Journal of Physics: Conference Series **656** (2015) 012101 doi:10.1088/1742-6596/656/1/012101

On the relation between tonal and broadband content of hull pressure spectra due to cavitating ship propellers

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Abstract. Cavitating propellers generate pressure fluctuations on the hull of the ship. These pressure fluctuations are usually analyzed in the frequency domain using FFTs and the spectrum is composed of tonals at multiples of the blade passage frequency and a broadband part. The two are often considered separately but a relation between the two exists which has been investigated by theoretical signal analysis. It will be shown that the broadband part is related to the variability of the signal between blade passages and a simple procedure is proposed to quantify the variability in terms of amplitude and phase angle. The procedure has been applied to a data set obtained at sea trials.

1. Introduction
Cavitating propellers may generate inboard noise and vibration hindrance for crew and passengers on-board ships. Often, it is the hull pressure spectrum that is analyzed to evaluate the hull-excitation force of the cavitating propeller. This spectrum is composed of tonal components at harmonics of the blade passage frequency and a broadband part. The broadband part is known to be caused by the variability of the pressure signal [1][2], but a quantitative description is missing in literature. The present paper presents a formulation that can fill this gap although mathematical proof is still to be completed. Before presenting the formulation some basic aspects of the spectrum are discussed.

2. Stylised spectra of a single blade passage
Stylised hull pressure time traces and corresponding spectra of a cavity collapse during a single blade passage are shown in figure 1. The signal was generated using the absolute value of a sine for a number of cycles. Signal 1a and 1b have no damping and the width of the broadband hump in the spectrum, with a centre frequency at 10 Hz which is the repeat frequency of the pressure pulse, is inversely proportional to the number of cycles. For signal 2a and 2b an exponential damping was added to the time trace which leads to a smooth variation of the spectrum from frequency zero to the broadband hump.

3. Stylised spectra of multiple blade passages
Analytical formulations for the effect of amplitude and phase angle modulation on spectra of pulse trains, referred to as blade passages in the present paper, have been presented by McFarlane [3] and the theory is briefly restated here. Consider that the amplitude density spectrum of a single blade passage at time zero is given by $G(\omega)$.

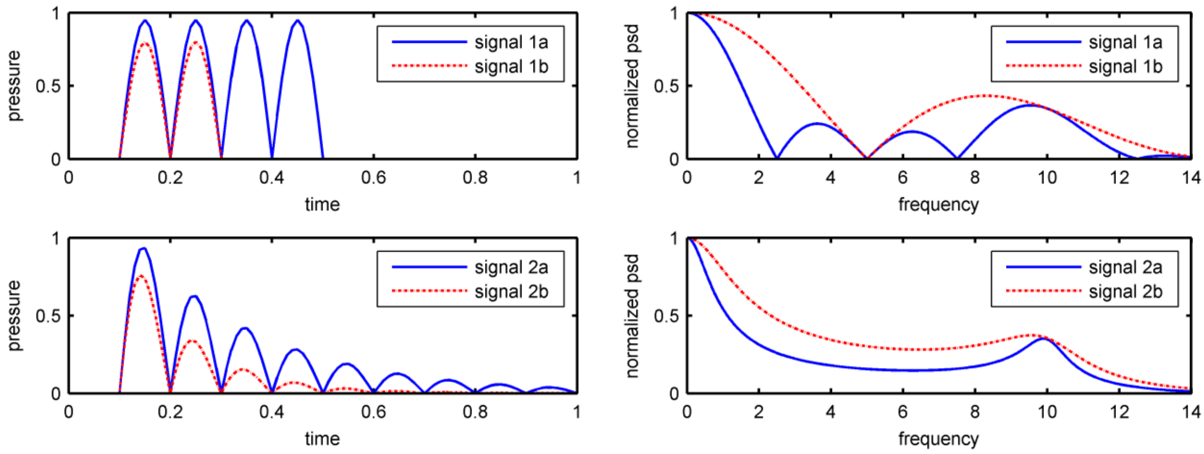


Figure 1. Stylised pressure time traces (left) and normalised power spectral densities (right) due to a cavity collapse and rebounds during a single blade passage

The spectrum, S, of a sum of $2N+1$ (non-overlapping) blade passages spaced T apart, with amplitude a_n and relative time shift t_n, is then given by

$$S(\omega)=G(\omega)\sum_{n=-N}^{N} a_n \exp\left[i\omega\left(nT+t_n\right)\right] \tag{1}$$

and the power spectral density (psd) is given by $R(\omega)=\left|S(\omega)\right|^2/(2N+1)T$.

