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Acting Force Mechanism of a Body Crossing the Water Layer of a Cavity

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Abstract. The acting force mechanism of the water layer of the cavity on a body when it crosses the water surface is studied. A simplified mechanical model is proposed to explain the mechanism of the impact of the water layer onto the body. The estimating formula for the maximum pressure in the water layer and on the body surface impacted by the water layer is derived. It is proved that the maximum pressure is always in proportion to the square of the moving velocity of the maximum pressure position.

1. Introduction

The collapse of an underwater cavity or bubble can cause damages to underwater structures such as propellers and ships. Many works have been done to understand the damage mechanism of the cavity collapse. Besant [1] was the first to consider the problem of bubble dynamics in history and Rayleigh [2] established the dynamic equation for a spherical bubble in an unbounded flow field, as

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{p_c-p_\infty}{\rho} \tag{1}$$

Subsequently, Plesset [3] built the well-known Rayleigh-Plesset (RP) equation by means of adding the viscosity coefficient μ and the surface tension coefficient σ to equation (1), as

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{p_c-p_\infty}{\rho}-\frac{2\sigma R}{\rho}-\frac{4\mu\dot{R}R}{\rho} \tag{2}$$

RP equation is only applicable for describing the expansion or collapse of a single spherical bubble in an unbounded flow field. It is found that when the bubble moves near to the body surface, the collapse will lead to the bubble deformation and develop a jet flow. The impact force by the jet flow is considered as the main reason for the body damage. Plesset [4] simulated the collapsing process of a bubble near the solid wall surface with a special numerical method, and the results showed that the bubble will develop a jet directed towards the plane solid wall before the bubble radius decreases to its minimum. Nakajama and Shima [5] analyzed the behavior of a non-spherical bubble in a viscous incompressible liquid under axial conditions with finite element method, and made it clear that a jet formed on the bubble is decelerated by the effect of liquid viscosity.

The researches mentioned above have just focused on the impact of the bubble collapse onto the the body outside the bubble, while few considered the impact of the bubble collapse onto the body inside the bubble which is crossing the bubble surface. This kind of cavity can be seen in the process of water exit and entry. When a body enters water, the splash sheath is gathering on the water surface to form a closed cavity, which is called the surface closure. However, if the afterbody is long enough, the splash sheath may close on the body surface (figure 1a). Similarly, when the body exits from water with attached cavity, the part of cavity bulging water surface will also close on the body surface (figure 1b).

These two kinds of cavities will also collapse under the atmospheric pressure and produce very high pressure on the closure position. Based on this point, we considered these two cases as the acting force problem of a body crossing the cavity near the water surface. As the liquid layer of the cavity near the water surface is thin and the collapsing time is very short, it is impossible to observe the movement of the water layer of the collapsing cavity on the body by experiments. To understand the mechanism of the impact of the liquid layer of the cavity onto the body surface, this paper proposed a simplified theoretical model to explain the formation of the collapsing pressure of the cavity near the water surface and to estimate its value.

The characteristics of cavity collapse without body crossing the cavity are analyzed in section 2. Then, in section 3 the acting force mechanism of a body crossing the cavity near the water surface is studied. At last, the conclusion is given in section 4.

2. Collapse of a single bubble with finite thickness water layer

As the upper part of the cavity near the water surface is formed by the closure of the splash sheath, the thickness of the water layer is finite. For convenience of the mathematical treatment, the upper part of the cavity near the water surface can be simplified as a spherical cavity and the thickness of the water layer outside the cavity is finite, as seen in figure 2.

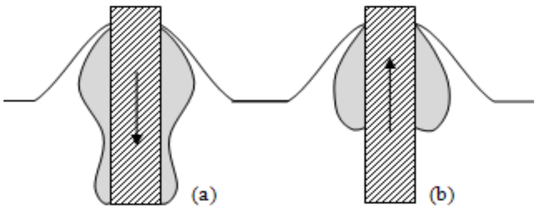

Figure 1. The cavity formed on the water surface in the process of water exit and entry.

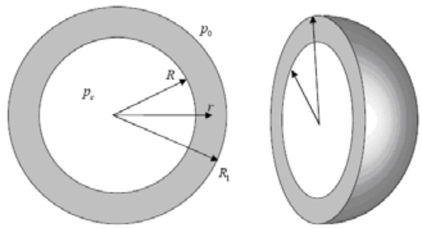

Figure 2. Schematic view of a spherical bubble closed by water sheath with finite thickness.

