

# Influence of artificial viscosity on the formation of vortex cascades in the shear layer

S V Fortova<sup>1</sup> and A P Pronina<sup>2</sup>

<sup>1</sup> Institute for Computer-Aided Design of the Russian Academy of Sciences, Vtoraya Brestskaya 19/18, Moscow 123056, Russia

<sup>2</sup> Moscow Institute of Physics and Technology, Institutskiy Pereulok 9, Dolgoprudny, Moscow Region 141700, Russia

E-mail: [sfortova@mail.ru](mailto:sfortova@mail.ru)

**Abstract.** Numerical analysis of development of vortex cascades in a three-dimensional inviscid shear layer depending on the method of determining and the value of artificial viscosity is done. At the developed stage of turbulence the spectral analysis of kinetic energy is carried out and the law  $-5/3$  of Kholmogorov is confirmed. Pulsation and correlation functions of the flow are investigated.

## 1. Introduction

Numerical simulation of turbulence allows to consider very complex systems of equations and to carry out calculations of flows in domains of various geometries. An important simplification in this case is the separation the computational domain into zones, in each of which simpler and faster to solve the systems of equations can be used [1]. According to the theory of Kolmogorov–Obukhov [2], at great numbers of  $Re$  turbulent movement consists of several stages: energy range, the inertial interval and the interval of dissipation. To describe viscous flows in all three ranges we must use the system of Navier–Stokes equations. For a wide class of spatial tasks at high  $Re$  numbers in the energy and inertial intervals of turbulent motion the effect of molecular viscosity on the general characteristics of the flow is insignificance. Therefore, the numerical study of flow dynamics can be investigated on the basis of ideal fluid models, i.e., the Euler equations [2, 3]. For problems with high Reynolds numbers, it is possible to compare the results obtained by the aforementioned two models and to define their advantages or disadvantages in relation to a specific task. This was done in the work [4], where it was shown that the main flow parameters for Rayleigh–Taylor instability are the same for both models. Since the difference between Navier–Stokes equations and Euler equations is the availability of terms describing the viscosity, the interest is in the construction of a numerical model, in which so-called “artificial viscosity” is included in the Euler equations, and in the investigating an effect of this viscosity on a behavior of the solution.

In the present work, turbulent flows developing at very high Reynolds numbers away from the body surface in shear layers are investigated. Representatives of such currents are turbulent traces of moving objects in the atmosphere and oceans, the problems on the interaction of the injected jet with the main stream, the interaction of powerful laser radiation with matter, and many atmospheric phenomena.



The formation of a turbulent flow at various hydrodynamic parameters and sizes of computational domains for the shear layer of ideal compressible medium with and without constant external power was analyzed in detail in the works [5–7].

The main idea of this paper is to study the effect of “artificial viscosity”, entered into the equations in the form of the second derivative of the horizontal velocity component along the vertical coordinate (approximation of Prandtl), on the development of the vortex cascades in a three-dimensional inviscid shear layer.

Free developed turbulence was simulated using monotone dissipative stable difference schemes with a positive operator, which are well proven for the calculation of large-scale flows. This scheme has second-order accuracy for smooth solutions and is monotone, so it does not involve any artificial viscosity, smoothing procedures, or flux limiters, which are frequently used in modern CFD schemes. In this work, numerical simulation was performed on the base of parallel programming technology MPI using this difference scheme on grids up to 1000000 cells.

## 2. Problem

The initial stage of the onset of turbulence in three-dimensional compressible inviscid shear flow is studied. The integration domain is shaped as 3D parallelepiped in Cartesian coordinates XYZ. Boundary conditions along X and Y coordinates are periodic, along Z coordinate, we used conditions of impermeability. Inside the shear layer the initial perturbations of the velocity  $V(u, v, w)$  components were specified as:  $u = \text{Ampl} \cdot \sin(2\pi x) \cos(\pi y)$ ,  $w = \text{Ampl} \cdot \sin(2\pi x) \cos(\pi y)$ . For the streamwise velocity, we set linear profile:  $v = 2V(z - \pi)$ , where  $V = 5$ . The initial particle concentration was set equal to 1 in the entire computational domain. The initial streamwise velocity was specified as  $v = -5$  below the shear layer and  $v = 5$  above the shear layer, other components were equal to 0. The size of computational domain is  $Lx = 2\pi$ ,  $Ly = 2\pi$ ,  $Lz = 2\pi$ . All calculations were made in SI units.