With amplitude modulation the values for a_n are statistically described by the mean $\bar{a}$ and standard deviation σ_a while there is no relative time shift $(t_n=0)$. After some algebraic manipulations in which an infinite series of blade passages is assumed, we find the psd for amplitude modulation,

$$R(\omega)=\left[\sigma_a^2+\bar{a}^2\omega_r\,\Delta_{\omega_r}(\omega)\right]\frac{\left|G(\omega)\right|^2}{T} \tag{2}$$

where $\omega_r=2\pi/T$ corresponds to the blade passage frequency (bpf) and $\Delta_{\omega_r}(\omega)$ to a Dirac comb with frequency spacing ω_r. The spectrum is composed of a broadband part with amplitude σ_a^2 summed with a tonal part with amplitude $\bar{a}^2$ weighted with the frequency bin width ω_r.

Phase modulation is defined here as an arbitrary variation in arrival time $nT+t_n$ of the pressure signal for each blade passage n while the amplitude does not vary in time $(a_n=a)$. The probability of t_n is given by a Gaussian distribution q with zero mean and standard deviation σ_p,

$$q(t_n)=\frac{1}{\sigma_p\sqrt{2\pi}}\exp\left(-t_n^2/2\sigma_p^2\right). \tag{3}$$

After lengthy algebraic manipulations, the following expression for the psd for phase angle modulation is obtained,

$$R(\omega)=a^2\left[1-\exp\left(-\sigma_p^2\omega^2\right)+\exp\left(-\sigma_p^2\omega^2\right)\omega_r\,\Delta_{\omega_r}(\omega)\right]\frac{\left|G(\omega)\right|^2}{T} \tag{4}$$

The effect of the amplitude and phase angle modulation is illustrated in figure 2, following [2]. A stylised spectrum is sketched of a single blade passage together with the spectrum of a series of blade passages. The frequency is non-dimensionalized by the bpf. If the signal repeat perfectly there is no modulation and the spectrum consists of tonals at harmonics of the bpf. Random variations in amplitude alone between different blade passages hardly affect the amplitudes of the tonals. However, the spectrum now does contain a broadband part of which the magnitude is proportional to the standard deviation of the amplitude variations. Random changes in time of arrival of the signal lead to random phase angle variations of which the magnitude increases linearly with frequency. This random phase modulation decreases the amplitude of the tonals and redistributes the power over a broadband region. The spreading is most pronounced at higher frequencies and may cause the complete disappearance of the tonal.

