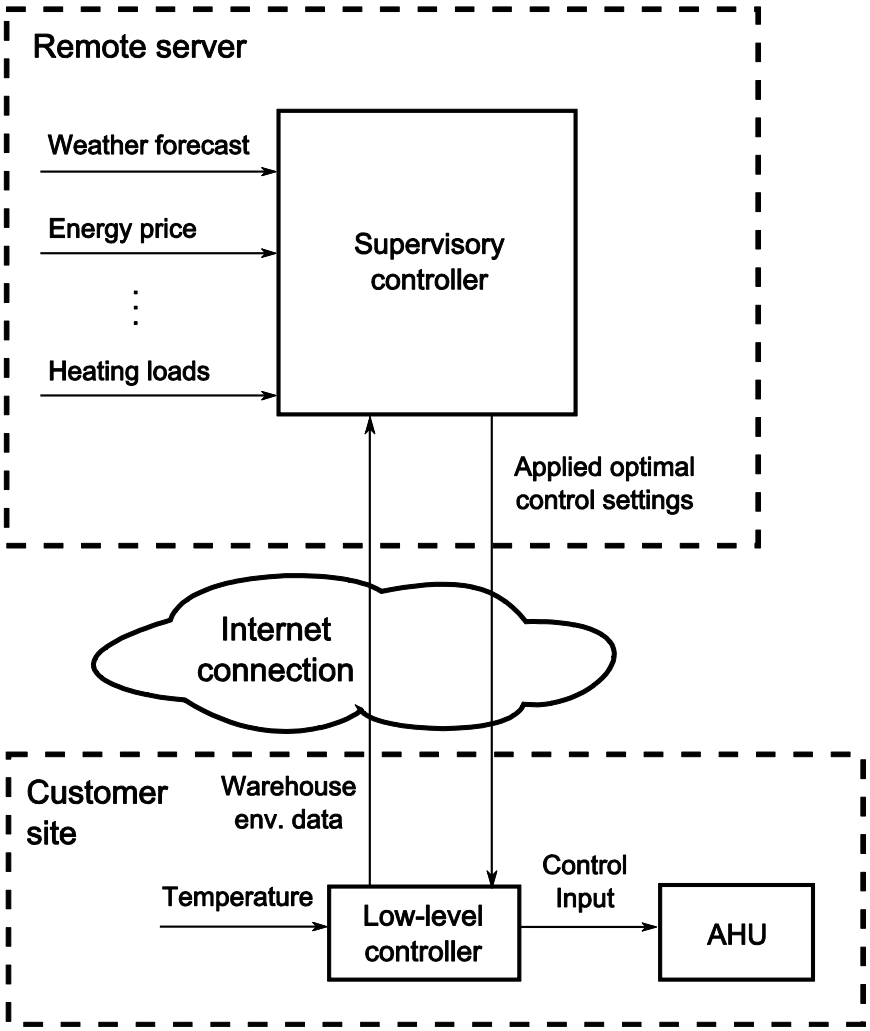

Figure 3. Proposed advanced control system architecture.

4. Supervisory control system

The supervisory controller is based on model predictive control architecture, which is the *state-of-the-art* for indoor climate control systems. The architecture is depicted in figure 4 and composes of three essential components, which are a mathematical model of the building (including AHU and low-level control logic), cost function and optimization algorithm. First, consider the mathematical thermal model of the building, which is used for computer simulation.

1) *Mathematical model*: Having the model it is possible to simulate and subsequently predict the indoor air temperature of the building, which is then utilized by the cost function. To obtain the prediction of the indoor air temperature the latest measurement of building environmental data (indoor and outdoor air temperature) is required as well as the prediction of future heating effects acting on the building. The heating effects are mainly defined by the outdoor weather conditions such as ambient air temperature, solar irradiation and wind speed. For this purpose the weather forecast data are required. Other possible cause of future heating load is the personnel and machinery activity within the building. The prediction of the indoor air temperature is obtained over a so called prediction horizon, denoted h_p.