Let $R(t)$ and $R_1(t)$ be the radius of the inner and outer sphere respectively, ρ be the fluid density, p_c and p_0 be the pressure inside and outside the bubble respectively, $p(r,t),(R \le r \le R_1)$ be the pressure in the water layer. Obviously, $p(R,t) = p_c$ and $p(R_1,t) = p_0$ .The fluid is assumed to be ideal, incompressible, and irrotational.

Due to a spherical symmetrical flow, the momentum equation can be written as [6]

$$\ddot{R}\frac{R^2}{r^2} + 2\dot{R}^2\left[\frac{R}{r^2} - \frac{R^4}{r^5}\right] = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{3}$$

For a single bubble with finite thickness water layer, integrating (3) from R to R_1 and taking the boundary conditions that $p(R,t) = p_c$ and $p(R_1,t) = p_0$ into account, we get

$$\left(R_1\ddot{R}_1 + \frac{3}{2}\dot{R}_1^{\,2}\right) - \left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = \frac{p_0 - p_c}{\rho} \tag{4}$$

The pressure distribution in water layer can be described as

$$\frac{p(r) - p_c}{\rho} = \left(\frac{R}{r} - 1\right)\left(R\ddot{R} + 2\dot{R}^2\right) - \frac{1}{2}\dot{R}^2\left(\frac{R^4}{r^4} - 1\right) \tag{5}$$

Let $r_{\max}$ represent the position with maximum pressure $p_{\max}$ .If the cavity is an vapor bubble, namely $p_c = p_v = const$, it can be proved easily by mathematical derivation that the following equality holds when the cavity radius tends to be zero ($R \to 0$), as

$$r_{\max}/R \to 4^{1/3}, \quad r_{\max}/R_1 \to 0, \quad \dot{r}_{\max}/\dot{R} = R^2/r_{\max}^2 \to 1/2^{4/3} \tag{6}$$

According to (4) and (5), the maximum pressure can be expressed as

$$\frac{p_{\max} - p_0}{\rho} = \frac{\dot{r}_{\max}^2}{2}\left(3 - \frac{4r_{\max}}{R_1} + \frac{r_{\max}^4}{R_1^4}\right) \tag{7}$$

So, if R is small enough, the maximum pressure can be estimated by substituting (6) into (7), as

$$\frac{p_{\max} - p_0}{\rho} \approx \frac{3}{2}\dot{r}_{\max}^2 \tag{8}$$

In reference [7], the formula to calculate the maximum pressure in the unsteady cavity closure region in the infinite flow is proposed, as

$$\frac{p_{\max} - p_0}{\rho} = \frac{1}{2}\dot{r}_{\max}^2 \tag{9}$$

Equation (8) and (9) have the same form but with different coefficients for the $\dot{r}_{\max}$ velocity item.

3. A body crossing the cavity near the water surface

When a body crosses the cavity near the water surface, it will certainly influence the liquid layer of cavity. Therefore, it can't be described directly by the spherical cavity model. Figure 3a shows a simplified flow scheme of the liquid layer of cavity on a cylindrical body.

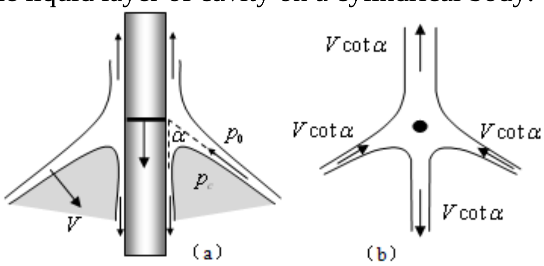

Figure 3. Scheme of the cavity flow on the body surface.

Obviously, the impact of the cavity onto the body is only related with the local flow of the liquid layer on the body surface, rather than with the shape of other cavity parts. The cavity shape near the body surface can be assumed to be a taper with half cone angle of α . The thick black line in figure 3 shows the position with the maximum pressure. On the condition that the cavity contacts under the pressure difference Δp , the maximum pressure position of $r_{\max}$ will move down along the axis of the cylindrical body with the velocity of $\dot{r}_{\max}$ .For convenience, we proposed three hypotheses: (A) the moving velocity of the maximum pressure position is constant ($\dot{r}_{\max} = const$), (B) the thickness of the water layer is constant, (C) the cone angle is constant ($\alpha = const$). So in the inertial coordinate system moved with $\dot{r}_{\max}$, we can get the steady flow state as shown in figure 3b. This flow state has some similarities with the symmetrical jet flow in the shaped charge theory [8].