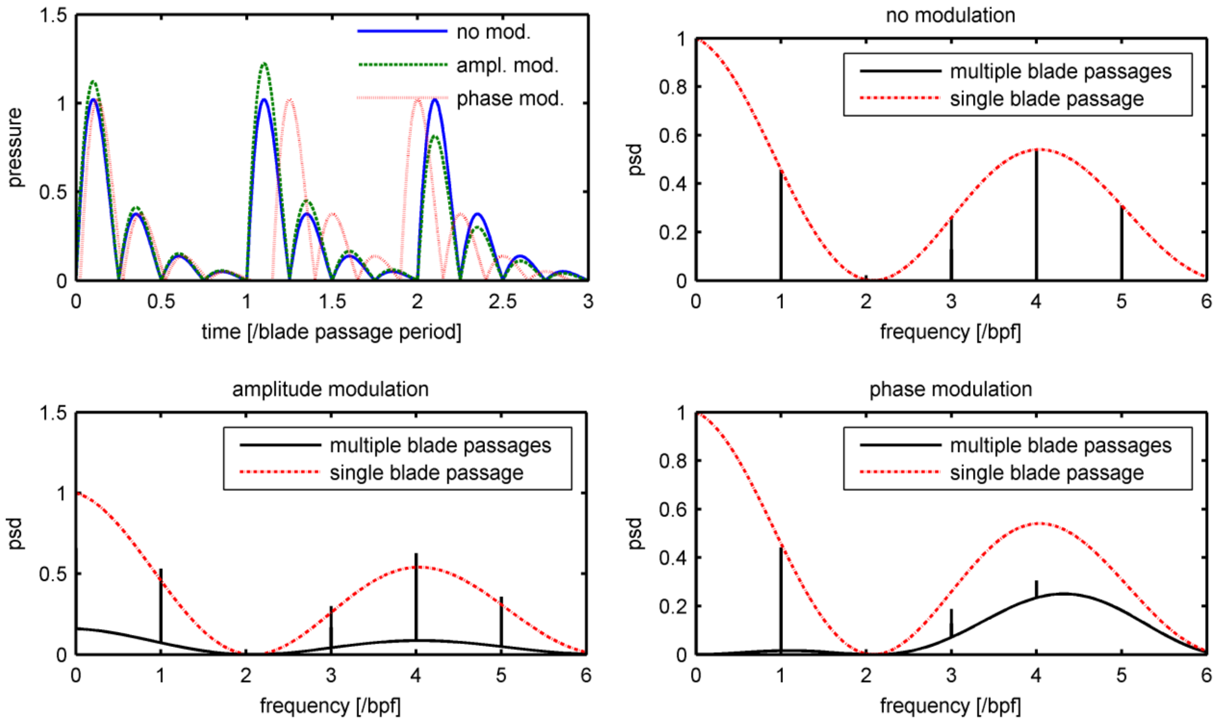


Figure 2. Stylised pressure time traces and resulting spectra for multiple blade passages with amplitude and phase modulation

4. Practical example of combined amplitude and phase angle modulation

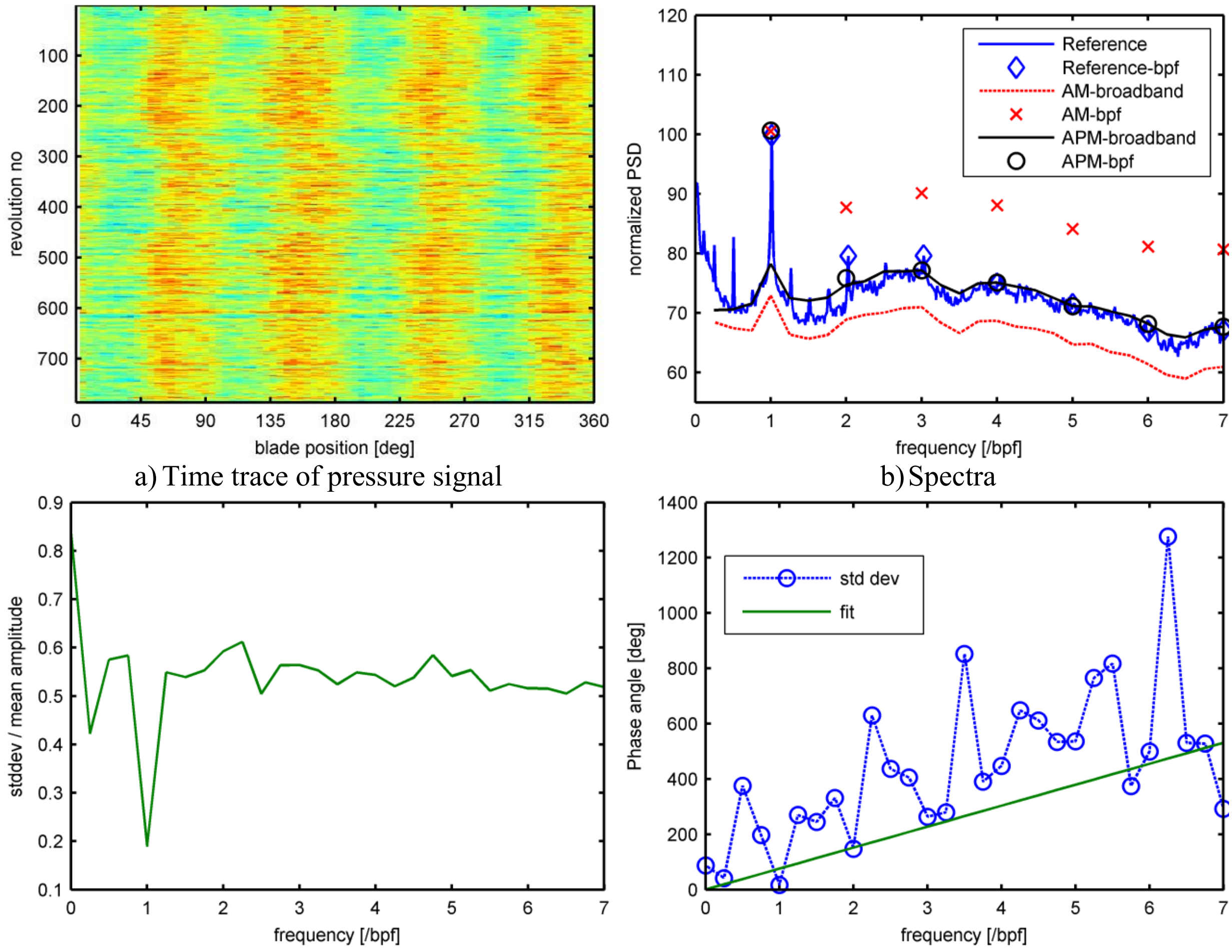
For the combination of amplitude and phase angle modulation, the equations (2) and (4) are combined in a heuristic manner with mathematical proof still to be completed. The formulation is given by

