Nonlinear diffusion waves in high magnetic fields

V I Oreshkin[1,2], S A Chaikovsky[1], N A Labetskaya[1], I M Datsko[1], D V Rybka[1], N A Ratakhin[1] and K V Khishchenko[3]

[1] Institute of High Current Electronics of the Siberian Branch of the Russian Academy of Sciences, Akademichesky Avenue 2/3, Tomsk 634055, Russia
[2] National Research Tomsk Polytechnical University, Lenin Avenue 30, Tomsk 634050, Russia
[3] Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13 Bldg 2, Moscow 125412, Russia

E-mail: oreshkin@ovpe.hcei.tsc.ru

Abstract. The nonlinear diffusion of a magnetic field and the large-scale instabilities arising upon an electrical explosion of conductors in a superstrong (2–3 MG) magnetic field were investigated experimentally on the MIG high-current generator (up to 2.5 peak current, 100 ns current rise time). It was observed that in the nonlinear stage of the process, the wavelength of thermal instabilities (striations) increased with a rate of 1.5–3 km/s.

1. Introduction

The electrical explosion of conductors (EEC) has been studied for a rather long time and has got a number of applications [1–3]. Several EEC modes are distinguished, which are defined by the proportions between the time of energy deposition in a conductor and other characteristic times [4, 5]. In particular, in a skin-effect mode, which is the object of investigation in this paper, the time of energy deposition in the conductor is less than or comparable to the time of magnetic field diffusion. The main processes that characterize the EEC in a skin-effect mode [6–9] are a shock wave and a nonlinear magnetic diffusion wave propagating in the conductor material [10–12], formation of low-temperature plasma at the conductor surface, and growth of thermal instabilities [13–15]. The nonlinear diffusion is characterized by abnormally high, compared to conventional diffusion, rate of penetration of an electromagnetic field into a conductor [6,7]. The increased diffusion rate is related to the decreased conductivity of the metal due to its heating by electric current. Nonlinear diffusion shows up in rather strong magnetic fields (some hundreds of kilogauss) [6, 7].

2. Experimental setup

Experiments were carried out on the MIG terawatt high-current generator [16] at a peak current of up to 2.5 with a rise time of 100 ns. The diagnostic complex of the MIG generator consisted of Rogowski coils, magnetic probes, voltage dividers, and an HSFC Pro four-frame optical camera providing a 3-ns exposure time per frame. The generator load was cylindrical titanium conductors. The load design is shown schematically in figure 1. It can be seen that the load consisted of two parts: a rod 3 mm in diameter and a tube of the same diameter and 250 μm wall thickness.

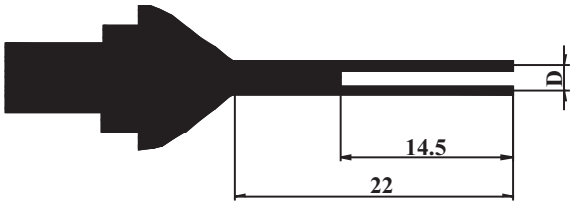
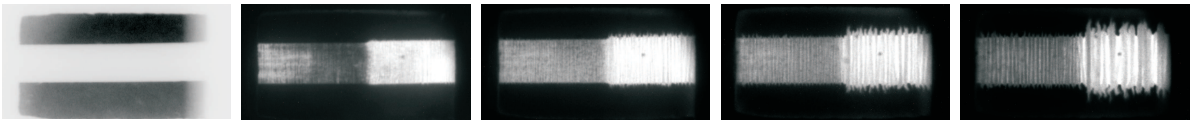


Figure 1. Schematic view of the MIG generator load. The cathode is on the left-hand-side.



$t < 0$ $t = 130$ ns $t = 190$ ns $t = 230$ ns $t = 270$ ns

Figure 2. Photographs of a titanium conductor of external diameter 3 mm and hollow part wall thickness 250 μm taken in self-radiation at different times t from the onset of the generator current.

Figure 2 gives optical frames of a titanium load taken at different times from the onset of current flow. It can be seen that instabilities of wavelength several hundreds of micrometers develop after the 130th nanosecond all over the conductor surface, including both the rod and the tube. The instabilities show up as striations: alternating dark and bright layers normal to the direction of current flow. The dark layers consist of a material of lower temperature and higher density, and the bright layers consist of a material of higher temperature and lower density. On the hollow part of the conductor, the instabilities arise much earlier and remain more pronounced throughout the process.

