# Time of relaxation in dusty plasma model

**A V Timofeev**[1,2,3]

[1] Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13 Bldg 2, Moscow 125412, Russia

[2] Moscow Institute of Physics and Technology, Institutskiy Pereulok 9, Dolgoprudny, Moscow Region 141700, Russia

[3] Moscow Institute of Electronics and Mathematics, National Research University Higher School of Economics, Tallinskaya 34, Moscow 123458, Russia

E-mail: timofeevalvl@gmail.com

**Abstract.** Dust particles in plasma may have different values of average kinetic energy for vertical and horizontal motion. The partial equilibrium of the subsystems and the relaxation processes leading to this asymmetry are under consideration. A method for the relaxation time estimation in nonideal dusty plasma is suggested. The characteristic relaxation times of vertical and horizontal motion of dust particles in gas discharge are estimated by analytical approach and by analysis of simulation results. These relaxation times for vertical and horizontal subsystems appear to be different. A single hierarchy of relaxation times is proposed.

## 1. Introduction

The dusty plasma consists of ionized gas and solid particles of the size of a few micrometers. Dust particles can acquire a significant negative charge due to the different mobility of electrons and ions. The number of dust particles in standard dusty plasma experiments [1–6] can be from one to thousands. Dust particles can acquire kinetic energy of 10 $eV$ and even more, which is much higher than the temperature of ions and electrons in gas discharge and the temperature of dust particles substance [7,8]. The dust particles levitation is provided by the fact that gravity is compensated by the electric force. This force is conditioned by the influence of the electric field in the near-electrode layer on the charged dust particles. Average kinetic energy of the dust particles vertical motion may differ significantly from the average kinetic energy of the horizontal motion of dust particles due to the preferred direction of gravity and discharge. The mechanisms of energy transfer between the degrees of freedom is studied in [9–13]. The relaxation processes of horizontal and vertical degrees of freedom and equilibrium of these subsystems and the whole system of dust particles raise many questions in dusty plasma scientific area. Relaxation processes in ideal plasma and gas have been studied in detail. At the same time, there is no such good study for the relaxation processes in nonideal plasma, liquids, dusty plasma and the like. Theoretical study of two-temperature relaxation in nonideal plasma [14–16] offers an extrapolation of the Landau-Spitzer theory and also shows the quantitative difference between the relaxation rates for the ion and electron temperature, which gives a direct reference to the dusty plasma with several temperatures. The relaxation processes in the systems simulated by molecular dynamics method has been studied in [17, 18]. The relaxation in dusty plasma is mentioned in [19] for the evolution of temperature during recrystallization. The influence of external perturbation on the inter-particle interaction is studied in [20, 21]. In those papers,

dusty plasma is considered as whole system without separation into the subsystems, each of which has its own equilibrium. The study of relaxation and system equilibrium is essential for the use of thermodynamic functions for the dusty plasma system description.

In this article, the study of relaxation of horizontal, vertical motion and their connection in plasma-dust system is carried out using a theoretical approach and confirmed by simulation. The system of dust particles in plasma is simulated [8,13,22] by molecular dynamics with account of the particle charge fluctuations and the dependence of the charge and the electric field on the distance from the electrode and from the other dust particles.

In the second section, the dusty plasma model is described. The third section deals with the discussion of method of the relaxation time estimation. In the fourth section is divided into three subsections where the relaxation of the vertical motion, horizontal motion of dust particles and the connection between them are discussed. Each subsection includes relaxation time estimation based on theory and estimation based on analysis of simulation results.

