

The effect of nonreciprocal interaction on the redistribution of kinetic energy in the system of particles

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Abstract. Dynamical properties of particles in the chain-like and layered structures are studied with the help of Langevin MD approach. We considered infinite, confined chain-like structures, and extended layered structures with vertical (chain-like) and hexagonal ordering of particles in adjacent layers over a wide range of parameters, corresponding to the experimental conditions in the laboratory dusty plasmas. The nonreciprocal anisotropic interparticle interaction leads to a growth of kinetic energy of stochastic motion of particles especially its vertical component. Efficacy of the heating depends on the thermal particle motion, the anisotropy of the considered interparticle interaction and with decreasing the friction coefficient.

1. Introduction

A dusty plasma is ubiquitous in nature and is generated in different technological processes. In laboratory dusty plasma a variety of anisotropic configurations of dust arrangement is observed [1–5]. So, for example, in experiments with capacitive high-frequency (rf-) discharge plasma the forming monolayer and multilayer extended dust crystals have hexagonal lattice structure in the horizontal direction (perpendicular to gravity). However, in the vertical direction the adjacent dust layers can form either vertically aligned strings or a hexagonal close packing [1]. Isolated dust chains comprising up to several tens of particles, are often observed in inductive rf-discharge and direct-current discharge [6–8]. The charge-dipole interaction as an approximation of particle-wake interaction is commonly used to explain the vertical ordering of dust grains observed in the sheath region of capacitive rf-discharge, due to attractive forces induced by ion focusing [1], and the polarization of a trapped ion cloud around a dust particle. Numerical studies of crystalline and liquid structures of charged particles with quasi dipole-dipole interaction in the external electric field confirmed the validity of this approach [9–12].

Under certain conditions dust particles in plasma can acquire stochastic kinetic energy ~ 15 eV, which is much higher than the surrounding gas temperature. A basic mechanism of the anomalous dust heating is usually associated with the temporal and spatial fluctuation of charges [13, 14]. Dust particles can gain energy from the surrounding plasma, there are some well known cases: if the charge is a function of the coordinates, if the electric field cause



the effect of delayed charging, or energy may be acquired by the random fluctuations of the charge. Yet abovementioned theoretical models do not allow us to explain the increase in particle kinetic energy (> 0.5 eV) and some aspects of heating process under typical conditions of laboratory complex plasma [1, 15, 16]. The first theoretical works on ion focusing in a dusty plasma were published in the mid nineties [9, 17]. Papers [17, 18] contain analytical study of the wake structure, which arise downstream from the point charge in a collisionless plasma flow, in particular, it was shown that in a certain range of angles wake potential is oscillatory, whereas outside the Mach cone potential decreases exponentially. Later, numerical studies of a relationship between wake potential and the Mach number (the ion streaming velocity), the electron-to-ion temperature ratio, the Landau damping, the grain size, and its dielectric constant, as well as ion-neutral collisions [19–21]. Collisions and Landau damping may restrict the perturbed plasma region and reduce the oscillations of wake field to the first maximum only [19]. Herewith, neighboring dust grains, as is commonly believed, interact both with negatively charged dust grain, and with a positive ion cloud. Currently, the most widespread model used to describe interaction of dust particles in the plasma is the model of isotropic screened Coulomb (Yukawa) potential [15, 22–24]. In some cases, the screened potential model should be corrected by reason of anisotropy of plasma around dust particles. For the model of interparticle interaction in a sheath region of laboratory gas discharge it is necessary to take into account the value of the ion flow velocity relative to a stationary dust, which effect on distribution of ions near a dust particle [1].

2. Model

We use quasi-dipoles to simulate the anisotropic interaction. For this purpose, we put the virtual positive massless charge q at a fixed distance d below each particle with negative charge Q . The electrostatic potential distribution around the quasi-dipole may be described as follows:

$$\varphi(l_{kj}, l_{dj}) = \frac{Q}{l_{kj}} \exp\left(-\kappa_1 \frac{l_{kj}}{l_p}\right) + \frac{q}{l_{dj}} \exp\left(-\kappa_2 \frac{l_{dj}}{l_p}\right). \quad (1)$$