The solid line in figure 3 represents the typical waveform of the current flowing through the conductor, and the dots indicate the maximum ($D_{\max}$) and minimum diameter ($D_{\min}$) of the conductor in its different parts (in the rod and in the tube). The instability amplitude is determined by these quantities as $(D_{\max} - D_{\min})/2$.

3. Discussions

Let us discuss the experimental results, using for analysis a numerical model describing the propagation of a nonlinear magnetic diffusion wave. Nonlinear magnetic diffusion into a conductor is described by the Maxwell equations written in a quasi-stationary approximation (not taking into account displacement currents) and complemented with Ohm's and Joule's laws:

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{1}$$

$$\mathbf{E} = \mathbf{j}\delta, \tag{2}$$

$$\frac{\partial Q}{\partial t} = \mathbf{j}^2\delta, \tag{3}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and the magnetic field strength, respectively; $\mathbf{j}$ is the current density; Q is the thermal energy density; δ is the resistivity, and c is the velocity of light in vacuum,

$$\delta(T) = \delta_0\left(1 + \frac{\partial \delta}{\partial T}T\right), \tag{4}$$

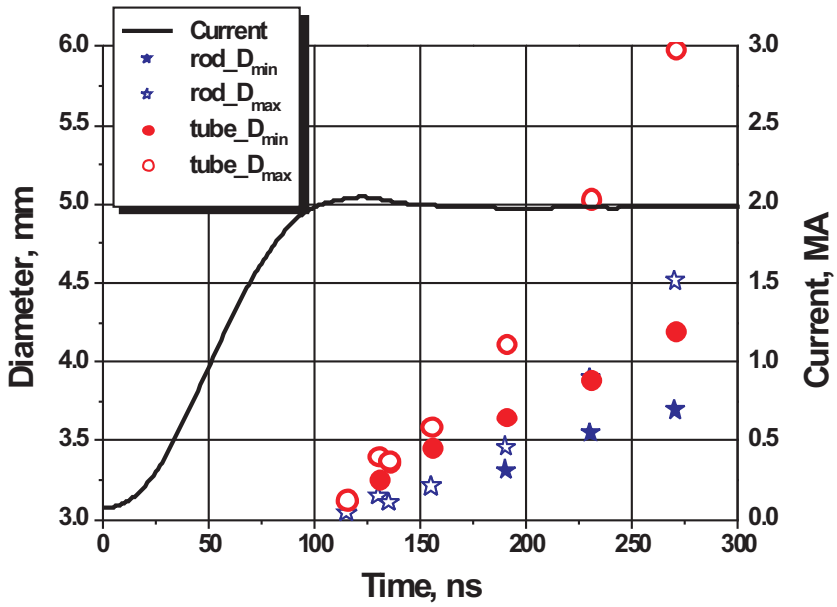


Figure 3. The generator current (solid line), the conductor maximum diameter $D_{\max}$ (empty asterisks—rod; empty circles—tube) and minimum diameter $D_{\min}$ (bold asterisks—rod; bold circles—tube).

where δ_0 is the resistivity under normal conditions; $\partial\delta/\partial T$ is the temperature derivative of resistivity. Since $Q = \rho c_V T$, where ρ is the density of the conductor material and c_V is its heat capacity at constant volume, the resistivity can be expressed in terms of thermal energy density as

$$\delta(Q) = \delta_0(1 + \beta Q), \tag{5}$$

where $\beta = (\rho c_V \delta_0)^{-1}\partial\delta/\partial T$. The characteristic field H_0 corresponds to the doubled conductor resistivity (for titanium, $H_0 \approx 310$ kG). Thus, assuming that $Q = H_0^2/(8\pi)$, we obtain $H_0 = \sqrt{8\pi/\beta}$.

The system of equations (1)–(5) has been solved numerically for the nonlinear magnetic diffusion wave propagating through the solid part of the titanium conductor, and the results are presented in figure 4. In the calculations, the experimental current waveform given in figure 3 was used. Figure 4 presents the current density profiles at different points in time.