We studied the influence of the artificial viscosity on the formation of vortex cascade. Average velocity obtained by averaging the instantaneous velocity on a horizontal plane XY. Pulsations are defined as the difference between instantaneous and average velocity. Correlations present averaged over the horizontal plane XY compositions of pulsations, taken in different directions.

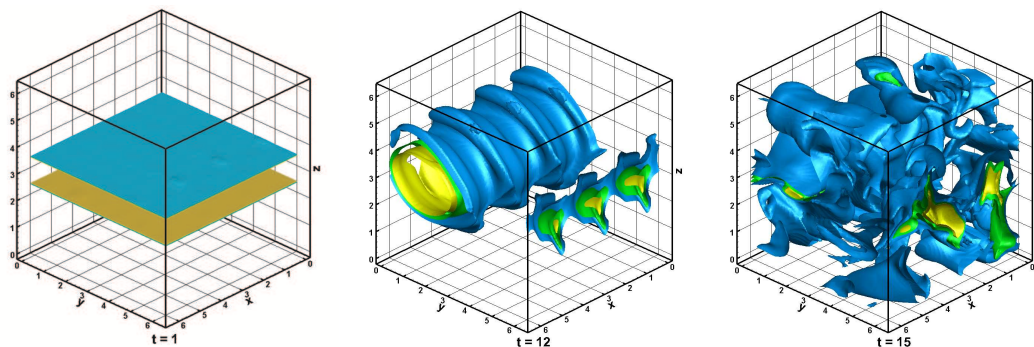
## 3. Results

In this work, we give general results concerning the influence of various factors on the formation of turbulent flow in the shear layer of inviscid compressible medium [4–7]. When the transverse size of the domain is small, the flow represents a large stable structure (secondary flow), which agrees with Batchelors two-dimensional theory. Transition to the turbulent phase occurs at a certain mode perturbation of two velocity components within the mixing layer at the initial time. The main factor of the onset of the turbulence in this case is the three-dimensional formulation of the problem.

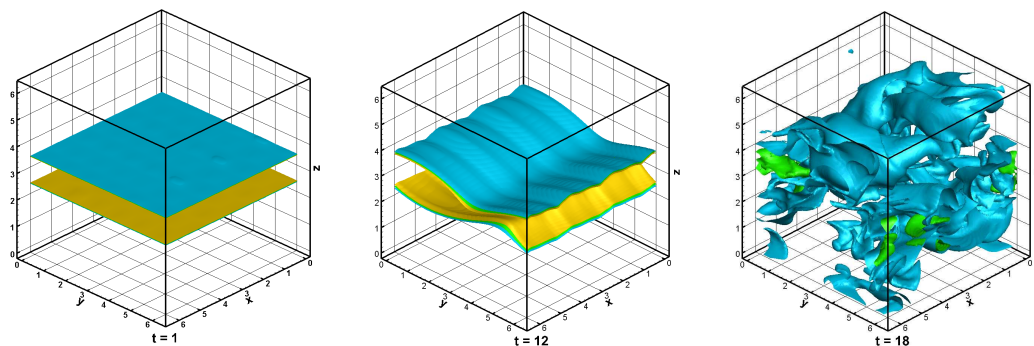
Analyzing only the flows in which there is the presence of a vortex cascade, we consider in detail the evolution of the structure of the shear flow depending on the value of artificial viscosity. The calculations were carried out with coefficients of viscosity  $\mu = 0.1, 0.01$  and  $0.001$ .

The development of the vortex cascade of instabilities in the absence of artificial viscosity and in the case of the presence with coefficient  $\mu = 0.001$  has the same time of development and the formed flow structure. Specifically, at the initial stage of the turbulence development, a single large structure is formed (as in the two-dimensional case) similar to a vortex roll smoothly streamlined by flow. Then, after the large structure formation, on the roll surface the incident “laminar” flow collapses into the bundles of smaller diameter than the diameter of the primary roll. Thus, this process leads to the main flow losing its stability, so, in a result, in this case the vortex cascade also exists and the flow transforms to the turbulence (figure 1). It should be noted that the formation of a single large structure for the flows in the absence of artificial

viscosity and in the case of its presence with coefficient  $\mu = 0.001$  occurs by the time  $t = 12$ , and transition to turbulent phase—by  $t = 15$ . When the artificial viscosity coefficient increases up to 0.01 the development of vortex is also observed. However, the evolution of this process is slower (figure 2). By this means, a large structure at the time  $t = 12$  only germinates, but it is completely formed by the time  $t = 13$ . The development of the vortex cascade is delayed similarly. Flow turbulence develops by the time  $t = 18$ . Further increase in the artificial viscosity coefficient up to 0.1 leads to the fact that a vortex cascade is not formed at a very long time calculation intervals. This does not mean that the cascade cannot be formed later, but underlines the fact that the artificial viscosity stabilizes the flow.