Here $l_{kj} = |\vec{l}_k - \vec{l}_j| \equiv (z_{kj}^2 + x_{kj}^2 + y_{kj}^2)^{1/2}$ is the distance between the k -th (probe) and the j -th (test) interacting particles ($l_{kj} > d$), $l_{dj} = |\vec{d}_k - \vec{l}_j|$ is the distance between the j -th particle and virtual charge of the k -th, l_p is the average interparticle distance, and $\kappa_1 = l_p/\lambda_1$, $\kappa_2 = l_p/\lambda_2$. The Mach number for ion flow, the ion-neutral collisions in plasma and amount of trapped ions near dust particles, could significantly affect on the distribution of anisotropic interaction potential [25–27], which should be properly taken into account when considering parameters of the anisotropic potential (1), d/l_p and q/Q .

The main benefit of proposed approach (that is applying of virtual massless charges) is that it enables us to construct various distributions of anisotropic potential, including proposed by theoretical works [17, 18, 25] wake electric field distribution around the point charge, simply by changing the parameters d/l_p and q/Q in equation (1). We also add that the model considered here reflect in an adequate way the generation of a restoring force acting on the downstream particle [28].

Numerical and analytical studies of spatial distribution of electrostatic potential around a dust particle in an anisotropic plasma show that the location and density of the ion cloud is mainly dependent on the position of the upstream particle and weakly depend on the position of the downstream one [19, 25, 28]. Thus, to analyze the formation conditions of dust structures in systems of particles with anisotropic pair interaction arising due to ion focusing, we can consider two simplified limiting cases: (i) surrounding dust particles affect the ion cloud (thus its density and position is a result of joint action of the ion flow and the internal electric forces); (ii) surrounding dust particles do not influence the ion distribution behind an individual (probe)

dust particle, while the electric field produced by the ion cloud affects all the dust particles, except its parent particle. In the case of nonreciprocal interaction (ii) virtual positive charge influence only surrounding grains with negative charge Q . But surrounding particles themselves should not influence virtual charges, and the pair interaction energy can be written in the following form

$$U_{kj} = Q\varphi(l_{kj}, l_{dj}). \quad (2)$$

Therefore, if $q \neq 0$ and $z_{kj} \neq 0$, we have $U_{kj} \neq U_{jk}$. Similar numerical model of nonpotential interaction was considered in papers [9, 29] in order to explain the anomalous heating of dust grains in multilayer dust structures which are formed in a sheath of the capacitive RF discharge. We should add that, the possible cause of nonreciprocal interaction in the system of dust particles (where the Newton's third law is violated, in contrast to reciprocal interaction) can be fluxes of recombining ions; the effect of ion collisions on characteristics of dust-plasma structures is discussed in detail in Ref. [26, 27].

We note also that in the view of recent experimental and numerical work on dust grains aligned with the ion flow, the charge of grains in the chain is reduced downstream, and the corresponding ion focus can be modified. In laboratory experiments, magnitude of the particle charge can depend on the ion flux, mutual positions of particles, on external field distribution, local plasma density. For a clearer study the effect of the anisotropy of interparticle interaction on the stability of the particle vertical chain, in our model we assume that the charge of dust grains is independent of their spatial position and vertical ordering is due to external field configuration.

3. Simulation details

Numerical investigation of dynamics and conditions of layered and chain-like structures formation was based on Langevin molecular dynamics approach, detailed in [30]. MD is based on the solution of ordinary differential equations—the equations of motion. MD well describes a change in transport properties of the system while it is approaching to equilibrium state. Moreover, Langevin approach takes into account the dissipation of stochastic kinetic energy of dust particles due to viscous friction with the buffer gas, and the statistical equilibrium in the system is maintained by energy exchange with the thermostat particles. Therefore, Langevin MD is used to study such physical phenomena as phase transitions, thermal diffusion of macroparticles, viscosity and thermal conductivity, nonequilibrium dynamics of particle system, etc. When integrating the equations of motion, an important issue is how to control the integration time step gently, and to define the characteristic equilibration time correctly [31]. The choice of integration time step depends on characteristic frequencies of the physical process under consideration.