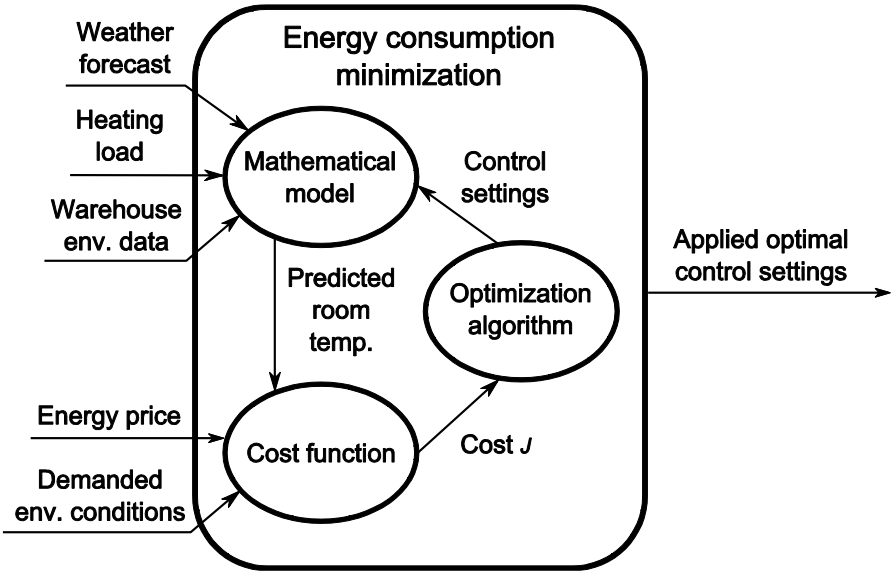

Figure 4. Model predictive control architecture of the supervisory controller designed for minimal energy utilization. The computed optimal control settings are then cascaded down and applied to the low-level, local, controller.

The indoor air temperature prediction is realized for a given low-level control setting (whose value is being optimized), i.e. specific configuration of the HVAC system such as cooling unit enabled or main fan disabled. Note, that different low-level control setting would cause indoor air temperature prediction to change despite using the same forecast of heating loads. The mathematical model representing the thermal response of the building to heating and cooling loads can be obtained through data analysis (black box modelling approach) or with the use of physical laws (white box modelling approach). The model derived using first principles is briefly introduced in appendix, see [9] for full reference.

Second, consider the cost function component in figure 4, which takes as an input the indoor air temperature prediction.

2) *Cost function*: The operating cost of the HVAC system (and with that the energy consumed) is minimized by means of minimizing the cost function, denoted J, defined in equation (1). This means that the cost function is defined such that its value directly relates to the energy consumed to achieve a particular prediction of indoor air temperature. The cost function arguments u_c and u_h are vectors comprising predicted cooling and heating loads over the prediction horizon h_p, respectively. The arguments u_c and u_h are weighted by respective future energy prices per unit of energy, p_c and p_h.

$$J = \sum_{t=1}^{h_p} p_c(t) \cdot u_c(t) + \sum_{t=1}^{h_p} p_h(t) \cdot u_h(t) \tag{1}$$

The achieved cost is a sum of products of HVAC unit activity and the price of the energy spent to run the unit over a selected prediction horizon.

Subsequently, the calculated cost, to achieve a given indoor air temperature over a given prediction horizon, is passed to the optimization algorithm. This is considered in the next third step.

3) *Optimization algorithm*: The optimization algorithm uses the latest computed cost J to compute a new set of low-level control settings, which will be used by the mathematical model in next iteration of the optimization procedure. The optimization algorithm seeks such a value of low-level control settings, which will cause the cost J to reduce. Note, that optimization methods are discussed in more detail in next subsection.

