Study on Pressure Wave Propagation in a Liquid Containing Spherical Bubbles in a Rectangular Duct

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Abstract. Pressure wave propagation in a liquid containing several bubbles is numerically investigated. We simulate liner plane wave propagation in a liquid containing 10 spherical bubbles in a rectangular duct with the equation of motion for N spherical bubbles. The sound pressures of the reflected waves from the rigid walls are calculated by using the method of images. The result shows that the phase velocity of the pressure wave propagating in the liquid containing 10 spherical bubbles in the duct agrees well with the low-frequency speed of sound in a homogeneous bubbly liquid.

1. Introduction

The presence of gas bubbles dispersed in a liquid affects profoundly the acoustic behaviour of the liquid. For example, the low-frequency speed of sound in a bubbly liquid is lower than that in a pure liquid. In the previous studies [1, 2, 3], the physical mechanism of the reduction of the low-frequency speed of sound has been explained in a liquid containing bubbles where the assumption of homogeneous medium holds. In general, the speed of sound in a continuum medium is defined as $a = \sqrt{1/\kappa\rho}$, where κ is the compressibility and ρ is the density of the medium. The compressibility of a homogeneous bubbly liquid is primarily determined by the amount of bubbles and the density by the amount of liquid. Bubbles greatly increase the compressibility of a homogeneous bubbly liquid, but hardly affect the density. High compressibility and density yield a low speed of sound.

The purpose of the present paper is to answer the question: *will the low-frequency speed of sound be reduced in a liquid containing several bubbles where the assumption of homogeneous medium no longer holds?* In the present study, we simulate pressure wave propagation in a liquid containing 10 spherical bubbles in a rectangular duct with the equation of motion for N spherical bubbles, in which the bubble-bubble interactions are taken into account. We assume that bubble size is much smaller than the wavelength of the pressure wave; hence, we restrict ourselves to low-frequency wave propagation in a bubbly liquid [3]. We compare the low-frequency speed of sound in a homogeneous bubbly liquid with the phase velocity of the pressure wave propagating in the duct.

2. Problem Statement

Let us simulate pressure wave propagation in a liquid containing 10 spherical bubbles in a rectangular duct [Fig. 1(a)]. The bubbles have the same initial radius R_0, and are placed at

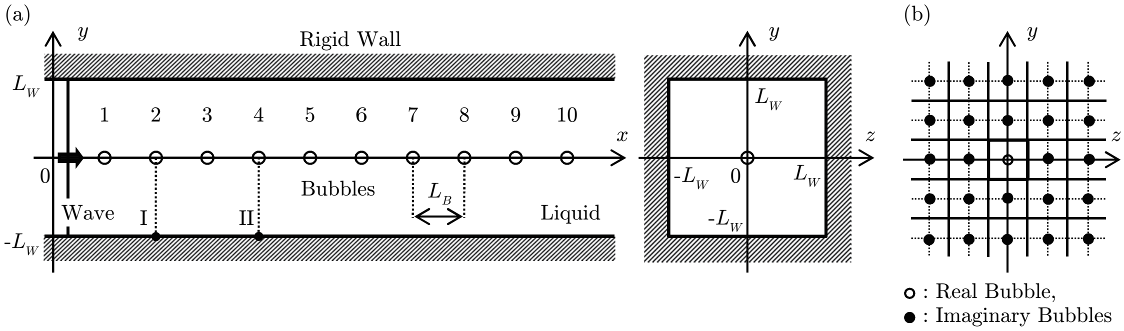

Figure 1. (a) 10 spherical bubbles are placed at equal intervals in a rectangular duct. (b) Schematic of distribution of imaginary bubbles in the $y-z$ plane.

equal intervals on the center axis of the duct. The bubbles are excited by a linear plane wave started to propagate from $x=0$. The input pressure wave having $p_{\ell 0}+P_{\rm in}(t,x)$ propagates at the speed of sound in a liquid-phase, where t is the time and $p_{\ell 0}$ is the pressure of the liquid in the initial undisturbed state. The sound pressure of the input pressure wave $P_{\rm in}(t,x)$ is $P_A[1-\cos\omega(t-x/a_\ell)]$, where P_A is the amplitude of the input pressure wave ($P_A/p_{\ell 0}\ll 1$), ω is the angular frequency of the input pressure wave, and a_ℓ is the speed of sound in a liquid-phase. The delay time x/a_ℓ denotes the time it takes for the pressure wave to propagate the distance x at the speed a_ℓ. The angular frequency ω is much smaller than the natural angular frequency of the single bubble.

