On Rayleigh-Plesset based cavitation modelling of fluid film bearings using the Reynolds equation

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Abstract. In the 'universe' of the general cavitation phenomena the issue of cavitation in bearings, due to its particular application and the mostly non-homogeneous working fluids associated with it, has presented a rather specialized challenge. The present paper models the phenomenon of pseudo-cavitation in fluid film bearings and offers a physics-based approach that conserves mass while solving the Reynolds (RE) and Rayleigh-Plesset (RP) equations in a coupled, fully transient environment. The RP solution calculates a time dependent void fraction synchronized with the RE transient solution, where density and viscosity are (re)calculated at every grid point of this homogeneous two-phase fluid. The growth and evolution of the cavitation zone expanse is physics-based and thus can accommodate evaporation, diffusion, or pseudo-cavitation as separate processes. This is a step beyond the present available cavitation models both for the RE and the Navier-Stokes equations.

1. Introduction

Following the early non-mass-conserving bearing cavitation models of Gümbel, and Swift-Stieber, mass conserving models have emerged. Coyne and Elrod [1], Mori et al. [2], Birkhoff and Hays [3] accounted for fluid transport above and beneath the bubble and used mass conservation to define the cavity interface. Floberg [4, 5], and Jakobsson and Floberg [5], assumed striated flow with the liquid being transported in-between gas fingers that extended across the clearance, implemented a mass conservation model and subcavity pressures. Many authors have offered analytical and numerical solutions, the list of excellent contributions being too long to do it justice here. The interested reader is further referred to review publications of Dowson et al. [6], Brewe et al. [7], and Braun and Hannon [8]. Zuber and Dougherty [9] in 1982 were the first to propose a model for a homogeneous two-phase lubricating film mixture in conjunction with the RE. Natsumeda and Someya [10] in 1987 followed the same concept of a homogeneous two-phase flow RE by introducing the RP equation as the source of void production for the homogeneous two-phase lubricant film. The authors proved experimentally the existence of tensile subcavity pressures, (as high as 1.2MPa), and used a modified RP equation that retained the surface tension, ($\frac{2S}{\rho_l R_B}$), viscous, ($4\nu_l \frac{\dot{R}_B}{R_B}$), and viscous surface dilatation effects ($\frac{4\kappa^s}{\rho_l R_B^2}\dot{R}_B$). The latter term importance in modeling and sustenance of the subcavity pressures was clearly defined and recognized both by [10] and Gehannin et al. [11]. Solving for the growth in the bubble radius through the offices of RP provides the mechanism for the calculation of the void fraction, α, as well as the calculation of the transport properties for the two-phase homogeneous RE.

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2. Scope of Work

The present work addresses specifically the development of pseudo-cavitation in a fluid film bearing, that is the formation and evolution of the cavitated region in the divergent section of the bearing through the growth of a collection of kernel bubbles due to depressurization, but without mass addition; this situation is particular to lubrication oils which contain large numbers of air kernel bubbles and have rather low hydrocarbon components evaporation pressure thresholds. The model presented herein is a step forward compared to other models available in that the RP retains all physics-significant terms (viscosity and surface dilatational viscosity) and calculates at every grid point a time-dependent void fraction that allows discrete (re)calculation of local density and viscosity in synchronicity of the RE transient solution for this homogeneous two-phase fluid.

3. On the Surface Dilatational Viscosity

For most flows, the thermodynamic and mechanical pressures are sufficiently close, to cause the bulk viscosity κ to be negligible in magnitude, thus validating the Stokes's hypothesis. Donnan in his preface to McBain [12] defined an interface as "*two-dimensional molecular world the dynamics of which is analogous to that of ordinary three dimensional world of homogeneous phases in bulk*". Therein lies the analogy between κ and κ^s ,whereby the latter provides the source for the dynamic interfacial stress (e.g. oil and vapor, or oil and gas) similar to the one provided by the three-dimensional bulk viscosity in an isotropic fluid that expands, contracts, or goes through a shock wave. The dynamic equation for the thin spherical surface between the fluids inside and outside the bubble reduces to a dynamic stress jump condition (body forces and inertia are neglected)

$$\Delta\sigma_{fluid-bubble} = \frac{2\sigma}{R_B} + \frac{4\kappa^s}{R_B^2}\dot{R}_B \tag{1}$$

The surface dilatational viscosity stress is an excess stress and its action offers a dissipative resistance, acting towards restoring equilibrium temporally. In a moving (deforming) situation when the dynamic tension at the interface exceeds the equilibrium tension, $\frac{2\sigma}{R_B}$, by the quantity $\frac{4\kappa^s}{R_B^2}\dot{R}_B$, the latter through its derivative $\dot{R}_B$, always exerts an action *opposing the motion of the bubble wall* resulting into an interface stress jump that causes the deviation from the static case.