**Figure 1.** Vortex cascade when the viscosity coefficient  $\mu = 0$ .

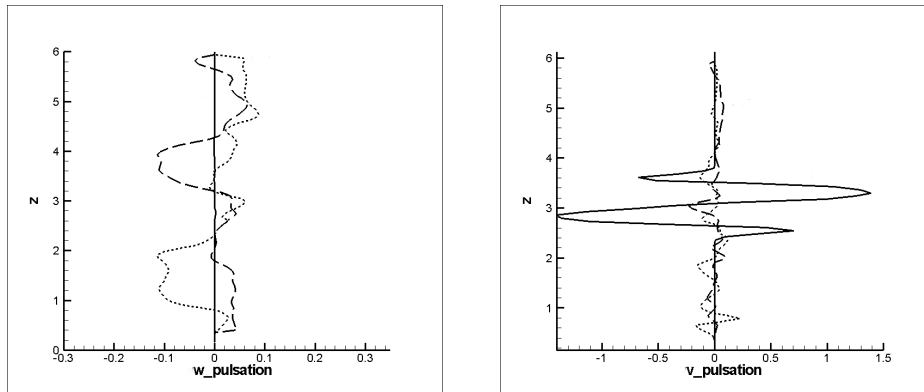


**Figure 2.** Vortex cascade when the viscosity coefficient  $\mu = 0.01$

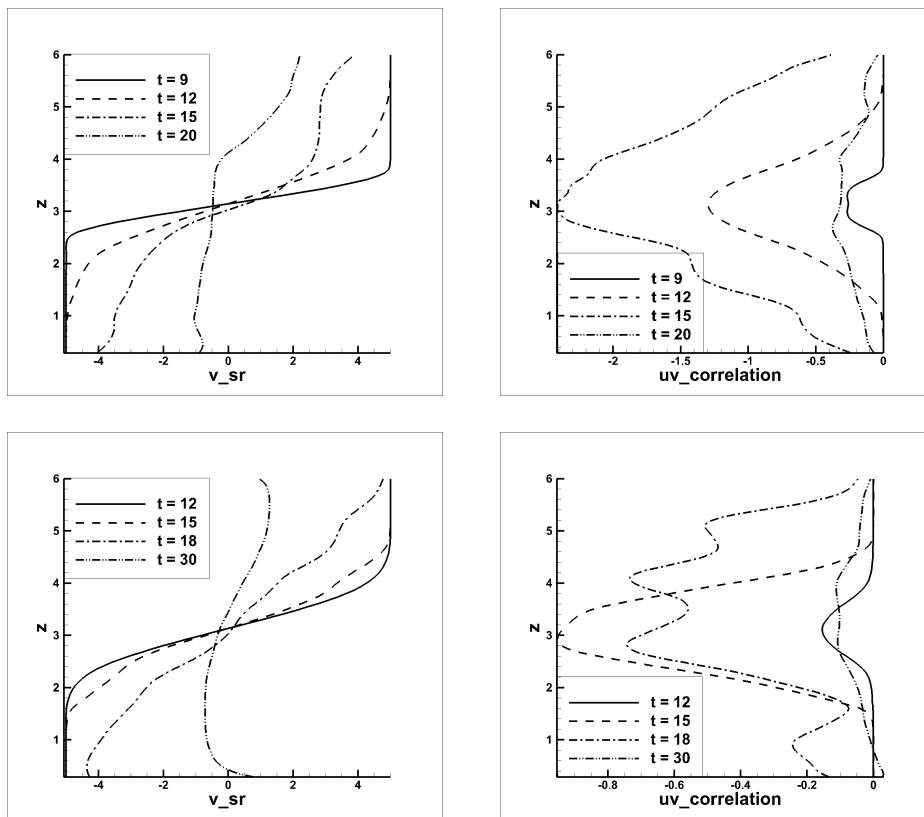
Turbulent flows, despite its chaotic, have a number of stable integral characteristics (fluctuations and velocity correlations, energy spectrum, etc.) [1–3]. Let us examine in more detail mentioned flow characteristics.