In the present study, the following assumptions are made: (i) the liquid flow is a potential flow. (ii) The translation and deformation of the bubbles are ignored. (iii) The bubble-bubble interactions are taken into account. (iv) The bubbles are filled with a non-condensable gas, which follows the isothermal process, without phase changes. (v) The bubble radius $R(t)$ is large enough so that the surface tension is negligible. (vi) All attenuations including the acoustic radiation are ignored.

We shall use the equation of motion for N spherical bubbles in a sound field [4]. Under the present assumptions, we linearize the equation of motion,

$$\ddot{R}'_I(t)+\sum_{J=1,\ J\neq I}^{N} R_0\frac{\ddot{R}'_J(\zeta_{JI})}{r_{IJ}}+\omega_B^2 R'_I(t)=-\frac{P_{\rm in}(t,x_I)}{\rho_\ell R_0}, \tag{1}$$

where $R'(t)=R(t)-R_0$ is the small fluctuation of the bubble radius ($|R'|/R_0\ll 1$), the subscripts I and J are the bubble indices ($I,J=1,2,\cdots,N, I\neq J$), x_I is the x coordinate of the center of bubble I, r_{IJ} is the distance between the centers of bubbles I and J, ρ_ℓ is the density of a liquid-phase, and $\zeta_{JI}=t-[r_{IJ}-R_J(\zeta_{JI})]/a_\ell$. The delay time $[r_{IJ}-R_J(\zeta_{JI})]/a_\ell$ denotes the time it takes for the radiative wave generated by the radial oscillation of bubble J to propagate the distance r_{IJ} at the speed a_ℓ. The natural angular frequency of the single bubble ω_B is $\sqrt{3p_{\ell 0}/(\rho_\ell R_0^2)}$ ($\omega/\omega_B\ll 1$). The second term in the left-hand side of Eq. (1) represents the bubble-bubble interactions. In the limit as $r_{IJ}\to\infty$, or $N\to 1$ (when the bubble-bubble interactions are ignored), Eq. (1) is reduced to the equation of linear oscillation for a single spherical bubble [4].

The radiative waves generated by the radial oscillations of the bubbles in the rectangular duct are reflected from the rigid walls. We calculate the sound pressures of the reflected waves by using the method of images. We place imaginary bubbles in a two-dimensional array in the $y-z$ plane as shown in Fig. 1(b).

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Now, we calculate the pressure field at the point $Q(x, y, z)$. The total sound pressure P_Q is given by the summation of $P_{\rm in}(t, x)$ and the sound pressures generated by the radial oscillations of the real and imaginary bubbles. It should be emphasized here that during a given period of time T, the radiative waves generated by the radial oscillations of the bubbles propagate only for a distance $a_\ell T$; therefore, we calculate only the sound pressures generated by the radial oscillations of the real and imaginary bubbles in the distance $a_\ell T$ from the point Q. The sound pressure generated by the radial oscillation of a real, or imaginary bubble k is written by

$$P_{Qk}(t) = -\rho_\ell \frac{\partial \phi_k}{\partial t} = \rho_\ell R_0^2 \frac{\ddot{R}'_k(t - r_k/a_\ell)}{r_k}, \tag{2}$$

where ϕ_k is the velocity potential for the flow field generated by the radial oscillation of bubble k given by ref. [5] and r_k is the distance between the center of bubble k and the point Q. Hence, the total sound pressure P_Q can be calculated by using the following equation:

$$P_Q(t) = P_{\rm in}(t, x) + \sum_k \rho_\ell R_0^2 \frac{\ddot{R}'_k(t - r_k/a_\ell)}{r_k}. \tag{3}$$

The total sound pressure in a water at 20°C is calculated by numerically solving Eqs. (1) and (3) under the conditions with $p_{\ell 0}$, a_ℓ, and ρ_ℓ of 101.3 kPa, 1483 m/s, and 998.2 kg/m^3, respectively. The bubble 1 is placed at $x = L_B$. The distance between adjacent bubbles L_B is 50.0 mm. The nondimensional amplitude $P_A/p_{\ell 0}$ and the nondimensional angular frequency ω/ω_B of the input pressure wave are 1.82×10^{-3}. The ordinary differential equations (1) are solved numerically by using the 4th order Runge-Kutta method.