The value of the maximum pressure can be estimated according to the shaped charge theory. Obviously, the maximum pressure position of $r_{\max}$ is the stagnation point that the upper and lower jets intersect at. Let the cavity contracting velocity be V , then we get the jet velocity as $V\cot\alpha$ and the moving velocity of the coordinate origin as $\dot{r}_{\max} = V/\cos\alpha$. Based on the Bernoulli equation, we get the maximum pressure expression, as

$$\frac{p_{\max} - p_0}{\rho} = \frac{1}{2}\left(V\cot\alpha\right)^2 = \frac{\sin^2\alpha}{2}\cdot\dot{r}_{\max}^2 \tag{10}$$

Compared to (8) and (9), it can be found that (10) has the similar form. But the coefficient in (9) is decided by the half cone angle of α , which means that the smaller the angle between the water surface cavity and the body surface, the smaller the maximum pressure.

The model (8) and (10) are very simplified and mainly used to explain the mechanical mechanism of the cavity collapse pressure near the water surface, so the model results can be only used to estimate the order of magnitude. In experiment, the maximum pressure $p_{\max}$ and its position $r_{\max}$ were measured by the pressure sensors. Figure 4 shows the measured $t \sim r_{\max}$ curve within $T = 0.05$ s, which is approximated as a straight line. As the cavity collapses so fast that it is very difficult to obtain the speed $\dot{r}_{\max}$ for every moment, only the average speed is got (the slope of line in figure 4). Similarly, the average cone angle α can be obtained by high-speed photography. The scope of maximum pressure can be estimated by taking the average speed and angle into model (8) and model (10).The comparisons between calculation results and experimental results are shown in figure 5.

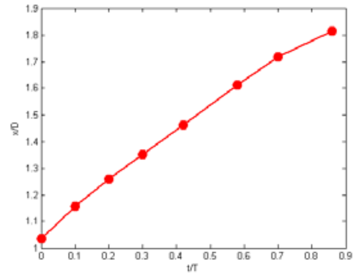

Figure 4. The position of the maximum pressure from experiments.

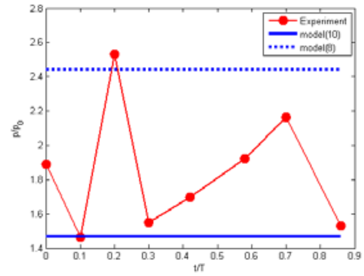

Figure 5. Comparison between model results and experimental results.

According to the experiment results, the hypothesis (A) and (B) are approximately acceptable while the hypothesis (C) is defective. It needs to be pointed out that as the pressure in the cavity is different form that outside the cavity, the flow velocity on the internal boundary and the external boundary are different too. So the jet velocity mentioned above should be understood as the mean velocity.

4. Conclusion

If the body crossing the cavity is relatively small compared to the cavity itself, which means that the interference of the body to the cavity can be ignored, the acting force on the body can be calculated by maximum pressure formula (8). However, if the body crossing the cavity changed the flow state in the water layer of the cavity, the acting force needs to be calculated by formula (10). In both cases, the maximum pressure is always in proportion to the square of the moving velocity of the maximum pressure position, and the difference of coefficients just reflects the different action way between the body and the cavity.

References

[1] W. B. 1859 *Hydrostatics and Hydrodynamics* (Cambridge University Press)
[2] Rayleigh L 1917 *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science.*, **34**(200) 94-98
[3] Plesset M S 1949 *Journal of applied mechanics.* **16** 277-282
[4] Plesset M S, Chapman R B 1971 *Journal of Fluid Mechanics* **47**(02) 283-290
[5] Nakajima K, Shima A 1977 *Ingenieur-Archiv.* **46**(1) 21-34
[6] Franc J P, Michel J M 2004 *Fundamentals of Cavitation* (Kluwer Academic Publishers)
[7] Chen W.Q, Wang B.S, Yi S.Q, et al. 2012 *Chinese Journal of theoretical and applied mechanics* **44**(4) 701-708 (in Chinese).
[8] Batchelor G K 1994 *An introduction to fluid dynamics* (London: Cambridge University Press)