$$R(\omega)=\left\{\sigma_a^2+\left[1-\exp\left(-\sigma_p^2\omega^2\right)+\exp\left(-\sigma_p^2\omega^2\right)\omega_r\,\Delta_{\omega_r}(\omega)\right]\bar{a}^2\right\}\frac{|G(\omega)|^2}{T} \tag{5}$$

In reality, the shape of the spectrum also varies with the blade passages and the standard deviation for amplitude and phase is frequency dependent. The formulation has been applied to the hull pressure signal generated by a four bladed cavitating propeller during a sea trial.

Data sets are analyzed by subdividing the time trace into ‘shaft revolutions’ with a shaft revolution defined by a constant number of samples which does not change in time, see figure 3a. This implies that variations of the shaft revolution rate in time due to sea state are interpreted as phase angle modulation. Each shaft revolution is analyzed with an FFT, after which the mean and standard deviation of both amplitude and phase angle can be computed for each harmonic of the shaft rate frequency. The variation of standard deviation of amplitude and phase angle with frequency is given in figure 3c and 3d. The phase angle was ‘unwrapped’ which explains values for the standard deviation of the phase larger than 360 deg.

The spectrum that is generated from the amplitude information alone by using equation (2) is referred to as AM and the result is given in figure 3b. To simplify the graph, the tonals are given for the harmonics of the bpf only. Using equation (5), we can reconstruct the spectrum with amplitude and phase modulation (designated APM in figure 3b) and compare this with the reference spectrum that is obtained by taking an FFT over a large number of revolutions. For simplicity, a linear fit of the standard deviation of the phase angles at harmonics of the bpf is taken except for the first bpf. The pressure amplitude at the first bpf is also due to the passage of the non-cavitating blade which repeats well as shown by the small values of the relative standard deviation. Figure 3b shows reasonable agreement between the two spectra with some differences at the 2nd and 3rd bpf suggesting that, after some further improvement, the proposed formulation and analysis procedure may be useful for quantifying the relation between higher order tonals and broadband content of hull pressure spectra.



a) Time trace of pressure signal
b) Spectra
c) Variation of standard deviation of the amplitude with frequency
d) Variation of the standard deviation of the phase angle with frequency

Figure 3. Analysis of the hull pressure signal of a (four bladed) cavitating propeller as measured on a sea trial

References

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