On the calculation of forces in case of planar and axially symmetric flow around the cavitating body

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Abstract. The planar and axially symmetric problems of cavitational flow around the body are considered by the Riaboushinsky scheme. The running-on flow is considered to be established for vortex-free ideal and incompressible fluid. In order to find the flow around the body the boundary elements numerical method is used which incorporates the quadrature formulas without saturation. To find the free boundary the gradient descent method based on the Riabushischinsky method is proposed. The resistance force acting on the cavitator is expressed in terms of the Riabushischinsky function, enabling us to calculate the force with rather high precision for small cavitation numbers. The dependence of the resistance coefficient for cavitators of different shape is studied: for wedge and cone, the circular arc and the spherical segment.

1. Introduction

The cavitation problems and the related numerical methods can be called classic. A considerable number of works is devoted to them, and their reviews may be found in monographs [1,2]. With the new technology development the more effective and precise methods for the cavitational flow problems are needed. In the present paper the planar and axially symmetric problems of the cavitational flow around the body are considered according to the Riabushischinsky scheme. The flow is considered to be established for vortex-free, ideal and incompressible fluid. Symmetric cavitators are considered and the flow around them is also supposed to be symmetric. The boundary is considered to consist of a cavitator of a given form, its mirror image and the free boundary, on which the fluid velocity is constant.

For the flow around problem the boundary elements numerical method is applied to the integral equation of force on the flow boundary. The numerical scheme of the boundary elements method is applied to the integral equation of the velocity problem.

The distinguishing features of the solution:

- when solving the integral equation the quadrature formulas without saturation for the integral with logarithmic singularity are used [3];
- The Riabushischinsky variation principle is used to find the free surface and the gradient method is used to find the function extremum;

- The formula, that expresses the force on the cavitator through the Riabushischinsky function alows to calculate the force even at small cavitation number.

2. The cavitational flow on the wedge

To test the numerical scheme the exact solution of the cavitational flow around problem for the wedge according to the Riabushischinsky scheme [4] can be used. The solution is built with the help of the conformal mapping on the motion hodograph plane in the same way as the classical Riabushischinsky solution for the plate in [5] (pp. 304-306). A quarter of the flow plane CBAM in the complex plane z is mapped to the motion hodograph plane t as shown in figure 1. On the free surface MA the fluid velocity is constant $v_0 = v_\infty \cdot \sqrt{1+\sigma}$ where v∞ is the velocity in a point at infinity C, σ is the cavitation number, figure 1. The complex potential of the flow W is expressed through the variables z and t as follows:

$$\frac{dW}{dz} = v_0 t^\alpha, \quad W = G v_0 w, \quad w = \frac{1-t^2}{\sqrt{(1/t_0^2 - t^2)(t_0^2 - t^2)}\left(1-t_0^2\right)^2 t_0}, \quad t_0 = (1+\sigma)^{-1/(2\alpha)}$$

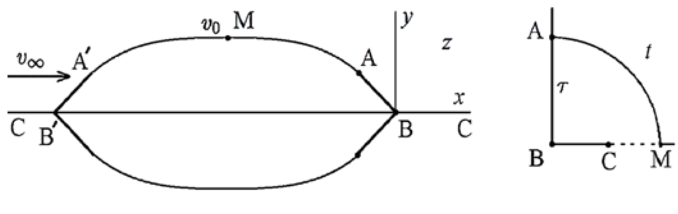

Figure 1. The conformal mapping scheme.

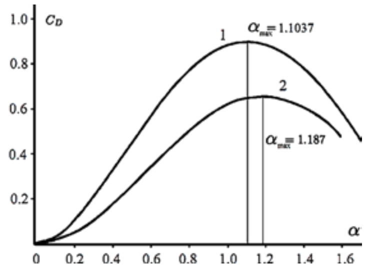

Figure 2. The resistance coefficient for the wedge (1) and for the cone (2).