To study anisotropic interaction (2), we considered systems (consists of particles having a weight M of 10^{-10} grams) with the following parameters of potentials (2): $d/l_p \sim 1/6 \div 2/3$, $q \sim (0.05 \div 0.5)Q$, $\kappa_1 \equiv \kappa_2 = 0 \div 2$. We simulated finite ($N_p = \text{const}$) and homogeneous (infinite) chains of particles and layered structures with vertical (chain-like) and hexagonal ordering of particles in adjacent layers (see figure 1). In the first case the system of N_p (of 2 to 16) particles was confined in a linear electrostatic trap. For the case of homogeneous chains we used periodic boundary conditions imposed on the system of particles in the $\mathbf{z}$ direction, and electrostatic trap in the $\mathbf{r}$ direction, $E_r = \alpha r$. The number of particles in the central simulation cell was varied from 20 to 50; the potential cut-off length was $\sim 10\text{--}25 l_p$. As for extended layered structures, during the simulation we observed crystals with from 1 to 5 layers of interacting particles. We set the number of layers by varying gradient β and number of particles N_p . In the horizontal directions ($\mathbf{x}$ and $\mathbf{y}$) periodic boundary conditions were applied. In the vertical direction $\mathbf{z}$ the linear electric field $E_z = E_z^0 + \beta z$ act on the particles, where $E_z^0 = g/Q$ compensate the gravity field $M\mathbf{g}$. Here β is the magnitude of gradient of external electrical field. The number of

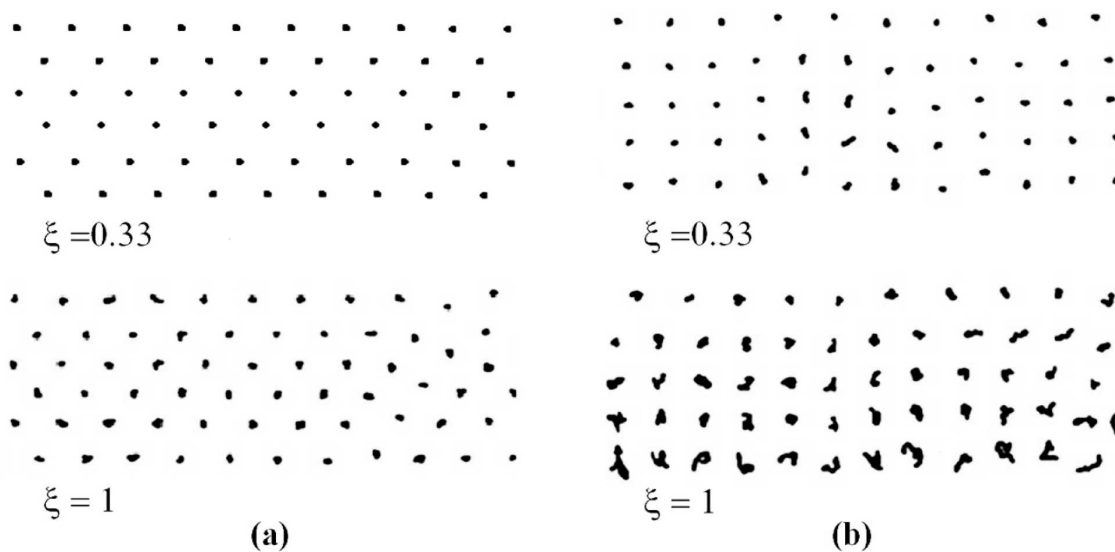


Figure 1. Fragments of vertical cross section of five-layer structures of particles with anisotropic pair interaction energy (2) with parameters $\kappa = 1$, $q = 0.16Q$, $d/l_p \approx 0.25$ (a) and $d/l_p \approx 0.37$ (b) and parameter ξ . Particle tracks are plotted for time interval $t = 1000\Delta t$.

particles in the central simulation cell was varied from ~ 600 to ~ 10000 ; the potential cut-off length was $\sim 10-20l_p$. The results of numerical modeling of five-layer particle structures with different parameters of the expression (2) for anisotropic pair interaction energy are shown in figure 1. It is easy to see that vertical ordering is observed only for certain values of parameters of the pair interaction energy (2). The assigned temperature T of particles was valued to provide the effective coupling parameter $\Gamma^* = l_p^2 U_1^{(2)} / (2T) \gg \Gamma_c^*$, here $\Gamma_c^* \approx 100$ is the melting point of systems with isotropic pair potentials [1], $U_1^{(2)}$ is second derivative of the first term of the expression (2) at the point l_p . The scaling factor $\xi = \omega^* / \nu_{fr}$ was changed from ~ 0.4 to ~ 4 , which corresponds to typical laboratory conditions [1]; here $\omega^* = \left(U_1^{(2)} / \pi M \right)^{1/2}$.