## 2. Model

The dust particles forming a monolayer structure in plasma is considered as three-dimensional system. The dust particles interaction potential is assumed to be the screened Coulomb potential $\mathbf{U}_{ij}^{int} = \frac{Q_i Q_j}{r_{ij}} e^{-r_{ij}/r_D}$ [7,8,23,24], where $Q_i$ is a charge of i$^{th}$ dust particle, $r_{ij}$ is a distance between i$^{th}$ and j$^{th}$ dust particles, $r_D$ is a screening radius. The number of dust particles $N$ is below than two hundreds, as in standard laboratory experiment, so there is no need in potential cutoff. The gravity force on dust particle $\mathbf{F}^{gr} = m\mathbf{g}$ is compensated by the vertical electric field $\mathbf{E}$ acting on charged dust particle. The dust particles are assumed to be of the same size of a few micrometers. The influence of neutral gas $\mathbf{F}^{fr}$ is modeled by the Langevin thermostat. Trap-potential is considered to be parabolic $U_{trap} = \alpha r_i^2$, where $\alpha$ is a constant parameter of trap potential, $\mathbf{r}_i$ is radius vector of $i$th dust particle from the minimum of trap potential. The dependence of particle charge on time, distance to electrode and distance to other dust particles is taken into account. Charge is determined by the equilibrium of electron and ion fluxes onto the surface of dust particle. Dust particle charge fluctuations are conditioned by the fluctuations of these flows and the fluctuations of local plasma parameters near the particle. Hence, the dust particle charge fluctuates on time. In the sheath gas discharge electron and ion densities vary considerably in height. That leads to the charge dependence on vertical coordinate. Dust particles in the gas-discharge plasma acquire big charge and affect the surrounding plasma. Thus, two adjacent dust particles charge depend on the distance between each other.

Thus, the system of equations of motion of the dust particles is given by

$$m\ddot{\mathbf{r}}_i = \mathbf{F}_{ij}^{int} + \mathbf{F}_i^{fr} + \mathbf{F}_i^{trap} + \mathbf{F}_i^{gr} + \mathbf{F}_i^{el}, \ i = \overline{1,N}. \tag{1}$$

The parameters of the system are chosen to be close to the conditions of standard laboratory experiment: $dt = 10^{(-6)}$ s, screening radius $r_D = 0.025$ cm, average charge $Q = 10^4 e$, parameter of trap potential $\alpha = 0.08$ CGS units, friction coefficient $\gamma = 3.0$ s$^{-1}$.

## 3. Method of the relaxation time evaluation

The number of dust particles in laboratory experiment on dusty plasma is usually from one to thousands. The small number of dust particles leads to the necessity for the statistics to be collected not only in space but also in time. The ergodicity hypothesis is usually valid for molecular dynamics simulation and allows getting enough data for statistics. Duration of the statistics collection for the construction of the Maxwell distribution can often exceed the characteristic relaxation time of the system, so the classical method for the determination of the relaxation time of the system from the evolution of the Maxwellian velocity distribution of dust particles is not applicable to dusty plasmas.

The estimation of relaxation time as the time of the moments evolution is proposed in [17,25]. The nonideal system relaxation can be divided into two stages. In the first stage, the energy increase to the maximum is due to the translation of residue potential energy into kinetic energy. Relaxation is accompanied by damped oscillations of temperature. So the rate of oscillations damping in the case of system relaxation to the equilibrium does not depend on the form of the interparticle potential [17].

The simulation of dusty particles system in plasma shows that the shape of the dependence of the average kinetic energy on the time is identical to the one suggested in [17]. Both initial rise to a maximum and the subsequent exponential decay are observed. This exponential decay provides the relaxation time.

So the basic method for the detection of partial equilibrium in dusty plasma simulation is the analysis of moments of Maxwell distribution $M_n = \int_0^\infty v^n f(v) dv$, where $f(v)$ is the Maxwell distribution and $n$ is a positive even number and an order of moment. In the case of MD calculation moment should be calculated by the formula $M_n = \frac{1}{N} \sum_{i=1}^N v_i^n$, where $N$ is the number of particles. The moments in an equilibrium system are related to temperature as $M_n = (T/m)^n (n+1)!$ The moments of Maxwell distribution tend to their equilibrium values by exponential law with the same power [17]. In the case of dusty plasmas, this assumption is incorrect. The reason for this is that the dusty plasma is a set of related systems in which relaxation occurs in different ways and the whole system is in partial equilibrium. Exchange of energy between the degrees of freedom distorts different moments. Only the second-order moment can be used for the study of relaxation in dusty plasmas. The second-order moment is proportional to the kinetic energy. This leads to an additional uncertainty in the determination of the characteristic relaxation time in comparison with moments of higher orders.

The determination of the relaxation time by the molecular dynamics method is performed by creating of various disturbances in the subsystem and observation of the second moment relaxation to equilibrium. The rate of exponential decay times gives the characteristic relaxation time of our subsystem.