The wedge segment AB (symmetric to the initial wedge segment A'B') corresponds to the segment on the imaginary axis $t=\mathrm{i}\tau$, $0<\tau<1$. The segment AB is defined as the dependence of the complex vector $z(\tau)=\mathrm{i}^{2-\alpha}L_\tau$ on the parameter τ, the unit vector $\mathrm{i}^{2-\alpha}$ is directed along the AB, and L_τ is the distance from the edge of the wedge to the observation point. The dependences of L_τ and the velocity v on the parameter τ are:

$$L_\tau = \frac{L}{G_0}\int_0^\tau \frac{\left(1-\tau^2\right)\tau^{1-\alpha} d\tau}{\left(\tau^2 + t_0{}^2\right)^{3/2}\left(\tau^2 t_0{}^2 + 1\right)^{3/2}}, \quad \frac{v^2}{v_0^2} = \tau^{2\alpha}, \quad G_0 = \int_0^1 \frac{\left(1-\tau^2\right)\tau^{1-\alpha} d\tau}{\left(\tau^2 + t_0{}^2\right)^{3/2}\left(\tau^2 t_0{}^2 + 1\right)^{3/2}}$$

Here L stays for the segment AB length. The force acting on the wedge may be defined as:

$$F = v_0^2 \int_0^L \rho\left(1 - \frac{v^2}{v_0^2}\right) dL_\tau \sin(\pi\alpha/2) = \rho v_\infty^2 L C_D$$

For the wedge resistance coefficient C_D the exact expansion can be obtained

$$C_D = (1+\sigma)\frac{\sin(\pi\alpha/2)}{G_0}\int_0^1 \frac{(1-\tau^{2\alpha})\tau^{1-\alpha}\left(1-\tau^2\right)d\tau}{\left(t_0^2+\tau^2\right)^{3/2}\left(t_0^2\tau^2+1\right)^{3/2}} = C_{D0}(1+\sigma)(1+k(\sigma^2-\sigma^3)+O(\sigma^4)) \tag{1}$$

By (1) the resistance coefficient can be calculated with the precision up to 5 significant digits for cavitation numbers less than 0.3. For the plate α=1, σ=0 we obtain the known formula (see [5], p. 303):

$$C_D = 2\pi/(\pi+4) \approx 0.87980$$

For the wedge with α=1/2 the exact value of the resistance coefficient can be found

$$C_D = 2\pi/[12+\sqrt{2}\pi+\sqrt{2}\log(3-2\sqrt{2})] \approx 0.45041$$

As the cavitation number is zero the dependences of α on the resistance coefficient of the wedge and cone are presented in figure 2. The wedge resistance coefficient achieves its maximum 0.89478 as α=1.1038, which corresponds to the wedge apex angle 180°·α=198.7°.

3. The flow around problem calculation scheme

Following [6] the numerical solution scheme of the flow around problem is described in the case of a two-dimensional planar or axially symmetric flow of the ideal fluid. The equation of the flow around boundary is presented in the parametric form $x(s)$, $y(s)$, where s is the coordinate along the solid boundary, calculated from the critical point. According to the boundary elements method for the velocity distribution $v(s)$ on the flow around boundary the integral equation $Av(s)=2(1+n)\pi v_\infty (y(s))^{1+n}$ is solved, where n=0 for the planar problem, n=1 for the axially symmetric one, A is the integral operator $Av(s) = -\int_0^l G(s,s')v(s')ds'$, l – is the full length of the boundary in the planar case and the length of the generating line in the axially symmetric one. Green function for the planar problem is expressed through the distance $r=\sqrt{(x(s)-x(s'))^2+(y(s)-y(s'))^2}$ between the points of the boundary $(x(s), y(s))$ and $(x(s'), y(s'))$. For the planar problem $G(s,s')=\ln(r)$, and for the axially symmetric problem it is the flow function of the vortex ring and is expressed through elliptical integrals. To perform the numerical calculations the sampling of the boundary with a finite number of points, such that the value $\zeta=\zeta_i=i/N$, $i=1..N$ corresponds to the point M_i is introduced. The parameter ζ and the coordinate s are coupled by the following relation $ds=J\, d\zeta$, where J characterizes the point density on the boundary. With the help of quadrature formulas [3, 6] the integral equations are reduced to a system of linear equations.

4. The iteration process to determine the form of the cavity

According to the Riabushischinsky principle, the solution should maximize the function $U=(p_\infty-p_0)V-T$, where V is the surface or the volume, limited by the surface ∂V; T is the kinetic energy of the fluid when the solid in it is moving translationally with the velocity v_∞, p_∞ is the pressure at infinity. The free boundary is found using the gradient descent method that maximizes the function U. The variations of the points coordinates of the free boundary and the cavitator are set as follows $\delta x=\varepsilon x$, $\delta y=k(v_0^2-v^2)+\varepsilon x\cdot dy/dx$, $\delta l_x=\varepsilon l_x$. Then the variation of the function U in the planar problem is strictly positive $\delta U=\frac{\rho}{2}\int_{\partial V} k(v_0^2-v^2)^2 dx$. The parameters k,ε can be found using that the velocity at the end-point of the cavitator is v_0 and the scheme stability. The formulas have the second order of accuracy by the subinterval length. The obtained forms of the cavities for spherical cavitators are shown in figure 3.

