Numerical evaluation of cavitation void ratio significance on hydrofoil dynamic response

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Abstract. The added mass effects on a NACA0009 hydrofoil under cavitation conditions determined in a cavitation tunnel have been numerically simulated using finite element method (FEM). Based on the validated model, the effects of averaged properties of the cavity considered as a two-phase mixture have been evaluated. The results indicate that the void ratio of the cavity plays an increasing role on the frequency reduction ratio and on the mode shape as the mode number increases. Moreover, the sound speed shows a more important role than the average cavity density.

1. Introduction

At present, FEM-based acoustic fluid approach to simulate the added mass effect of surrounding water has been widely used in hydraulic turbine runners with sufficient accuracy as for example by Liang, et al. [1]. Nevertheless, this method has not yet been used to estimate the added mass effects on hydrofoils under cavitation conditions. In fact, the experimental results indicate that the actual macroscopic hydrodynamic cavities are composed of a mixture of water and vapour phases. As a result, averaged properties must be considered in the model that are difficult to quantify and that vary from the expected effects of pure liquid water flow. Moreover, new boundary conditions at cavitation regions appear that are not fully understood.

Consequently, the research purpose of this work is to use a numerical Fluid Structure Interaction (FSI) approach for evaluation of the significance of cavitation void ratio on the dynamic response of an hydrofoil. For that, the numerical results for a NACA0009 hydrofoil with partial cavitation have been compared with the experimental ones obtained by De La Torre, et al. [2] assuming that the cavity is composed of a single fluid phase of pure vapour. With the validated model and based on the possibilities of the numerical approach, a study of the effects that the two-phase flow properties have on the hydrofoil modes of vibration (natural frequencies and mode shapes) of the coupled system have been explored.

2. Formulations of fluid structure interaction (FSI)

In FSI problems, the structural dynamics equation must be considered along with the Navier-Stokes (NS) and the continuity equations. The discretized structural dynamics equation can be formulated based on the structural elements. The NS and continuity equations are simplified to get the acoustic wave equation assuming that the fluid is compressible, which means that its density changes due to pressure variations (i.e. acoustic medium), and that there is no fluid flow [3].

The governing finite element matrix equations of FSI are expressed in Equation (1):

$$\begin{bmatrix} M_s & 0 \\ \rho_0 R^T & M_F \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_F \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{P} \end{Bmatrix} + \begin{bmatrix} K_s & -R \\ 0 & K_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F_s \\ F_F \end{Bmatrix} \quad (1)$$

where M_F, C_F and K_F are the fluid mass, damping and stiffness matrices, respectively. R is a coupling matrix that represents the effective surface area associated with each node on the fluid-structure interface. F_F is the applied fluid pressure vector at the interface obtained by integrating the pressure over the area of the surface. P and U are the fluid pressure and the structure displacement vectors, respectively. M_S, C_S and K_S are the structure mass, damping and stiffness matrices, respectively, and F_S is the structural load vector.

3. FEM model

The simulated case corresponds to a NACA0009 hydrofoil, as shown in **Figure 1**, under various cavitation conditions experimentally tested at the EPFL High-Speed Cavitation Tunnel. On the left of **Figure 3**, a photograph of the typical attached partial cavitation as described in De La Torre, et al. [2] is shown. The foil is made of aluminum alloy with a Young's modulus of 0.54 GPa, a Poisson's ratio of 0.33 and a density of 2770 kg/m^3.

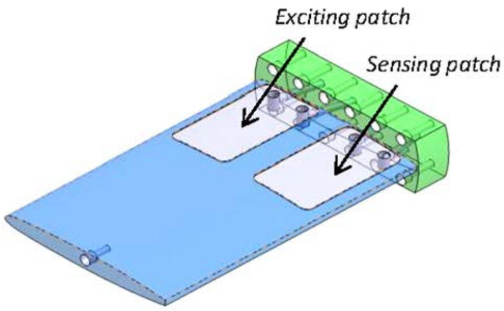

Figure 1. Truncated NACA0009 hydrofoil

The FSI domain, which comprises the whole test section of the cavitation tunnel, was meshed with hexahedral elements as shown in **Figure 2**. The hydrofoil was fixed with an incidence angle of 0º.

