

# A model for water-entry projectile with an open cavity

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**Abstract.** Study on dynamics of a projectile running with a cavity in the process of water entry closely relates to the development of new generation of air-to-water vehicle. It becomes a key concern to develop a flexible model of the high-speed water-entry projectile. The projectile is modelled using the unsteady cavity model from Serebryakov in the frame of the rigid-body dynamics theory. Based on the model, without loss of generality, oblique water-entry projectile running with an open cavity is simulated, and numerical results are in good agreement with measured data available. The model proves effective and has a great potential to investigate ballistics of an air-dropped vehicle at the early stage of water entry.

## 1. Introduction

Water entry has come into researchers' sight for over 100 years of history as a classical problem, and has made some great achievements [1]. Because supercavitation drag reduction technique has drawn more and more attention, water-entry projectile at high speed always becomes a research hotspot for improving and developing air-to-water vehicle.

Some very valuable images of supercavitating projectiles [2] were obtained at the speed of sound for analysis of projectile characteristics such as flight behaviour, cavity shape, and stability mechanism. The excellent findings confirmed that projectile can lean against cavity wall to produce planning force as a stable motion mode similarly to underwater supercavitating vehicle [3]. Compared with horizontal water entry, more extensive attention focuses on water entry of air-dropped object in high-speed motion. Vertical water entry was modelled by Aristoff [4], and cavity shape and pinch-off time are formulated for cavity evolution dominated by aerodynamic pressure. Cavity model based on Logvinovich's principle [5] is also a powerful approach to study vertical water entry [6], and besides, the algorithm [7] was developed to solve Navier-Stokes equations for subsonic and supersonic projectile. It is easy to understand that oblique water entry is a more general problem. Truscott et al. [8] conducted some beneficial work, providing design foundation for oblique water-entry projectile to avoid ricochet and improving Logvinovich's model.

Although great progress has been made, trajectory characteristic, motion stability and control of high-speed projectile remain to study further, and restrict its applications, i.e., developing intelligent air-to-water vehicle. It is urgent to establish a model of high-speed water-entry projectile applied to projectile's dynamics and control effectively. Hence, the projectile is modelled using Serebryakov's unsteady cavity model [9] and the rigid-body dynamics theory. Based on the model, oblique water-entry projectile is simulated at high speed before cavity closures, and the model is validated by the measured data available [10].



## 2. Water-entry projectile model

For a projectile entering water at high speed, a cavity appears around the projectile due to violent deformation of the free surface. The mathematical formulation of projectile consists of the cavity model and the dynamics model of a projectile.

### 2.1. Cavity model

Considered from the Lagrangian coordinate system, cavity section is reasonably assumed to expand radially in the plane perpendicular to trajectory in water entry. In such a case, cavity is merely open to atmosphere, and has a pressure drop owing to airflow following projectile. The unsteady cavity model [9] derived by Serebryakov, validated by lots of experiments, is applied to describe cavity evolution. The model is known as a modern presentation of Logvinovich's principle with very clear physical meaning as follows:

$$\mu(s) \frac{\partial^2 R_c^2(s,t)}{\partial t^2} + \frac{2\Delta p(s,t)}{\rho_w} = 0 \quad (1)$$

$$R_c^2(s,t) \Big|_{t=t_n(s)} = R_n^2(s), \quad \frac{\partial R_c^2(s,t)}{\partial t} \Big|_{t=t_n(s)} = R_n(s)V(s) \sqrt{\frac{2(C_d(s) - k(s)\sigma_c(s))}{k(s)\mu(s)}} \quad (2)$$

where  $R_c(s,t)$  and  $R_n(s)$  are the radii of cavity and cavitator on the trajectory  $s$ ;  $V(s)$  is the velocity of cavitator;  $\rho_w$  is the water density;  $\Delta p(s,t)$  is the pressure difference inside and outside cavity,  $\Delta p(s,t) = p_\infty(s,t) - p_c(s,t)$ ,  $p_\infty(s,t)$  is the ambient pressure,  $p_c(s,t)$  is the internal pressure under the influence of aerodynamic pressure [4],  $p_c(s) = p_a - \min(C_a \rho_a V_0^2, p_a)$ ,  $C_a$  is the coefficient of aerodynamic pressure, taken as  $C_a = 2.5$ ,  $p_a$  is the atmospheric pressure,  $V_0$  is the initial velocity of water entry,  $\rho_a$  is the air density;  $t_n(s)$  is the time when cavitation occurs;  $C_d(s)$  is the drag coefficient;  $\sigma_c(s)$  is the cavitation number;  $k(s)$  and  $\mu(s)$  are the parameters for assessments of cavity dimensions.