The steps 1), 2) and 3) are repeated until the optimization routine converges, i.e. a minimal value of the cost function J is achieved. Subsequently, the found optimal control setting is cascaded down and applied to a real low-level controller via the remote internet connection. It is required that the values of optimal low-level control setting are found within one sampling time instance, denoted Δt. This is the re-scheduling time of the controller at which the newly computed control inputs are applied to the AHU. Finally, the whole optimization routine repeats in the next sampling time interval, i.e. prediction and optimization steps until convergence is achieved. This gives an alternative name to MPC, namely receding horizon control.

Figure 5 shows an illustrative example of one realization of indoor air temperature prediction as part of one optimization step. The figure shows the past, measured, indoor and outdoor air temperatures, cooling and heating set-point values as well as the forecasted outdoor air temperature and predicted, corresponding, indoor air temperature. In this example, it is assumed that only the position of mixing damper blades needs to be optimized over the prediction horizon, where only open (fresh air supply only) or closed (full air recirculation) damper blades positions are considered. For this realization the optimization routine computes a particular combination of open/closed mixing damper blades positions over the prediction horizon $h_p = 7$, which is depicted by solid and empty circles in figure 5. After such a combination of mixing damper blades position is found, which results in the lowest value of the cost function J, then only the first damper blades position is actually applied to the low-level controller, while the rest is discarded. Subsequently, the whole optimization cycle repeats.

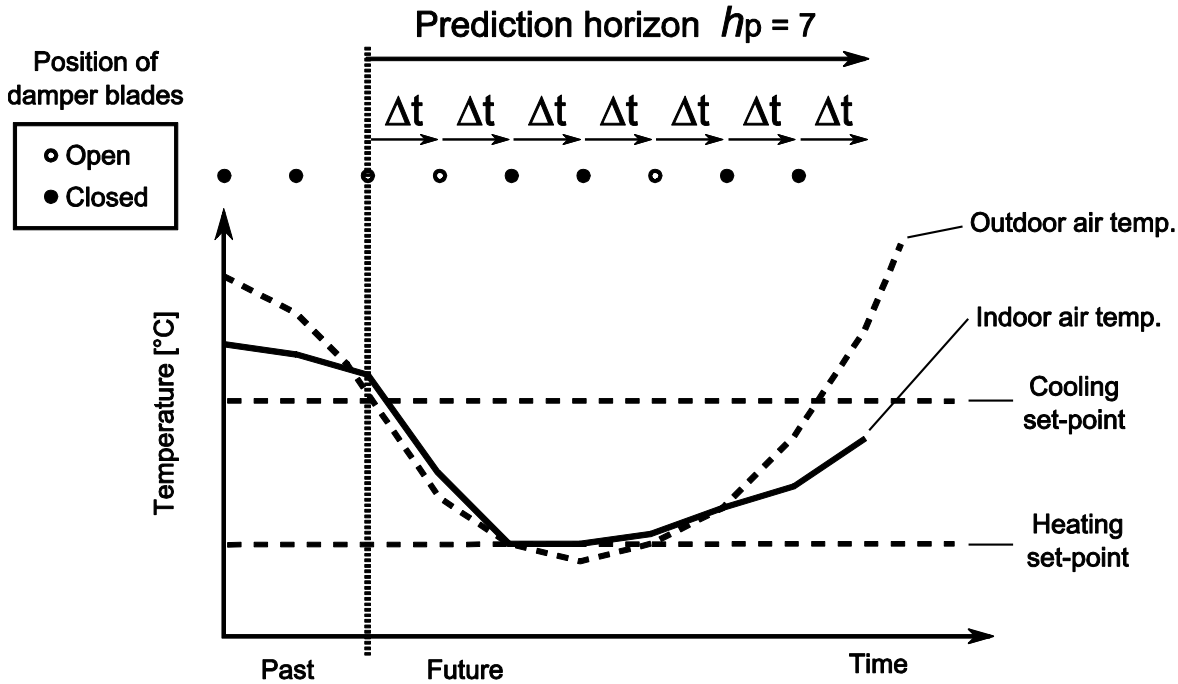

Figure 5. Conceptual Model Predictive Control of damper blades position. The predictive control strategy is either opening or closing the air mixing damper to force the indoor air temperature to remain in between cooling and heating set-point for maximum energy efficiency, i.e. no active heating cooling take place. Sampling time interval $\Delta \boldsymbol{t}$ is 15 min.

