

# The non-linear response of bubble clouds to pressure excitations

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**Abstract.** In this article we numerically investigate the non-linear response of a bubble cloud against a periodic pressure excitation. By exciting a planar bubble curtain with an external acoustic pulse of given amplitude and frequency, we characterize the global dynamic response of the system using phase diagrams representing the void fraction against the excitation pressure. Even in the absence of mass transfer, the void fraction around which the system oscillates increases when increasing the excitation amplitude. We show how the maximum pressures reached during the collapse of bubbles are higher in polydisperse bubble clouds than in monodisperse clouds for strong pressure pulses.

## 1. Introduction

The response of bubble clouds to external pressure pulses is a difficult problem with application in various fields such as military applications, noise reduction or sea life protection. In this latter case, the presence of a bubble curtain around an acoustic source is used to attenuate the pressure waves and minimize the damage of the noise induced in its surroundings. Various authors have tried to model the response of bubble clusters using Direct Numerical Simulations. However, the range of applicability of such codes is very limited due to the lack of enough computational resources. Thus, similar to the development of LES models in turbulence, new lines of research have been devoted to propose simplified models able to capture the influence of thousands or millions of small bubbles in large scales. In this manuscript, we use the model proposed in [1] to investigate the influence of non-linear bubble cluster response and polydispersity. To this end, we explore the response of large bubble cloud curtains to non-linear pulses in order to gain new insight into the effect of bubbles on global and local variables.

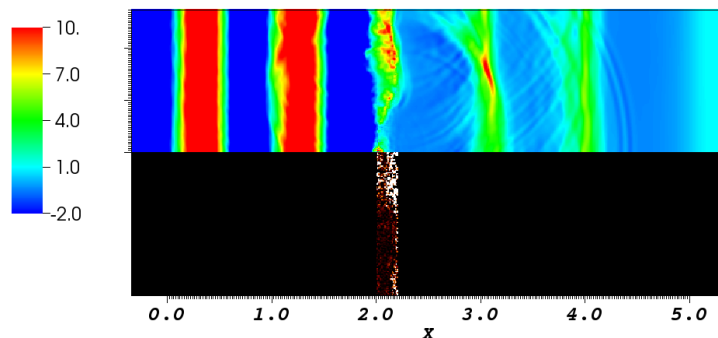
## 2. Physical model and numerical code

In this work we use the model presented in Fuster & Colonius [1] implemented in a compressible flow solver that utilizes advanced WENO shock-capturing techniques [4]. This code has been validated against the theoretical solution of linear wave propagation in diluted bubbly liquids in Fuster et al [2]. The model treats bubbles as punctual sources of mass and momentum that depend on the local dynamic response of the bubbles at the subgrid scale. To obtain the bubble oscillating motion, the model resorts to a potential decomposition at the subgrid scale in order to derive a Rayleigh-Plesset like equation that allows us to integrate the bubble radius evolution in time. This model makes possible to capture the influence of bubble clouds containing a large



number of bubbles in the pressure waves propagating through a bubble cloud. For more details about the model and the implementation in the code, the reader is referred to the publications mentioned above.

### 3. Numerical results



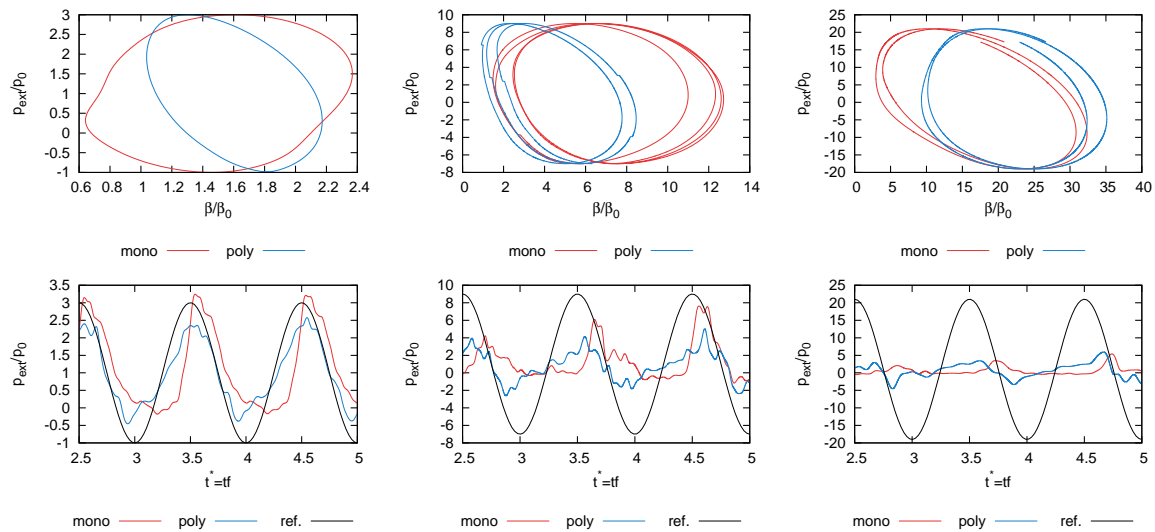
**Figure 1.** Snapshots of the pressure (top) and concentration contours (bottom) after some cycles. The presence of the bubble curtain significantly modifies the pressure field upstream, downstream and inside the cloud. Strong bubble collapses take place inside the cloud. The color scale for pressure is made dimensionless using the reference pressure.