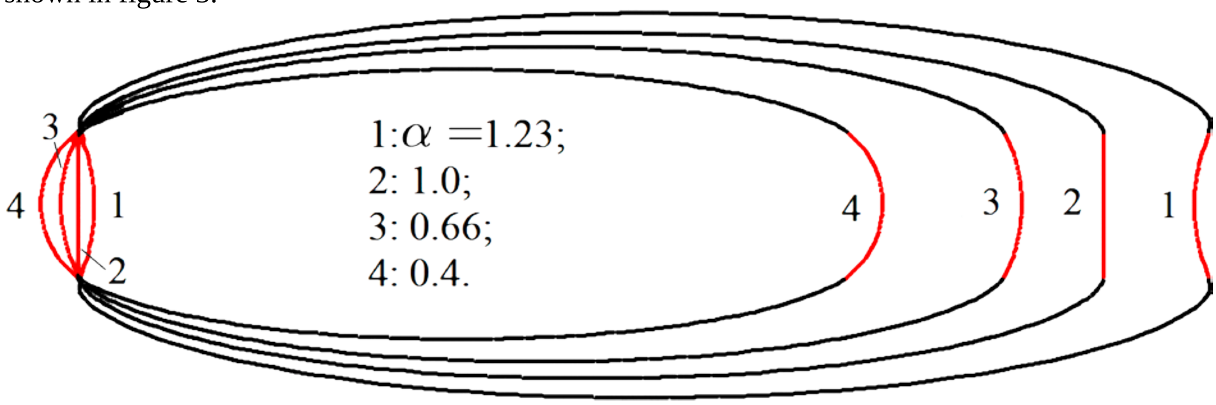

Figure 3. The forms of the cavities for spherical cavitators for different values of the angle parameter α. (α – is the apex angle of the descending flow divided by 180°)

5. The force formula

A solid flowed around by fluid with velocity v_∞ is considered by the Riabushischinsky scheme. The resistance force $F = -\int_\Sigma (p - p_0) n_x dS$ can be expressed through the Riabushischinsky function. For the planar problem it is

$$F = U / l_x + \Delta F, \quad \Delta F = (1/l_x)\int_\Sigma (p - p_0)(ydx - (x + l_x)dy) \tag{2}$$

For the wedge ΔF=0, and for the parabolic cavitator $x=-l_x+ky^2$ we have that $\Delta F = -(\rho / l_x)\int_0^{y_0} (v_0^2 - v^2) ky^2 dy$. The term ΔF is strictly negative. It constantly diminishes the resistance force for convex cavitators k>0 and increases it for concave ones (the cavitators in the shape of umbrellas). It is equally simple to calculate ΔF for the cavitators in the shape of circular arcs and for other shapes. In the axially symmetric case

$$F = 3U / (2l_z) + \Delta F, \quad \Delta F = (1/l_z)\int_\Sigma (v_0^2 - v^2)\pi r(rdz - (z + l_z)dr) \tag{3}$$

The direct calculation of the resistance coefficients by integrating the pressure on the cavitator boundary leads to substantial measurement errors for small cavitation number, as the relative number of computational points on the cavitator for the planar problem decreases proportionally to the square of the cavitation number. Formulas (2) and (3) are free of this limitation, because when they are used, the integral on the whole boundary of the whole computational domain is calculated. The extreme value of the Riabushischinsky function U*, obtained in the solution of the cavitation problem allows finding the resistance coefficients with high precision.

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References

[1] Birkhoff G and Zarantonello E HY 1957 *Jets, wakes and cavities* (New York: Academic Press)
[2] Terentiev A, Kirschner I and Uhlman J 2011 *The Hydrodynamics of Cavitating Flows* (Backbone Publishing)
[3] Petrov A G 2008 Quadrature Formulas for Periodic Functions and Their Application to the Boundary Element Method *Comput. Math. Math. Phys.* **48** 1266–1283
[4] Riabushischinsky D 1927 Sur une probleme de variation *C.R. Acad. Sci. Paris* **185** 840 – 841
[5] Milne-Thomson L M 1950 *Theoretical Hydrodynamics* (New York: Macmillan)
[6] Petrov A G, Potapov I I 2015 On the Calculation of Forces that Act on Solids for Planar and Axially Symmetric Cavitation Flow Around Problems *Comput. Math. Math. Phys.* **55**