4.1. Optimization methods

The optimization problem is solved using a class of heuristic optimisation techniques. The solver proposed for this purpose is Genetic Algorithm (GA), which belongs to a group of evolutionary algorithms. This nature inspired method mirrors the process of natural selection by reproducing in each generation the members carrying the best solutions to create next offspring and rejecting the worst ones. New generations are created combining two methods, which are crossover and mutation method respectively. In the first method, being a fundamental way of procreation, each member of the new offspring inherits some part of genetic information from each of the two parents (solutions). The second method is used to introduce random changes for two reasons: first, this is important to avoid local optimal solution, second, the changes are made in hope that a desirable solution is introduced [10].

4.2. Weather forecasting

The performance of model predictive methods fundamentally rely on the knowledge of future inputs (heating gains), where the outdoor air temperature is of a high importance. The established air temperature forecasting algorithms are based on Kalman Filtering techniques using so called random walk models. These algorithms use the latest weather forecast and local measurements of the ambient air temperature of the building. The forecasted outdoor air temperature is extrapolated in a stochastic framework based on recorded past and present air temperature data and analysis of forecasted trends. The optimal predictive control algorithm can be improved by incorporating other measured variables into predictions, e.g. relative air humidity and solar irradiance, to achieve the highest rate of energy savings.

4.3. Exemplary case study

For the purpose of this exemplary case study a simplified mathematical thermal model of the facility described in section 2 has been derived using first principles (see appendix). For simplicity, it is assumed that the indoor space temperature is uniformly distributed implying perfect air mixing. Furthermore a perfect control is assumed such that whenever the indoor air temperature would rise above the cooling set-point or below heating set-point the air temperature will remain at these values, i.e. instantaneous cooling or heating response of the AHU is assumed. The indoor climate requirements hold as stated in section 2.1, where $r_c = 20°\mathrm{C}$ and $r_h = 17°\mathrm{C}$. The supervisory controller is designed to select optimal, energy efficient, position of mixing damper blades, where only open (fresh air supply only) or closed (full air recirculation) damper positions are considered.

The simulation results are shown in figure 6. The outdoor air temperature, depicted by a solid black line in the lower plot, is designed to be a sinusoidal signal being the main input to the thermal building model. Note, that it is also the case in practice that the outdoor air temperature naturally follows the night and day cycle. The upper plot shows the simulated indoor air temperature response subjected to the mixing damper blades position. In general, the indoor air temperature tends to follow the same trend as the ambient air temperature, but is limited from above by the cooling unit regulating the room temperature to remain at the cooling set-point value.

Considering the case of closed mixing damper (full air recalculation) the room temperature response, depicted by black solid line in figure 6, is very mild and prolonged. In comparison, the room temperature response obtained by having the mixing damper fully open (fresh/outdoor air intake only) follows the outdoor air temperature trend very closely and drops nearly as low as the ambient air temperature does, while rising sharply back. The room temperature response obtained using the optimal, energy efficient, combination of mixing damper blades positions is depicted by dashed dotted black line. The corresponding optimal damper blades position is depicted by grey solid line in the lower plot.