## 4. Relaxation of dust structure degrees of freedom
### 4.1. Vertical motion
The characteristic relaxation time of the vertical motion of dust particles is determined by observation on relaxation after rapid heating of vertical motion of one or several dust particles. Mostly this time is determined by charge fluctuations and gas friction. The characteristic relaxation time of particle charge fluctuations corresponds to $\Omega^{-1} \approx 20~\mu s$ and a reverse coefficient of gas friction corresponds to $\gamma^{-1} \approx 0.2~s$ for the conditions of standard laboratory experiment. Relaxation time of the vertical motion is in the range of these two values. The determination of the exact relaxation time by the evaluation of the maxwellization time is usually impossible because the construction of the distribution requires a set of statistics with duration of several periods of dust particle oscillation at the equilibrium phase of the trajectory. The oscillation period of dust particles is close to the relaxation time.

The second estimation is based on the analysis of simulation results. For the estimation of vertical motion relaxation time one may consider the motion of a single particle in order to isolate the vertical motion from the horizontal one, since in this case the influence of the horizontal motion can be neglected. The characteristic relaxation time of our subsystem is evaluated from the rate of exponential decay of the second moment. The dependence of relaxation time on the friction coefficient appears to be like $\tau_v = (0.84 \pm 0.05)\gamma^{-1}$ A more detailed study shows that the influence of dust particle charge fluctuations is significant and can reduce the characteristic relaxation time by tens of percent. It is worth noting that for small values of the friction coefficient the characteristic relaxation time of the vertical motion begins to deviate from its dependence.

*4.2. Horizontal motion*

The average kinetic energy of the horizontal motion of dust particles is largely determined by the influence of the surrounding gas. The characteristic frequency of gas particles collisions with the dust particle exceeds $10^{14}s^{-1}$. The gas influence on dust particle is modeled by the Langevin thermostat. The characteristic collision frequency is reduced to the reciprocal of the integration step $10^{6}s^{-1}$ due to the simulation by molecular dynamics method. By the way, this approximation only increases the relaxation time. By analogy with the vertical oscillations the relaxation time is between $\gamma^{-1}$ and integration step $dt$: $\tau_h \in (dt, \gamma^{-1})$. For conditions of the standard laboratory experiment $\tau_v \leq \tau_h$, as the impact of the charge fluctuations on the vertical motion is greater than on the horizontal motion and charge fluctuations accelerate the relaxation of vertical motion.

The relaxation time of the horizontal motion is evaluated from simulation results by exactly the same method as for the vertical motion of the dust particles. The dependence of the relaxation time on the friction coefficient appears to be $\tau_h = (1.025 \pm 0.003)\gamma^{-1}$ The relation of relaxing time is proved to be $\tau_v < \tau_h$, like it has been suggested. The influence of other mechanisms for conditions of standard laboratory experiment is negligible for the considered conditions.

*4.3. The whole system*

Let's consider mechanisms of energy exchange between the vertical and horizontal motion [9,11, 13] of the dust particles for the determination of applicability of the term "partial equilibrium" for this system. One of the mechanisms [13] allows to evaluate the condition of the instability transmitting energy between the degrees of freedom. This mechanism also gives the estimation of the characteristic relaxation time of the system. In this mechanism asymmetry caused by the gravity, and the intersection of the spectral bands of vertical and horizontal motion lead to the parametric resonance. This resonance leads to the energy exchange between vertical and horizontal motion. Moreover, the horizontal motion is warming up mainly on the frequency $2\omega_z/n$ where $\omega_z$ is the frequency of the vertical oscillations, $n$ is a natural number. The larger is the number $n$, the lesser is the chance of developing resonance. Thus, the amplitude of the horizontal oscillation increase is due to influence of big amplitude of vertical motion of dust particles. On the basis of the mechanism the characteristic time of equilibration between the vertical and horizontal vibrations of dust particles can be estimated as $\tau_{hv1} \approx (h_x\omega_x/4)^{-1}$, where the frequency of oscillation is determined by the formula $\omega_x^2(t) = \omega_x^2(1 + h_x cos(2\omega_z t))$ and $h_x \ll 1$ [13]. Parametric resonance occurs when the resonance condition $\gamma < (h_x\omega_x/4)$ is satisfied. The decrease of $\gamma$ leads to the appearance of a second resonance, which is discussed below. Thus, the characteristic time of establishing equilibrium between the horizontal and vertical oscillations is approximately $\gamma^{-1}$. The characteristic time of establishing equilibrium between the vertical and horizontal vibrations of dust particles for second resonance is $\tau_{hv2} \approx (h_z\omega_z/4)^{-1}$, where the oscillation frequency is determined by the formula $\omega_z^2(t) = \omega_z^2(1 + h_z cos(2\omega_x t))$ and $h_z \ll 1$. This time is close to $\gamma^{-1}$ because of the condition of resonance is $\gamma < (h_z\omega_z/4)$. With further decrease of $\gamma$ short-range order is lost in the system and the mechanism of energy transfer expires.