4. The Model Governing equations

The cavitation model consists of a simultaneous solution of the transient RE and a simplified RP equations (inertia terms are neglected based on an order of magnitude analysis). Within a single global time step, the field of initial bubble radii is advanced through 10 sub-iterative steps of the RP equation

$$\frac{4\nu_L}{R}\frac{DR}{Dt} + \frac{2\sigma}{\rho_L R} + \frac{4\kappa^s}{\rho_L R^2}\frac{DR}{Dt} = \frac{P_{g0}}{\rho_L}\left(\frac{R_0}{R}\right)^{3k} - \frac{P}{\rho_L} \tag{2}$$

The advective velocities within the substantial derivative of bubble radius are directly obtained from the emerging pressure field and averaged across the film thickness. Following the update of the bubble radii, the void field is recalculated as the ratio $\alpha = \frac{\forall_{bubble}}{\forall_{cell}}$. Then the homogeneous bulk flow properties of the lubricant film are updated using the newly found void field

$$\rho = \alpha\rho_B + (1-\alpha)\rho_L \qquad \mu = \alpha\mu_B + (1-\alpha)\mu_L \tag{3a, b}$$

Finally, the new pressure field is computed using the new $\rho(\alpha)$ and $\mu(\alpha)$ in the transient RE

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial P}{\partial z}\right) = \frac{\partial}{\partial x}\left(\frac{\rho h U}{x}\right) + \frac{\partial(\rho h)}{\partial t} \tag{4}$$

The RE is discretized using the finite-difference method. It is solved using a marching technique which begins at an axial boundary of the bearing and solves an implicit tri-diagonal system at each subsequent axial station.

5. Results and Discussion

Figure 1 presents graphically a comparison between 1000 and 5000 [rpm] cases with the surface dilatational viscosity held constant at a baseline value of κ^s=7.85E-3 [N.s/m], [10] and the relative eccentricity held constant at ε=0.4. The results show that when the κ^s used in the RP equation is kept constant, the pressure curve exhibits a subcavity pressure region which varies moderately as rotational speed increases, Figure 1b. This new model also shows that the cavitation zone may extend considerably from the divergent zone into the convergent zone as rotational speed increases. The contour graphs in Figure 1a, show clearly by comparison with each other, the advance of the cavitation zone in the circumferential direction as the rotational speed increases. It is important to note that the contour of this zone was obtained 'organically' through the coupling of the physics of the RP and two-phase homogeneous RE (RP-RE model). No film rupture boundary conditions were necessary. The color bar indicates the pressure magnitudes. Figure 1b presents the pressure and void evolution around the circumference, as they are calculated by the model presented herein, superimposed on a baseline case provided by the RE using the Gümbel cavitation condition (RE-G model). While the RE-G model considers the convergent region containing only liquid of ρ_L density and μ_L viscosity, the RP-RE model allows for the existence of compressed gas in the liquid, contributing to weighted average of ρ and μ in the two phase mixture throughout the bearing. This fact ('softer fluid'), and the advection of void through the $\dot{R}_B = \frac{\partial R_B}{\partial t} + u_i \frac{\partial R_B}{\partial x_i}$ term, lowers the maximum pressure and pushes the cavitation zone further into the divergent zone, and even convergent zone, respectively. This differentiates the RP-RE from the RE-G models both qualitatively and quantitatively.