4. Results

Numerical investigations have shown that there is a swap of additional kinetic energy $\delta K = (K - T) > 0$ in systems with the nonreciprocal anisotropic interparticle interaction (2) (here T is the given particle temperature, $K = 2(K_x + K_y + K_z)/3$ is the kinetic energy of the particles obtained in our numerical simulations), and systems with reciprocal interaction cant generate additional kinetic energy. In all the observed cases the velocity distribution was close to a Maxwellian. It was found that the energy value $K_z \geq K_x = K_y$. Figure 1 shows vertical cross section of five-layer structures of particles with different parameters of the expression (2) for anisotropic pair interaction energy. It is easy to see that vertical ordering is observed only for higher values of parameters of the pair interaction energy (2), and particle motion intensity (length of tracks) depends on the scaling factor ξ (i.e. the coefficient of friction). In the case of infinite chain with interaction (2) at $\xi = 0.2$ and the initially set temperature of particles $= 0.1$ eV, their kinetic energy was changed slightly in the radial direction ($2K_x = 2K_y \approx T = 0.1$ eV), but it rose to $2K_z \approx 0.16 \div 0.2$ eV in the vertical direction.

For the case of confined chains the lower the location of the particle (in the direction of the

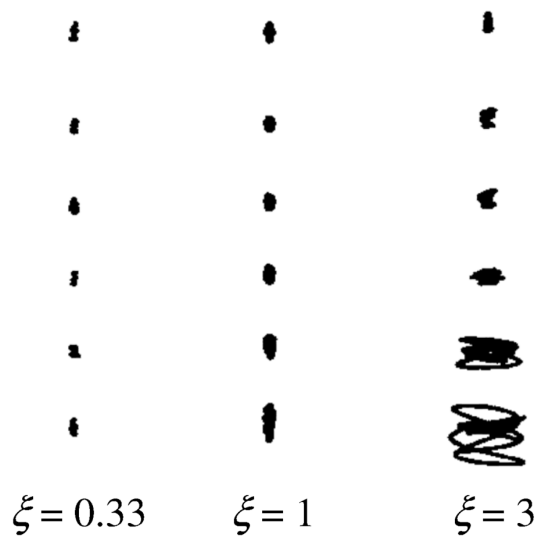


Figure 2. Illustration of particle tracks in confined chain of $N_p = 6$ particles with nonreciprocal interaction (2), when $q = 0.15Q$, $d/l_p \approx 1/6$, $\kappa = 0$, for different values of scaling factor are plotted all for the same time interval $t = 200(\omega^*)^1$, side view.

gravity field), the higher the measured value of its kinetic energy. Under some conditions, this effect led to total destruction of a chain due to development of amplitude instability. Figure 2 illustrates particle trajectories in a six-particle chain with different friction forces in the system (with increasing ξ up to 3.5 the particle system was completely destroyed.) Efficiency of additional kinetic energy δK redistribution between the degrees of freedom and between the particles increased with increasing the initially set temperature as well as the anisotropy of the considered potentials and with decreasing the friction coefficient.

The value of δK for bilayer and five-layer systems with different interactions (2) is shown in figure 3. The numerical results occur in proportion to the function $f(\xi) \propto A/\nu_{fr}^2$. Because of the “anomalous heating” of particles in case of vertically aligned structures there was observed their complete destruction.

Note that in real physical systems (such as, for example, dusty plasmas) there is a limit for growth of the particle kinetic energy due to existence of certain boundary conditions or spatial features of the medium providing a progressive increase in the dissipative loss, and also due to various dispersion effects (e.g., the phase mismatch) which phenomenologically play a dissipative role [3, 32, 33]. In all cases mentioned above, the dissipation (friction) can be described by a coefficient ν_{fr}^* greater than ν_{fr} . Figure 4 illustrates that $\delta K \propto 1/\nu_{fr}^2$ at high friction (at small ξ) in our simulations, but the growth of additional kinetic energy of the particles was limited at lower frictional forces. The effective friction coefficient ν_{fr}^* in the two-particle system with different interaction potentials becomes greater than ν_{fr} when $\xi > 1-2$.