The relaxation time of the horizontal and vertical motion of the dust particle system is evaluated from simulation results by the similar method as for the vertical and horizontal motion: vertical motion is warmed up and the observation on vertical and horizontal motion allows learning the effect energy transfer of relaxation. The monolayer of 11 dust particles is simulated in order to investigate the relationship of vertical and horizontal motion. The characteristic relaxation time of our subsystem (figure 1) is evaluated from the rate of exponential change of the second moment. Relaxation time $\tau_{hv}$ appears to be in the range $(0.5 \div 9.5)\gamma^{-1}$. The delayed growth of horizontal temperature begins in accordance with the resonance condition $\gamma < (h_x\omega_x/4)$ due to the requirement of sufficiently large amplitude of dust particles oscillations.

If growth is not observed, we can conclude that $\tau_{hv} > 40\gamma^{-1}$. The relaxation of vertical and horizontal motion of dust particles is associated with gas friction, but it is strongly influenced by additional mechanisms related to the energy transfer between the degrees of freedom of the non-ideal system.

Therefore, the single hierarchy of relaxation times has the form

$$\tau_v \leq \tau_h \ll \tau_{hv}. \tag{2}$$

The relaxation time of a system with vertical and horizontal motion is greater than the characteristic time of establishing equilibrium for horizontal and vertical motions separately. This fact leads to the suggestion of a partial equilibrium for the vertical and horizontal motion of the dust particles with different average kinetic energies.

## 5. Conclusions

A method for the relaxation time estimation in dusty plasma is suggested. The relaxation time for vertical and horizontal motion of dust particles are estimated by analytical approach and by analysis of simulation results. Difference between these times is detected and discussed. Relaxation time of whole dust particles system is also evaluated. The hierarchy of relaxation times is proposed. The vertical motion and horizontal motion of the dust particles are in partial equilibrium separately for the significant range of parameters.
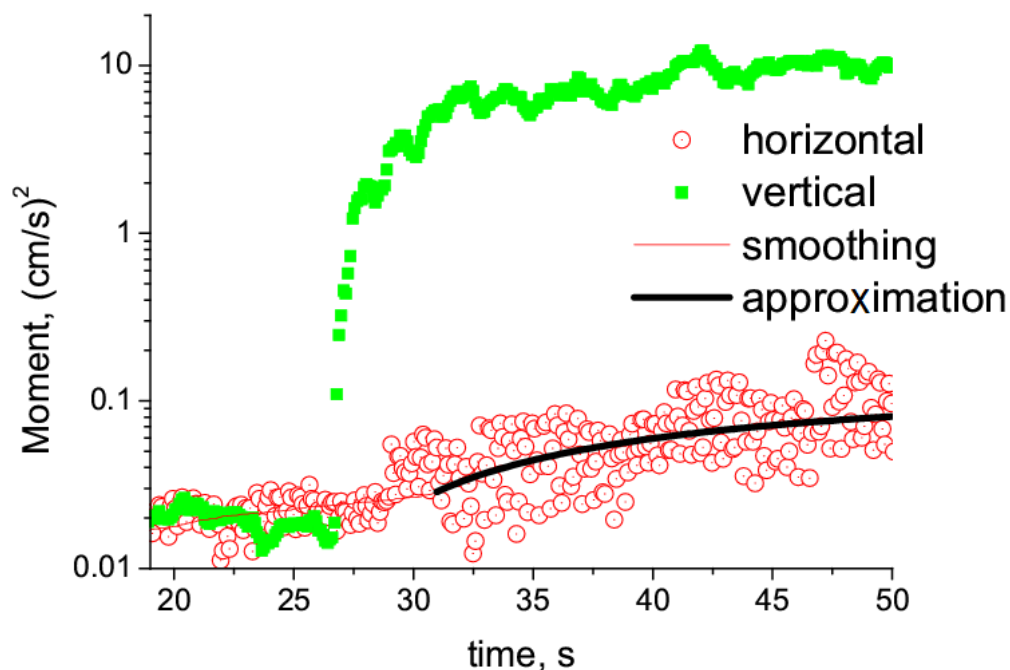