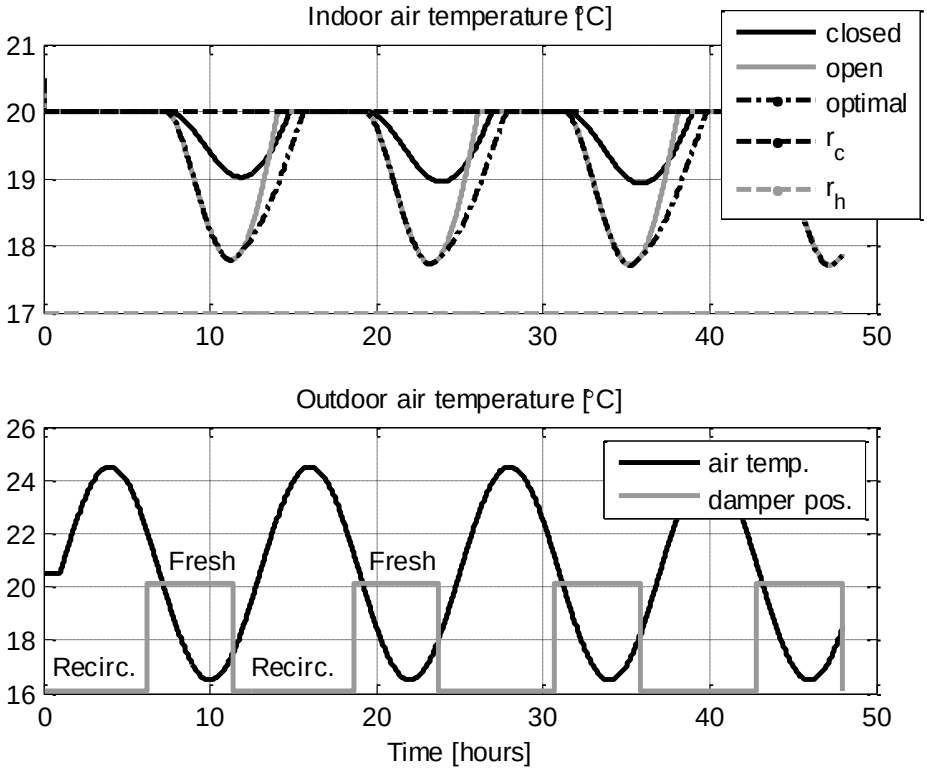

Figure 6. Simulation results of the simplified case study. The lower plot shows the selected outdoor air temperature, solid black line, and mixing damper position, solid grey line, where low state implies that the blades are closed, while the high state implies that blades are open. The upper plot shows the cooling and heating set-point values depicted by dashed black and grey lines, respectively. The black solid line shows the simulated room temperature for closed mixing damper (full recirculation), while the grey solid line shows the room temperature response for open mixing damper (fresh air supply only). The dashed dotted black line shows the simulated room temperature for the optimal combination of mixing damper blades positions depicted in the lower plot by solid grey line.

It can be observed that the room temperature response obtained by the optimal control strategy remains within the control dead-band the longest. This implies that minimal cooling is required, which, in return, reduces the HVAC system operating costs. The supervisory controller naturally selected such a strategy, which opens the mixing damper to cool down the room air temperature when ambient air temperature falls and closes the mixing damper when the ambient temperature rises. This strategy utilizes the natural cycling behavior of ambient air temperature to pre-cool the conditioned space when possible and prolong the periods where no cooling is required. In other words, the thermal storage capacity of the building is used to shift the heating loads for maximum energy efficiency.

Finally, assuming that the time for which cooling has been required directly relates to the energy consumed, then the energy savings can be numerically evaluated. The optimal control strategy reduced the cooling unit operating time by 10% when compared to the case of fully closed mixing damper. The cooling unit operating time has been reduced even more significantly, by 30%, when compared to the case of fully open mixing damper.

5. Case study: Pharmaceutical warehouse facility

In this case study, the potential for operating cost reduction of HVAC system installed in the existing pharmaceutical warehouse located in Midlands, UK, has been investigated. The results and detail description of the presented case study were originally presented in [9], while the general plant setup is introduced in section 2. The computer based simulation study has been chosen to compare the performance of the existing low-level controller with the proposed supervisory control strategy so that the HVAC system operation is not compromised. The thermal model of the building is similar in structure to the one used in subsection 4.3 and has been presented in detail in [9]. However, with the difference, that the model parameters have been correlated with measured local air temperature data collected in 2014. Additionally, the real measurements of ambient air temperature have been used during the simulation. This has resulted in more realistic case study, allowing for rapid changes in the ambient air temperature, which challenges the proposed control solution.