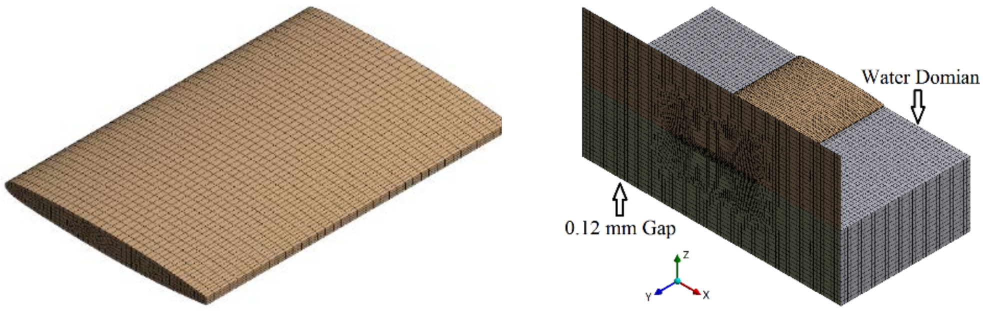

Figure 2. Discretized finite element mesh of the NACA0009 hydrofoil (left) and of the FSI domain with the surrounding water (right).

One lateral section of the hydrofoil was constrained in the model. The rest of hydrofoil surfaces were in contact with the fluid. A FSI condition was set at these interfaces so that the displacement of the structure is identical to that of the fluid in normal direction. Considering the coupling characteristics of vibration in water, the fluid boundaries were modeled as fully reflective. The water properties were set as 1000 kg/m^3 for density and 1450 m/s for speed of sound.

Regarding the modelling of the attached cavity, it was considered to be of a constant length along the whole span as shown on the right of **Figure 3**. Acoustic elements were used like for the water domain. At the interface of the vapor cavity with the surrounding water, the acoustic pressure and the normal component of the water/vapor velocity are continuous. The vapor properties were set as 0.174 kg/m^3 for density and 340 m/s for speed of sound.

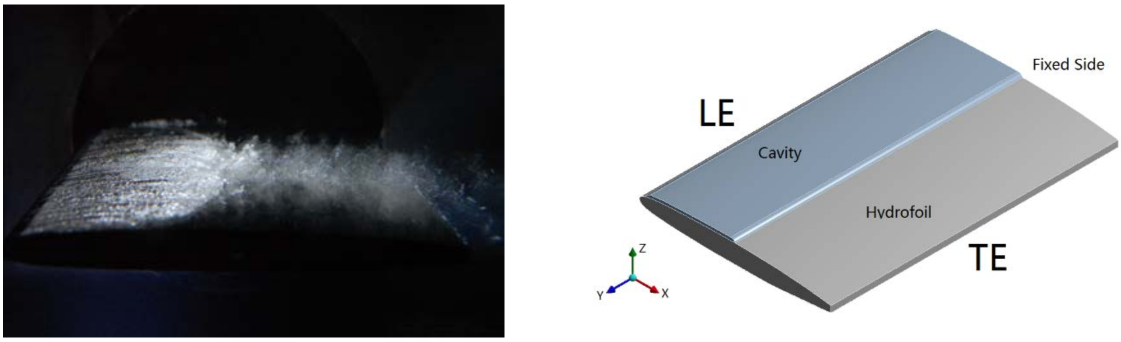

Figure 3. Lateral view of the attached cavity (left) and corresponding numerical model (right).

4. Results

The modal behaviour of the hydrofoil resembles that of a cantilever beam. The three first modes of vibration correspond to the first bending (f_1), the first torsion (f_2) and the second bending (f_3). To evaluate the changes of natural frequency with void ratio induced by the added mass effects, the Frequency Reduction Ratio in percent for mode *i*, *FRRi*, has been calculated as expressed in Equation (2):

$$\mathrm{FRR}_i = 100\% \cdot \frac{f_{i,\alpha=1} - f_{i,\alpha}}{f_{i,\alpha=1}} \tag{2}$$

where $f_{i,\alpha=1}$ and $f_{i,\alpha}$ are the mode *i* natural frequencies for void ratio 1 and for any void ratio α, respectively.

As it can be observed on the left of **Figure 4**, Brennen [4] states that the cavity density and the speed of sound in a mixture of water and vapor is a function of the void ratio, α, which is in turn the ratio between the vapor volume and the total cavity volume. Based on such data, a series of cases have been simulated considering the cavity domain void ratio in the range from 0.6 to 1, as shown on the right of **Figure 4**, while keeping the same cavity length and thickness and the same properties of the water domain around the cavity. As it can be observed from the results, both the *FRR* and the mode shape are affected by the changes in cavity properties. The significant effect of cavity void ratio on natural frequencies is parallel to a significant effect on mode shapes especially for mode f_3 and followed by mode f_2. Meanwhile, for mode f1 the effects on the shape are almost negligible.