### 2.2. Dynamics model of projectile

Water-entry projectile is subject to forces on cavitator, gravity and planing force [2] for motion stability. Paryshev's planing model [11] is applied because it better agrees with experimental data. Importantly, ricochet may occur if geometry, speed and entry angle of projectile does not satisfy some criterions [8]. The case without ricochet, which is put down to reasonable designs, is discussed here.

Because projectile is generally an axisymmetric body, water-entry problem can be simplified into two-dimensional one with respect to the longitudinal plane of projectile. Considering that cavitator movement represents projectile's trajectory, it is a good strategy to build the body-fixed frame **B** at cavitator. All forces must be transformed into ones in this system by coordinate transformations. According to Newton's second law of motion and theorem of momentum moment in the inertial frame **E**, dynamics equations are written:

$$m(\dot{u} + wq - q^2 x_g) = F_x \quad (3)$$

$$m(\dot{w} - uq - \dot{q}x_g) = F_z \quad (4)$$

$$I\dot{q} + m(-x_g(\dot{w} - uq) + x_g^2 \dot{q}) = M_y \quad (5)$$

where  $u$  and  $w$  are the velocity components of projectile in the longitudinal axis and its vertical axis downward;  $q$  is the angular velocity in the longitudinal plane;  $x_g$  is the coordinate of center of mass (**CM**) in the longitudinal axis;  $I$  is the moment of inertia about

the axis perpendicular to the plane;  $F_x$ ,  $F_z$  and  $M_y$  are the resultant forces and moment in their respective directions, and are composed of forces on cavitator, gravity and tail wetted region, i.e.,  $F_x = F_{xc} + F_{xg}$ ,  $F_z = F_{zc} + F_{zg} + F_{zp}$ ,  $M_y = M_{yg} + M_{yp}$ .

$$F_{xc} = -D_c(C\delta_c C\alpha_c + S\delta_c S\alpha_c) + L_c(C\delta_c S\alpha_c - S\delta_c C\alpha_c) \quad (6)$$

$$F_{zc} = D_c(S\delta_c C\alpha_c - C\delta_c S\alpha_c) - L_c(S\delta_c S\alpha_c + C\delta_c C\alpha_c) \quad (7)$$

$$F_{xg} = -mgS\theta; \quad F_{zg} = mgC\theta; \quad M_{yg} = -mgx_g C\theta \quad (8)$$

$$F_{zp} = \pi\rho_w R_b^2 V^2 S\alpha_p C\alpha_p \left(1 - \frac{(R_c - R_b)^2}{(h_0 + R_c - R_b)^2}\right); \quad M_{yp} = -F_{zp} x_p \quad (9)$$

where  $F_{xc}$  and  $F_{zc}$  are the force components of cavitator in the longitudinal axis and its vertical axis;  $F_{xg}$ ,  $F_{zg}$  and  $M_{yg}$  are the force components and moment of gravity;  $F_{zp}$  and  $M_{yp}$  are the planing force and moment;  $D_c$  and  $L_c$  are the drag and lift in the velocity frame of cavitator,  $D_c = 0.5\rho_w V^2 S_n C_d C^2\alpha_c$ ,  $L_c = 0.5\rho_w V^2 S_n C_d C\alpha_c S\alpha_c$ ;  $\delta_c$  is the deflection angle of cavitator;  $m$  is the mass of projectile;  $R_b$  is the radius of projectile afterbody;  $h_0$  is the immersion depth in the tail;  $\alpha_p$  is the angle between projectile axis and tangent of cavity axis in the tail;  $x_p$  is the coordinate of acting point of planing force in the longitudinal axis.