3. Numerical Results and Discussion

We compare the low-frequency speed of sound in a homogeneous bubbly liquid with the phase velocity of the pressure wave propagating in the duct. The phase velocity is defined as follows. Let $P_{\rm I}(t)$ and $P_{\rm II}(t)$ be the total sound pressures at points I and II (points $x = 2L_B$ and $4L_B$ on the rigid wall), respectively. From the simulation results, we find t' that satisfies the equation $P_{\rm I}(t) = P_{\rm II}(t')$ as a function of t, and define the phase velocity as $a_{\rm p}(t) = 2L_B/(t' - t)$ in a time $t = 10L_B/a_\ell$ when the input pressure wave arrived at the center of the bubble 10. The low-frequency speed of sound in a homogeneous bubbly liquid depends on the void fraction. In the present study, we define the void fraction β as the ratio of the volume of the bubble and the domain around the middle point between adjacent bubbles:

$$\beta = \frac{4\pi R_0^3/3}{L_B \times 2L_W \times 2L_W}. \tag{4}$$

We conducted the simulations with various void fractions by changing either L_W ($R_0 = 1.00$ mm) or R_0 ($L_W = 10.0$ mm). Figure 2(a) shows the time evolution of the total sound pressures at the points I and II in the case of $R_0 = 1.00$ mm and $L_W = 10.0$ mm. The present result is normalized by the natural period of the single bubble: $2\pi/\omega_B$. It should be emphasized here that we investigate the pressure wave development in the period of the order of $2\pi/\omega_B$, which corresponds to the only 0.1% of the period of the input pressure wave. In the case of $\beta = 2.09 \times 10^{-4}$, the phase velocity of the pressure wave propagating in the duct is 639 m/s.

The low-frequency speed of sound in a homogeneous bubbly liquid is often modeled by an equation attributed to Wood's equation [2, 3, 6],

$$a_{\rm mlf} = \left[\frac{(1-\beta)^2}{a_\ell^2} + \frac{\beta^2}{a_g^2} + \beta(1-\beta) \frac{\rho_g^2 a_g^2 + \rho_\ell^2 a_\ell^2}{\rho_\ell \rho_g a_\ell^2 a_g^2} \right]^{-1/2}, \tag{5}$$

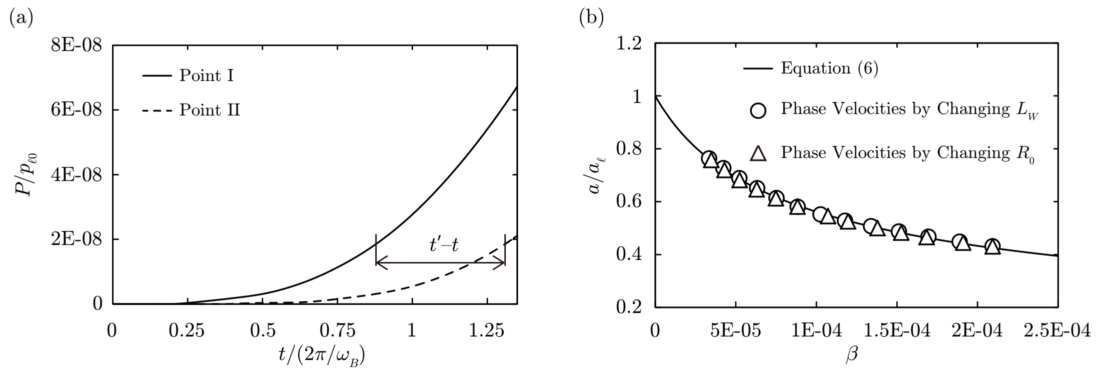

Figure 2. (a) Time evolution of the total sound pressures at the points I and II. (b) Relation between Eq. (6) and the phase velocities of the pressure wave.

where a_g is the speed of sound in a gas-phase and ρ_g is the density of a gas-phase. When the relation: $a_g^2 = p_{\ell 0}/\rho_g$ is used and the conditions: $\beta^2 \ll (\rho_g a_g^2)/(\rho_\ell a_\ell^2) \ll 1$ are imposed in Eq. (5), we derive the following equation,

$$a_{\mathrm{mlf}} = a_\ell \left(1 + \frac{\rho_\ell a_\ell^2}{p_{\ell 0}} \beta\right)^{-1/2}. \tag{6}$$

Figure 2(b) shows both the numerically obtained phase velocities and the low-frequency speed of sound by Eq. (6). The phase velocities of the pressure wave propagating in the duct agree well with Eq. (6). From these results, we conclude that in a liquid containing 10 spherical bubbles in a rectangular duct, the phase velocity of the pressure wave propagating in the media is reduced from the speed of sound in a liquid-phase with the increase in the void fraction defined by Eq. (4).

4. Conclusions

Pressure wave propagation in the liquid containing 10 spherical bubbles in the rectangular duct was simulated with the equation of motion for N spherical bubbles. The sound pressures of the reflected waves from the rigid walls were calculated by using the method of images. The phase velocity of the pressure wave propagating in the liquid containing 10 spherical bubbles in the duct agrees well with the low-frequency speed of sound in a homogeneous bubbly liquid.

Acknowledgments

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