The supervisory controller has been designed to optimise the position of air mixing damper blades and to select energy efficient control set-point values. Adaptive set-point adjustment aims for temporary extension of the control dead-band envelope by reducing the heating set-point value down to r_h = 16°C and increasing the value of cooling set-point up to r_c = 22°C, while not compromising the safe HVAC system operation. The maximization of the control dead-band envelope results in reduced heating and cooling demand. In order to see the full potential of proposed control solution the true value of future, forecasted, ambient air temperature has been made available to the MPC.

5.1. Results & discussion

The intelligent supervisory control system has been compared with the existing standard control system, which does not make use of predictive features and weather forecast data. The simulation study has shown that energy savings up to 7% in January, 57% in April and 76% in June can be theoretically achieved. Choosing representative data for cool, moderate and hot seasons, the obtained results indicate that energy reduction is possible at all times of the year. Nevertheless, visible differences in the level of energy savings are expected throughout the year as the typical seasonal temperatures vary. While almost continuous heating is required in a winter season, leaving little scope for reducing operational costs, the situation in summer season is more favourable and significant energy savings are possible.

6. Conclusions and further work

The conceptual control solution, which extends the capabilities of the existing low-level control strategies via remote internet connection, has been presented. A two-level scalable control topology has been proposed, where the high-level, supervisory controller, is based on Model Predictive Control (MPC) architecture feeding into existing low-level controller. The control design is targeted to work with established control solutions, e.g. provided by Trend Control Systems. The development of the supervisory controller goes alongside with the development of the control logic for the low-level controller. It is required that the control logic is implemented in a robust and reliable way and that this control logic is mated, by design, with the supervisory controller.

Results of simulation based case study have been presented. These results show potential for energy savings up to 7% in January, 57% in April and 76% in June when compared to equivalent simulation study where only existing, low-level, controller has been used.

In this work we are specifically targeting large volume buildings such as warehouse facilities, retail park stores or aircraft hangars. The control challenges include slow system dynamical response, air temperature stratification effects, large impact of solar irradiation and wind speed. Nevertheless, the applicability of the proposed control solutions is not limited to large volume buildings.

6.1. Next steps

The current work is focused on development and implementation of low-level controls and arrangement

of the supervisory controller for its use with low-level controls. Then, the real-time field studies on the building facility will follow to investigate the performance of the HVAC system and the actual energy saving capabilities of proposed concepts.

Acknowledgements

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Appendix

The simple air temperature model is based on first principles modelling methodology, see [3,11], where, in particular, a lumped parameter modelling approach has been considered. A schematic diagram of the room is shown in figure 7, where uniform air temperature distribution is assumed. It is also assumed that the air leaving the conditioned space, denoted $T_r(t)$ [K], is equal to the mean temperature of the whole room. The time index (t) is chosen to emphasize that the signals are not measured, but generated by physical based model. The final equations representing the physical system are reduced into a set of ordinary differential equations with a finite number of parameters.

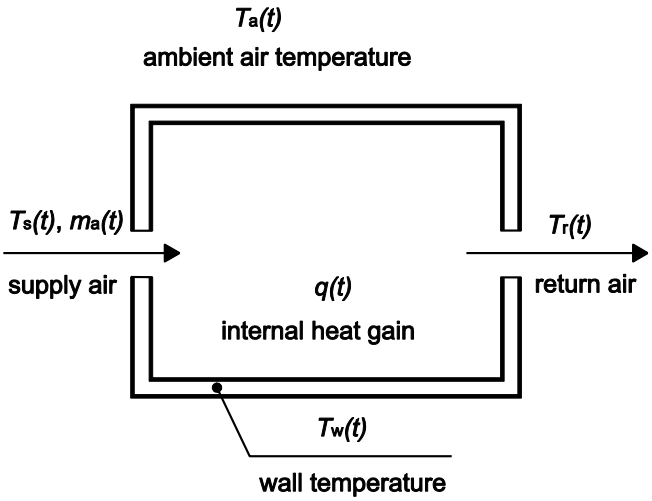

Figure 7. Schematic diagram of the room temperature model.