The nonlinear diffusion wave starts propagating through a conductor when the magnetic field at the conductor boundary reaches H_0; in our case, this occurs approximately in 35–40 ns. The velocity of the wave during its propagation through external layers is about 10 km/s, whereas at the axis it increases by an order of magnitude, to 100 km/s. The current density at the wave front also increases as the wave approaches the axis (see figure 4). The wave arrives at the axis approximately at $t \approx 120$ ns from the onset of current flow, and thereafter a reflected wave is generated that equalizes the current density over the conductor cross-section. The process of current density equalization takes about 10 ns. The second frame in figure 2 just corresponds to this point in time (130 ns). At this time, thermal instabilities start developing on the solid part of the conductor.

In the tube, the nonlinear diffusion wave propagates with a velocity of about 10 km/s and arrives at the tube internal surface within about 60 ns after the onset of current flow. In this case, the current density equalization also takes about 10 ns; i.e. it goes up to $t \approx 70$ ns. At

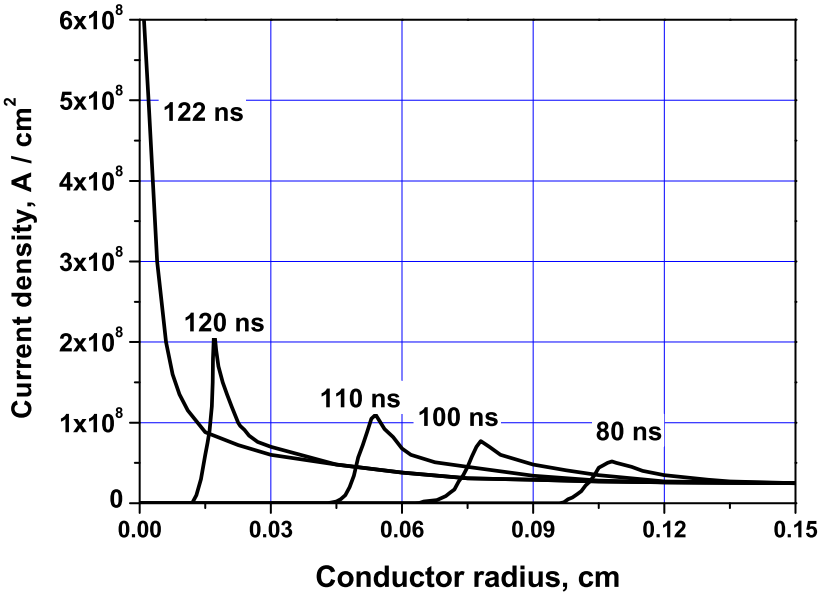


Figure 4. Propagation of a nonlinear magnetic diffusion wave through a titanium conductor of diameter 3 mm.

Table 1. The conductor expansion velocities and temperatures.

Conductor part	Rod	Tube
$V_{\min}$, km/s	2.2	3
$V_{\max}$, km/s	5.7	11
$T_{\min}$, eV	0.8	1.7
$T_{\max}$, eV	5.3	18.7

$t \approx 80$ ns, the conductor explodes. Therefore, in the second frame, of figure 2 (130 ns) we see developed thermal instabilities which has grown to the nonlinear stage.

The data given in figure 3 make it possible to determine the velocity of expansion of the conductor in its different parts: the rod and the tube. The maximum expansion velocities, $V_{\max}$, correspond to the hotter layers and the minimum ones, $V_{\min}$, to the cold layers. The expansion velocity values are given in table 1. Knowing the velocity of expansion of the plasma, we can estimate its temperature as $T = m_i V^2/3$, where m_i is the atomic mass. This formula implies that the plasma expansion velocity is the thermal velocity of ions; i.e. the effect of the magnetic field is not considered, and therefore the formula yields the lower estimate of the plasma temperature. The temperature values estimated by the plasma expansion velocity are given in table 1. It can be seen that the temperature in the striation hot region is 5–15 times more than that in the cold region.