6. Conclusions

The paper presents a coupled RP and homogenous two-phase RE equations where the RP equation is used as a source of void in the computation of the ρ and μ of the mixture. The core of the present approach eliminates the need for film rupture boundary conditions as used in [3, 4, 5, 1, 2], allowing cavitation to build up 'organically' on the basis of local void advection and growth. The weighted density influences significantly the magnitude of the pressure in the convergent zone causing the maximum pressure to be lower than the one that would be computed by a RE-G model. The present model allows also for the development of subcavity pressure zones based on the magnitude of the surface dilatational viscosity. As the latter increases, the capability of the fluid to sustain subcavity pressures (tensile stresses) increases. Based on the numerical results one can infer that the Sommerfeld and half-Sommerfeld (Gümbel-Swift-Stieber) curves represent limiting cases between a high and low surface dilatational viscosity values. Finally these differentiated results have potentially a significant effect on the static (load, attitude angle, torque) and dynamic performance (stiffness, damping, stability) of the fluid film bearings.

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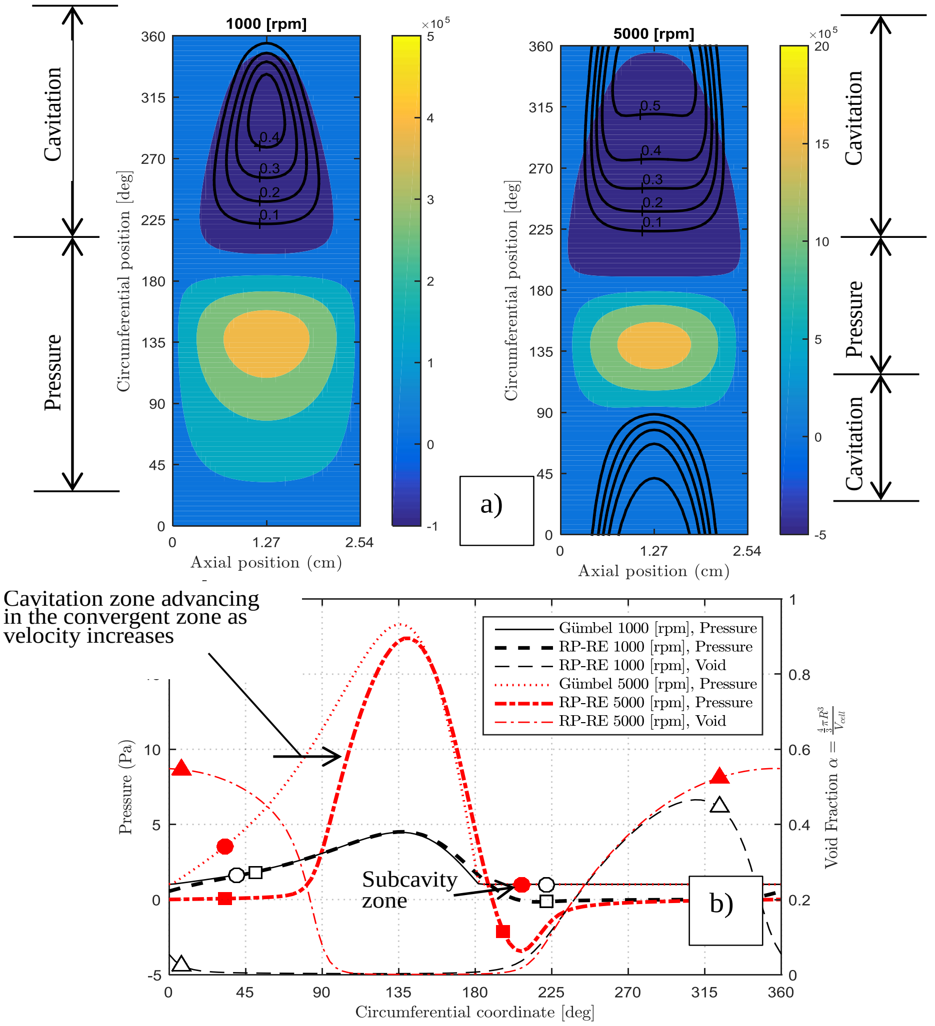

Figure 1. Pressure and void fraction at bearing midspan and corresponding contour plots of pressure and void fields at 1 and 5 krpm and ε=0.4. c/R=0.001; μ_L=7.1E-3 Pa.s; μ_B=1.81E-5 Pa.s; ρ_L=854 kg/m^3; ρ_B=1 kg/m^3; σ=3.5E-2 N/m^2; κ^s=7.85E-3 Ns/m; R_{g0}=3.85e-7 m