5. Conclusions

To summarize, we have presented numerical results for dynamical properties (distribution of kinetic energy of stochastic motion of particles) of chain-like and layered dust structures with anisotropic pair interaction potential, which is similar to wake arising by ion focusing. We showed that in the case of nonreciprocal anisotropic interaction dynamics of particles is nonconservative that leads to an increase of kinetic energy of stochastic motion of particles (anomalous heating) and systems with reciprocal interaction cannot generate additional kinetic energy. It can be assumed there are some non-potential forces arising due to the nonreciprocal interparticle interaction, which are the source of additional kinetic energy of particles in the simulated systems. Such forces are capable to make a positive work that compensates energy

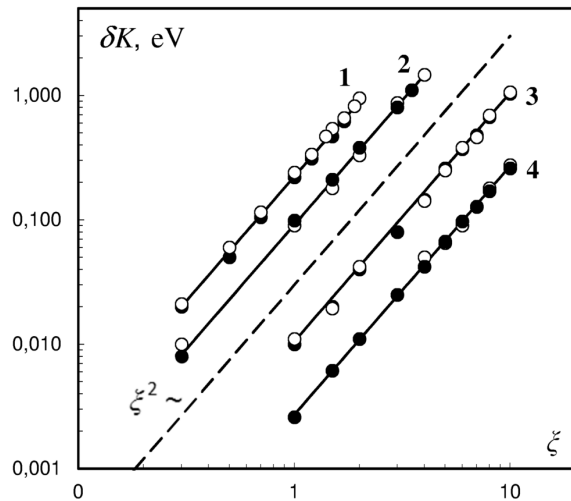


Figure 3. Dependence of additional stochastic energy δK on scaling factor ξ for five-layer (curves 1, 3) and bilayer (curves 2, 4) systems with anisotropic interaction energy (2) with $\kappa = 1$, $q = 0.16Q$, d/l_p : (curves 1, 2) 0.37 ; (curves 3, 4) 0.25; $\circ$ denotes $= 0.17$ eV; $\bullet$ denotes $= 0.51$ eV. The dashed line corresponds to a function, which is proportional to ξ^2 .

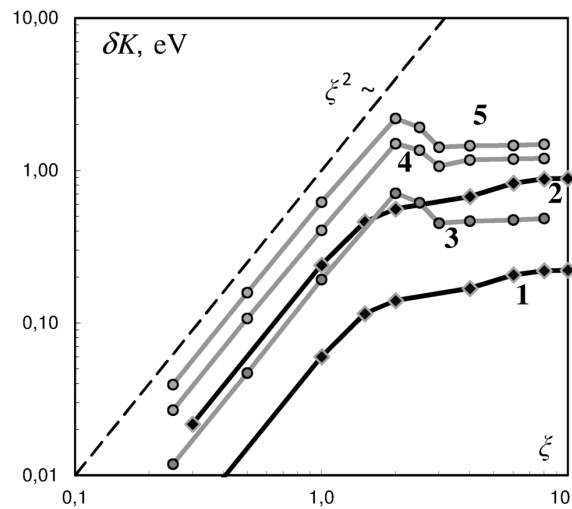


Figure 4. Dependence of the magnitude δK on the scaling factor ξ for two-particle system with nonreciprocal interaction (2) for the following parameters: $\kappa = 1$, $q = -0.15Q$, $d/l_p = 0.3$ (curve 1); $\kappa = 1$, $q = -0.3Q$, $d/l_p = 0.3$ (curve 2); $\kappa = 0$, $q = -0.1Q$, $d/l_p \approx 0.6$ (curve 3); $\kappa = 0$, $q = -0.3Q$, $d/l_p \approx 0.45$ (curve 4); $\kappa = 0$, $q = -0.5Q$, $d/l_p \approx 0.3$ (curve 5). The dashed line corresponds to a function, which is proportional to ξ^2 .

dissipation. Efficacy of the heating depends on the thermal particle motion, the anisotropy of the considered interparticle interaction and with decreasing the friction coefficient. The amount of additional energy is proportional to the inverse second power of friction coefficient. It was shown that structures with vertical ordering of particles are observed only for strong anisotropy of pair interaction potential. For systems of particles with chain-like ordering, the vertical component of kinetic energy is higher than the radial component of kinetic energy.

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