**Figure 1.** The dependence of the average kinetic energy of vertical and horizontal motion of dust particles on time. The data is obtained by study of moment relaxation after excitation of vertical motion of dust particle.

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## References

[1] Melzer A, Schweigert V A, Schweigert I V, Homann A, Peters S and Piel A 1996 *Phys. Rev. E* **54** 46–49
[2] Thomas H, Morfill G E, Demmel V, Goree J, Feuerbacher B and Möhlmann D 1994 *Phys. Rev. Lett.* **73** 652–655
[3] Williams J and Thomas E 2006 *Phys. Plasmas* **13** 063509
[4] Nosenko V, Goree J and Piel A 2006 *Phys. Plasmas* **13** 032106
[5] Zhukhovitskii D I, Petrov O F, Hyde T W, Herdrich G, Laufer R, Dropmann M and Matthews L S 2015 *New J. Phys.* **17** 053041
[6] Zhukhovitskii D I 2015 *Phys. Rev. E* **92** 023108
[7] Tsytovich V N, Morfill G E, Vladimirov S V and Thomas H M 2008 *Elementary Physics of Complex Plasmas* vol 731 (Berlin, Heidelberg: Springer Berlin Heidelberg)
[8] Vaulina O S, Petrov O F, Fortov V E, Khrapak S A and Khrapak A G 2009 *Dusty Plasmas: Experiment and Theory* (Moscow: Fizmatlit)
[9] Yaroshenko V V, Vladimirov S V and Morfill G E 2006 *New J. Phys.* **8** 201
[10] Ivlev A V, Zhdanov S K, Klumov B A and Morfill G E 2005 *Phys. Plasmas* **12** 092104
[11] Couëdel L, Nosenko V, Ivlev A V, Zhdanov S K, Thomas H M and Morfill G E 2010 *Phys. Rev. Lett.* **104** 195001
[12] Ivlev A V, Konopka U and Morfill G E 2000 *Phys. Rev. E* **62** 2739–2744
[13] Norman G E and Timofeev A V 2011 *Phys. Rev. E* **84** 056401
[14] Ohde T, Bonitz M, Bornath T, Kremp D and Schlanges M 1996 *Phys. Plasmas* **3** 1241
[15] Dharma-Wardana M W C 2001 *Phys. Rev. E* **64** 35401
[16] Dharma-Wardana M W C and Perrot F 2002 *Phys. Rev. B* **66** 14110
[17] Morozov I V, Norman G E and Smyslov A A 2008 *High Temp.* **46** 768–774
[18] Zwicknagel G 1999 *Contrib. Plasma Phys.* **39** 155–158
[19] Knapek C A, Samsonov D, Zhdanov S, Konopka U and Morfill G E 2007 *Phys. Rev. Lett.* **98** 015001
[20] Lisin E A, Timirkhanov R A, Vaulina O S, Petrov O F and Fortov V E 2013 *New J. Phys.* **15** 053004
[21] Lisin E A and Vaulina O S 2012 *JETP* **115** 947–952
[22] D'yachkov L G 2015 *Tech. Phys. Lett.* **41** 602–605
[23] Totsuji H and Totsuji C 2011 *J. Phys. Conf. Ser.* **327** 012037
[24] Vladimirov S V, Maiorov S A and Ishihara O 2003 *Phys. Plasmas* **10** 3867
[25] Morozov I V and Norman G E 2003 *J. Phys. A. Math. Gen.* **36** 6005–6012