The predicted mode evolutions are in fact induced by the particular sonic speed and averaged cavity density values considered in each case. These couple of variables are directly linked so it is difficult to determine their individual level of significance on the obtained results. To clarify this point, a series of simulations have been carried out by keeping one factor constant and changing the other although they are not physically accurate for large deviations from Brennen's data. To begin, the density has been reduced from 1000 kg/m^3 to 0.174 kg/m^3 while keeping the sonic velocity constant at 340 m/s. These results are plotted on the left of **Figure 5** and they indicate that the density of the cavity has a low influence on the natural frequency change of each mode. Only when the density is lower than 100 kg/m^3, the increase of the natural frequencies is larger, but still not significant. Contrarily, the role of the sonic velocity is dominant as it can be observed on the right of **Figure 5** when it is reduced from 1480 to 24 m/s while keeping the density constant at 0.174 kg/m^3. The results for all the modes of vibration show that the natural frequency changes significantly in a particular range of values and that outside such range it is almost constant. In particular, the frequency changes linearly with the sonic velocity following a common diagonal line for all the modes that we call “Mode Transition Line”.

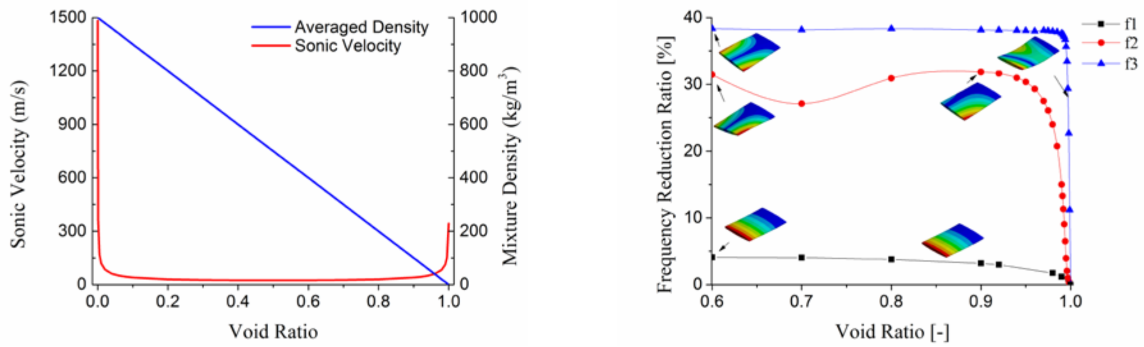

Figure 4. Speed of sound and cavity density as a function of void ratio (left); *FRR*’s and mode shape evolution as a function of cavity void ratio for f_1, f_2 and f_3 (right).

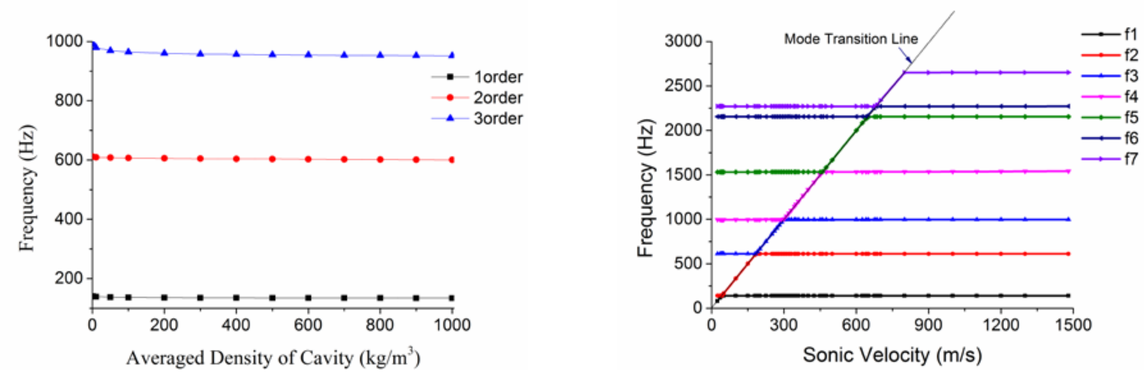

Figure 5. Natural frequency vs change in average density (left) and in speed of sound (right).

5. Conclusion

The added mass effects on a NACA0009 hydrofoil under cavitation conditions have been numerically calculated using acoustic FEM. The results show that the reduction of void ratio from 1 to 0.6 increases the added mass effects and modifies the mode shapes more significantly for higher order modes. In particular, the sonic speed is more relevant than the average density on the modal behavior.

Acknowledgements

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