In addition to the plasma expansion velocity, the average wavelengths of the thermal instabilities developing during the explosion were determined with the help of photographic images. The characteristic wavelengths of the instabilities, λ, were several hundreds of micrometers and turned out to be different for the solid and the hollow part of the conductor. Besides, it turned out that the average thermal instability wavelength increased with time (see figure 4) like in the case of exploding microconductors [17]. The rise rate of the instability

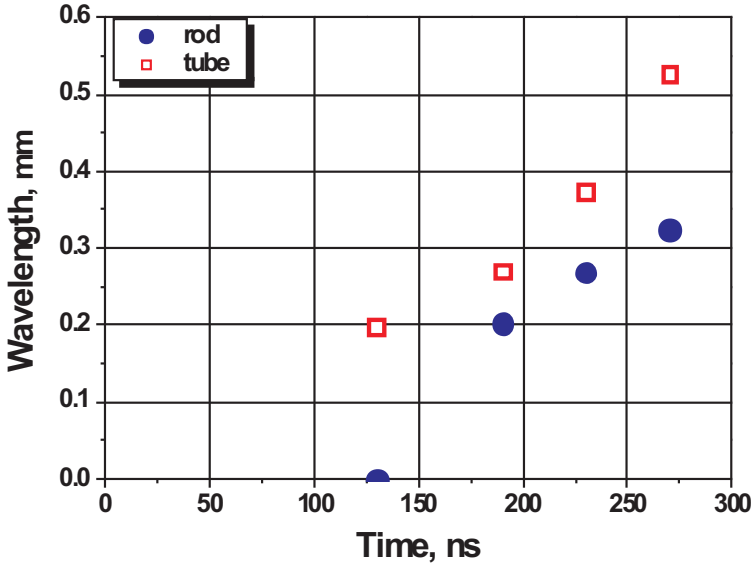


Figure 5. Time variation of the average wavelength of the thermal instabilities developing in exploded titanium conductors.

wavelength $d\lambda/dt$ has been estimated to be ~ 1.5 km/s for the solid part of the conductor and ~ 3 km/s for the hollow part.

4. Conclusion

Thus, the rise rate of the striation wavelength is close to the thermal velocity of the plasma ions.

Acknowledgments

The work was supported in part by the Presidium RAS under the programs "Fundamental problems of pulsed high current electronics" and "Study of matter at high energy densities" as well as by the Russian Foundation for Basic Research grant No. 14-08-00524.

References

[1] Mesyats G A 2000 *Ectons in a Vacuum Discharge: the Breakdown, the Spark, and the Arc* (Moscow: Nauka)
[2] Lebedev S V and Savvatimskii A I 1984 *Sov. Phys. Usp.* **27** 749–771
[3] Burtsev V A, Kalinin N V and Luchinskii A V 1990 *Electrical Explosion of Conductors and Its Applications* (Moscow: Energoizdat)
[4] Oreshkin V I, Barengolts S A and Chaikovsky S A 2007 *Tech. Phys.* **52** 642–650
[5] Oreshkin V I, Khishchenko K V, Levashov P R, Rousskikh A G and Chaikovskii S A 2012 *High Temp.* **50** 584–595
[6] Knoepfel H 1970 *Pulsed High Magnetic Fields* (Amsterdam: North-Holland)
[7] Shneerson G A 1973 *Zh. Tekh. Fiz.* **18** 419
[8] Krivosheev S I, Titkov V V and Shneerson G A 1997 *Tech. Phys.* **42** 352–366
[9] Chaikovsky S A, Oreshkin V I, Mesyats G A, Ratakhin N A, Datsko I M and Kablambaev B A 2009 *Phys. Plasmas* **16** 042701
[10] Oreshkin V I and Chaikovsky S A 2012 *Phys. Plasmas* **19** 022706
[11] Peterson K J S D B, Herrmann M C, Cuneo M E, Slutz S A, Smith I C, Atherton B W, Knudson M D and Nakhleh C 2012 *Phys. Plasmas* **19** 092701

[12] Peterson K J, Yu E P, Sinars D B, Cuneo M E, Slutz S A, Smith I C, Koning J M, Marinak M M, Nakhleh C and Herrmann M C 2013 *Phys. Plasmas* **20** 056305
[13] Valuev A A, Dikhter I Y and Zeigarnik V A 1978 *Zh. Tekh. Fiz.* **48** 2088
[14] Oreshkin V I 2008 *Phys. Plasmas* **15** 092103
[15] Oreshkin V I, Rousskikh A G, Chaikovsky S A and Oreshkin E V 2010 *Phys. Plasmas* **17** 072703
[16] Petin V K, Shlyakhtun S V, Oreshkin V I and Ratakhin N A 2008 *Tech. Phys.* **53** 776–782
[17] Rousskikh A G, Oreshkin V I, Chaikovsky S A, Labetskaya N A, Shishlov A V, Beilis I I and Baksht R B 2008 *Phys. Plasmas* **15** 102706