In a turbulent flow, chaotic fluctuations of major gas-dynamic variables take place. The transition to the turbulent motion occurs through the loss of flow stability and through the fluctuations development [1].

Figure 3 shows the evolution of pulsations of the streamwise velocity component. Over time, these pulsations develop and fluctuate around zero. This evolution demonstrates the establishment of turbulent profile in the flows in the absence of viscosity as well as in the availability. Interestingly to note that the magnitude of the fluctuations is gone from  $-0.1$  to  $0.1$ . Comparing this with the fact, that the flow is not developed when the artificial viscosity coefficient equals 0.1, it can be concluded that such a viscosity just suppresses the velocity fluctuations that are responsible for the promotion of turbulent motion.



**Figure 3.** Evolution of velocity component ( $v$  and  $w$ ) pulsations depending on time at  $t = 1$  (solid line),  $t = 15$  (dashed line),  $t = 17$  (dotted line).

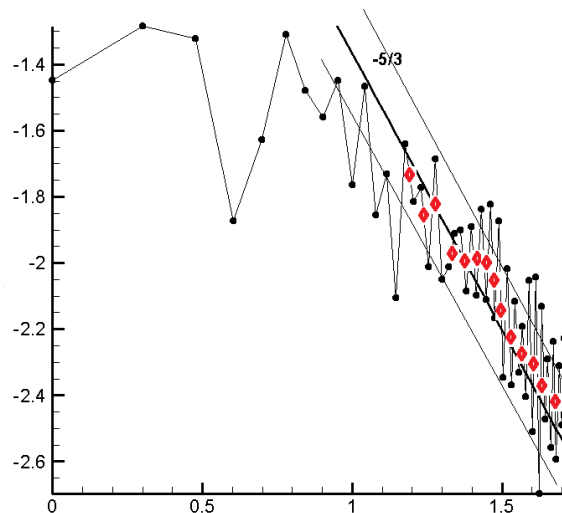


**Figure 4.** Averaged on the horizontal plane velocity component ( $v$ ) and correlations when the viscosity coefficient  $\mu = 0$  (upper images) and  $\mu = 0.01$  (lower).

In the figure 4, averaged streamwise velocity component and correlation are shown. In both cases (considering the artificial viscosity and excluding) the correlation profile is observed extended to the left. Correlation reaches its maximum amplitude at the moment corresponding to the formation of a single large structure. Further development of the flow in the vortex cascade generates correlation of smaller amplitude. Besides, it can be notice that in the presence of the artificial viscosity correlation value is smaller than in the absence of viscosity. The shape of the correlation is preserved in this case. Absence or presence of real (measured) turbulent stresses

(forces), generated by pulsations indicates whether a given medium is laminar or turbulent.

Large eddies in the turbulent flow not only determine the structure of the flow but also carry its basic energy. When the flow becomes turbulent, the energy is distributed over the scales of the eddy cascade. That is, in the course of the evolution, energy is gradually transferred from large to smaller eddies, and eventually dissipates into heat (not described by the Euler equations). Considering the spectral representation of kinetic energy over the vertical coordinate in the middle of the domain, we were able to trace the formation of a stable spectral segment for velocity component fluctuations. Figure 5 shows the fluctuation energy distribution over the wave numbers of streamwise fluctuations for the flow without viscosity. Analysis of the spectral characteristics of the flow revealed the presence of inertial interval in the energy spectrum and the implementation of the law  $-5/3$  of Kolmogorov. A similar situation can be also seen for the variant with the artificial viscosity coefficient  $\mu = 0.001$  and  $\mu = 0.01$ .



**Figure 5.** The kinetic energy spectrum when the viscosity coefficient  $\mu=0$ .

#### 4. Conclusions

Main results of numerical study of the vortex cascades are as follows. In free shear flow the birth of turbulence is connected with large vortex structures. We have identified conditions that lead to formation of the vortex cascade. The study of “artificial” viscosity parameter for Eulerian model shows the applicability of this model for the calculations of viscous flows in shear layers. We obtained special profiles for pulsations, correlations and spectrum of energy.

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