In this manuscript we investigate the influence of polydispersity in the response of a bubble cloud excited with a sinusoidal planar wave of given amplitude,  $\Delta P$ , and frequency  $f$  in a liquid with speed of sound  $c_l$ . The excitation is generated using a source in the energy equation far from the bubble cloud. We use the wavelength of the incoming wave  $\lambda = c_l/f$  as a characteristic length of the problem. The liquid properties are those of water. The initial void fraction  $\beta_0$  is set in all simulations equal to  $10^{-4}$  vol/vol. The dimensionless width of the bubble curtain in the wave propagation direction is  $\frac{L_x}{\lambda} = 0.4$ . We consider a situation with infinite number of bubbles in the two transversal directions. The averaged dimensionless radius is set to  $\frac{R_0}{\lambda} = 6.78 \cdot 10^{-4}$ . The actual bubble cluster is generated with a random number function obeying a log-normal distribution with a given dispersion  $\sigma$ . The bubbles are randomly distributed in the domain. Heat transfer effects between the bubble and its surrounding are taken into account but in this case mass transfer effects are neglected.

The simulation domain size in dimensionless units is  $6 \times 2 \times 0.1$ . Periodic boundary conditions are used in the  $y$  and  $z$  directions in order to mimic the presence of an infinite bubble cloud. The grid size is set to  $\frac{\Delta x}{\lambda} = 5 \cdot 10^{-3}$  except in the  $z$  direction, where only one grid is used in order to save computational time.

Figure 1 shows two snapshots of typical pressure fields after the arrival of the incoming wave. The bubble curtain significantly disturbs the pressure wave inducing a large attenuation. Inside the bubble cluster, bubbles react to the pressure pulse expanding first and finally imploding violently and generating large local pressures.

In order to characterize the global response of the bubble cluster we use void fraction versus excitation pressure diagrams (top of Figure 2). As expected the bubble cluster describes periodic orbits around a fixed point. The location of this point strongly depends on the amplitude of the

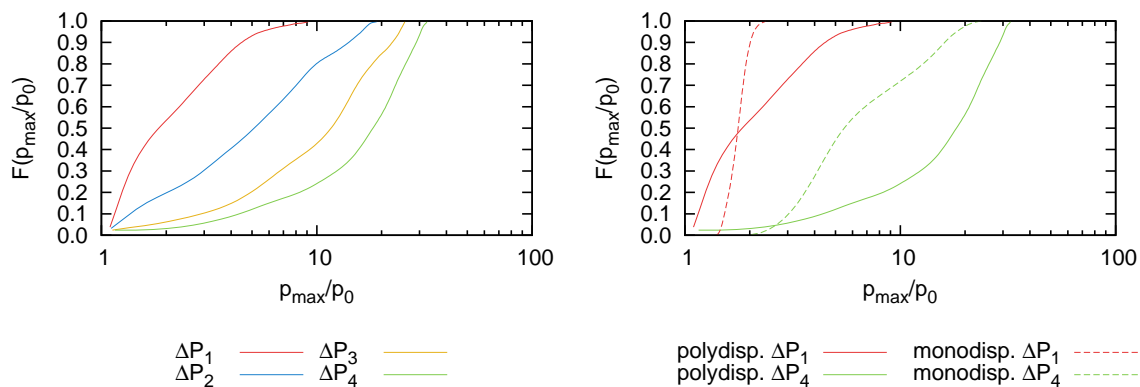


**Figure 2.** Top: Void fraction as a function of the excitation pressure for three different excitation amplitudes (from left to right: 2, 8, 20 atm). Bottom: Pressure sampled at a distance of 0.3 from the bubble cluster. Bubbles damp more effectively large amplitude waves due to the effective increase of the averaged void fraction during the excitation.