The continuous-time first principles room temperature dynamic model is based on energy balance equations, where two states are considered: the room air temperature, denoted $T_r(t)$, and the wall temperature, denoted $T_w(t)$. The energy balance equations for the temperature of the air within the conditioned space and adjacent walls, respectively, are

$$C_\mathrm{r}\frac{dT_\mathrm{r}(t)}{dt} = \mathrm{m_a}(t)\rho_\mathrm{a}c_\mathrm{a}[T_\mathrm{s}(t) - T_\mathrm{r}(t)] - (UA)_\mathrm{int}[T_\mathrm{r}(t) - T_\mathrm{w}(t)] + q(t) - n_\mathrm{v}\frac{\rho_\mathrm{a}V_\mathrm{r}c_\mathrm{a}}{3600}[T_\mathrm{r}(t) - T_\mathrm{a}(t)] \tag{A.1a}$$

$$C_\mathrm{w}\frac{dT_\mathrm{w}(t)}{dt} = (UA)_\mathrm{int}[T_\mathrm{r}(t) - T_\mathrm{w}(t)] - (UA)_\mathrm{ext}[T_\mathrm{w}(t) - T_\mathrm{a}(t)] \tag{A.1b}$$

where C_r [J/K] denotes the thermal air capacity of the room and C_w [J/K] the thermal capacity of the wall, c_a [J/(kg·K)] denotes the air specific heat capacity, $m_\mathrm{a}(t)$ [m^3/s] denotes the air volumetric-flow

rate, U_{int} and U_{ext} [W/(m^2·K)] are the heat transfer coefficients on the inner side and outer side of walls, respectively, A [m^2] denotes the effective surface area of the walls, $T_{\mathrm{w}}(t)$ [K] denotes the mean wall temperature, $T_{\mathrm{a}}(t)$ [K] denotes the ambient (outdoor) temperature, $q(t)$ [W] denotes the heat load disturbance within the room and $T_{\mathrm{s}}(t)$ [K] denotes the supply air temperature. The thermal capacities C_{r} and C_{w} are calculated as follows:

$$C_{\mathrm{r}} = \rho_{\mathrm{a}} V_{\mathrm{r}} c_{\mathrm{a}} \tag{A.2a}$$

$$C_{\mathrm{w}} = \rho_{\mathrm{w}} V_{\mathrm{w}} c_{\mathrm{w}} \tag{A.2b}$$

where ρ [kg/m^3] denotes the density of air, ρ_{a}, and wall material, ρ_{w}, c [J/(kg·K)] denotes the specific heat capacity of air, c_{a}, and wall material, c_{w}, V_{r} [m^3] denotes the volume of the air within the room and V_{w} [m^3] denotes the volume of the walls.

The left-hand side terms of equations (A.1a) and (A.1b) are regarded as the rate of change of energy in the room and the rate of change of energy within the walls, respectively. The rate of change must be in balance with the energy transfers placed on the right-hand side of the equations, as stated by the first law of thermodynamics. Analysing the first energy balance equation (A.1a), the first term represents the energy received from the HVAC system, the second term represents the transfer of internal energy into the wall, the third term introduces the heat loads, and the last term represents air infiltration. The ventilation air change rate is denoted n_{v} [1/h]. Note, that regulation of the position of damper blades will assume the supply temperature $T_{\mathrm{s}}(t) = T_{\mathrm{a}}(t)$ for damper open and $T_{\mathrm{s}}(t) = T_{\mathrm{r}}(t)$ for damper closed. Subsequently, the rate of change of energy in the walls is balanced with the heat transfer on the inner and outer side of walls.

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