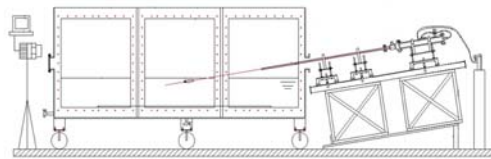
Then according to coordinate transformation between the frame **B** and inertial frame **E**, the projectile trajectory is in the inertial frame:

$$\dot{x}_e = uC\theta + wS\theta; \quad \dot{z}_e = -uS\theta + wC\theta \quad (10)$$

where  $x_e$  and  $z_e$  are the trajectory coordinates;  $\theta$  is the pitch angle,  $\dot{\theta} = q$ .

### 3. Validation of water-entry projectile model

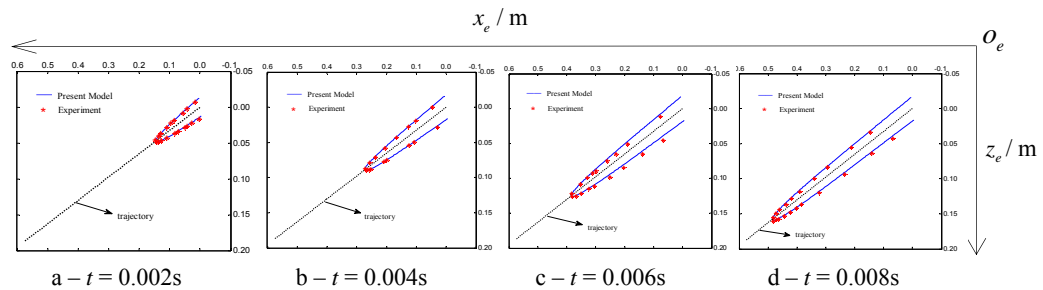
Based on the model, oblique water-entry projectile running with an open cavity is simulated as a general case to compare with results from the experiment [10] in figure 1 to validate the model using our self-developed algorithm [3]. The cylindrical projectile model has the truncated cone-shaped head and entry angle  $\theta_c = 18^\circ$  to prevent ricochet [8].



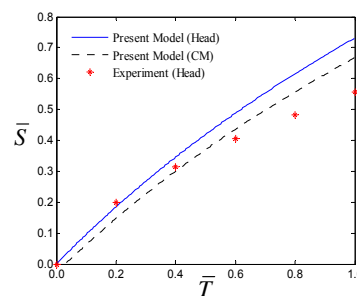
**Figure 1.** Schematic of experimental equipment [10]

Numerical results coincide well with experiment data, as is shown in figure 2, where  $x_e$  and  $z_e$  denote the horizontal and vertical directions. Cavity gradually elongates before closure, and its trajectory is rather stable and almost a straight line because of high speed and reasonable entry angle. Also, figure 3 shows the path lengths of head and CM of projectile in water, where  $\bar{T}$  and  $\bar{S}$  are the dimensionless time and path length,  $\bar{T} = t/T_0$ ,  $\bar{S} = s/(V_0 T_0)$ ,  $T_0$  is the total time during water entry,  $T_0 = 0.01s$ . The results are supported by experimental ones in the early stage of water entry, but as time goes on, some deviations display. One important reason accounts for it that viscous friction, which indeed accelerates speed attenuation and decreases path length, is not considered in the planing region. On the other

hand, it is also a factor to determine a reasonable coefficient of aerodynamic pressure. On the whole, these results from the model agree with experimental data.



**Figure 2.** Evolution of the cavity induced by a water-entry projectile ( $V_0 = 82.76 \text{ m/s}$ )



**Figure 3.** Path lengths of projectile in water

#### 4. Conclusion

The model of water-entry projectile with an open cavity is proposed, and is validated by the experiment available. In our further work, viscous force in the planing region will be focused to make the proposed model more precise, and then will combine our existing supercavity control algorithm [12] to develop intelligent air-to-water vehicle at high speed.

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