excitation. The larger the amplitude the higher the void fraction around which the cluster oscillates is. Note that we have neglected vaporization effects, so this effect cannot be attributed to a global change in the amount of gas/vapor along the simulation. Instead, this effect is attributed to the non-symmetry on the response of bubbles to pressure pulses in non-linear regimes. Bubbles tend to oscillate around an equilibrium point given by  $R_0$  and  $p_0$  only in the linear regime. For large amplitude excitations the increase on the void fraction induced during the bubble growth is more important than the compression taking place during the bubble collapse and we observe a *virtual* increase on the void fraction of the bubble cloud during the excitation of the cloud. This phenomenon can be interpreted in terms of potential energies [3], which show how the bubble expansion is energetically favored when the radius is larger than the Blake's radius ( $R_c \approx \sqrt{3}R_0$ ). The increase on the effective void fraction attenuates more effectively the incoming pressure waves as the amplitude increases (bottom of Figure 2) validating the hypothesis that bubble curtains are especially suitable to damp large amplitude pressure waves. We can see how polydispersity does not play a major role on these quantities.

In addition to the global response of the cluster, the model allows us to access to local quantities such as the maximum local pressures reached during the bubble collapse. In particular we obtain statistics of the maximum pressures reached at the liquid interface during the collapse of every single bubble. Figure 3 shows the cumulative distribution of the maximum pressures reached during the bubble collapse for different conditions. As expected, on the left of Figure 3 we can see how the maximum pressures increase as the excitation amplitude increases. Remarkably, polydispersity plays a major role on these distributions (Figure 3 right). At low amplitudes (red line), the cumulative distribution is broader showing how it is possible to induce large pressure during the collapse only due to the bubble size variability in the cluster. In this case the median value remains approximately constant. For large amplitudes the situation is different and we clearly show how polydispersity effectively increases the maximum pressures reached during the collapse of bubbles. This effect is attributed to the non-synchronization of

the bubble oscillation process enhanced by the presence of bubbles with different sizes. Note that these observations are consistent with the predictions in the low frequency limit obtained from the simplified theory presented in [3], although certainly dynamic effects play an important role in this situation.



**Figure 3.** Cumulative distribution of the maximum pressures reached in the liquid along the simulation. Left: Results obtained for a polydisperse bubble cloud excited at various amplitudes ( $\Delta P_1 = 2$ ,  $\Delta P_2 = 4$ ,  $\Delta P_3 = 8$  and  $\Delta P_4 = 20$  atmospheres). Right: Comparison between the cumulative distribution obtained in a monodisperse and polydisperse bubble cloud for low amplitude excitations (2 atm) and large amplitude excitations (20 atm).

#### 4. Conclusions

This paper presents numerical results used to investigate non-linear effects on the response of bubble clouds. For sinusoidal excitations we see how the system oscillates around a virtual fixed point with a *virtual* concentration larger than the equilibrium concentration without any excitation. This virtual concentration tends to increase when increasing the excitation amplitude for a fixed amount of gas and neglecting any mass transfer process. This effect is explained by the preference of bubbles to expand when they go beyond the critical Blake's radius.

The distribution of maximum pressures reached in the liquid have been shown to depend on the polydispersity of the bubble cloud. We systematically observe that the largest pressures are reached in the polydisperse case. Still, for low amplitudes, the median value remains constant irrespective of the polydispersity level. For large amplitudes, polydispersity significantly increases the maximum pressures reached during the bubble collapse.

#### References

- [1] D. Fuster and T. Colonius. Modeling bubble clusters in compressible liquids. *Journal of Fluid Mechanics*, 688:253–289, 2011.
- [2] D. Fuster, J.M. Conoir, and T. Colonius. Effect of direct bubble-bubble interactions on linear-wave propagation in bubbly liquids. *Phys. Rev. E*, 90(6):063010, 2014.
- [3] D. Fuster, K. Pham, and S. Zaleski. Stability of bubbly liquids and its connection to the process of cavitation inception. *Phys. Fluids*, 26:042002, 2014.
- [4] E. Johnsen and T. Colonius. Implementation of WENO schemes in compressible multicomponent flow problems. *Journal of Computational Physics*, 219(2):715–732, 2006.