Numerical study of the gas flow in nozzles and supersonic jets for different values of specific heats

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Abstract. The process of the issuing of a supersonic jet from a flat tapered nozzle has been investigated with two values of ratio of isobaric to isochoric specific heats ($\gamma = c_p/c_V$) and different parameters NPR (nozzle pressure ratio) on the basis of numerical modeling. NPR parameter represents the ratio of the pressure at the nozzle inlet to the pressure in the environment. As a result the solid state grid model have been created for the numerical simulations of the flow of gas inside the nozzle and jet formation of compressed gas from the nozzle, which can be used to analyze the data structure. For the same value of the parameter NPR, but at a smaller value of γ an intersection point shifts further away from the inlet section of the nozzle. The flow separation becomes substantially less.

1. Introduction

The study of supersonic jets is important in many engineering applications. Issuing of the supersonic jet from the nozzle may occur in overexpanded mode, when the pressure in the gas flow from the nozzle outlet is less than the ambient pressure. In a planar overexpanded nozzle flow two oblique shocks are created that start at the nozzle lips and are directed towards the symmetry plane. These incident shocks can reflect either the regular reflection or the Mach reflection [1]. These types of reflection are shown in figure 1. At reflection triple shock configuration appears, with three shocks and a slip stream.

Location of shocks in triple-shock configurations, and their intensity depend on the Mach number M_1 of the gas issuing from the nozzle, the initial angle of incidence ω_1 and the ratio of specific heats γ. For the calculation of triple shock configuration at Mach reflection three-shock theory may be used [2].

The scheme of triple-shock configurations is given in figure 2. It is assumed that near the triple point there is some neighborhood where all the waves are straight, at each of the waves conservation laws are valid and boundary conditions are as follows: the flow through the incident and the reflected wave is parallel to the flow through the Mach wave, the pressure on both sides of the slip stream AT is the same.

Typically three-shock configuration looks like in figure 2 (left), i.e. the reflected wave AR is located above the line of incoming flow This configuration will be called a configuration with a positive angle of reflection ($\omega_2 > 0$). Recently it has been shown [3–7] that in a steady supersonic flow of gas at high Mach number ($M_1 > 3$) and small values of $\gamma(\gamma < 1.4)$ the reflected wave must

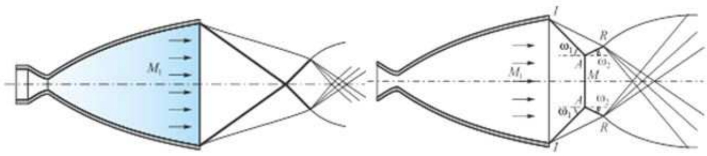


Figure 1. Supersonic nozzle flow. Left: regular reflection, right: Mach reflection. IA—incident wave. AR—reflected wave. AM—Mach wave. RF—rarefaction fan. ω_1—angle of incidence. ω_2—angle of reflection, A—triple point, M_1—Mach number.

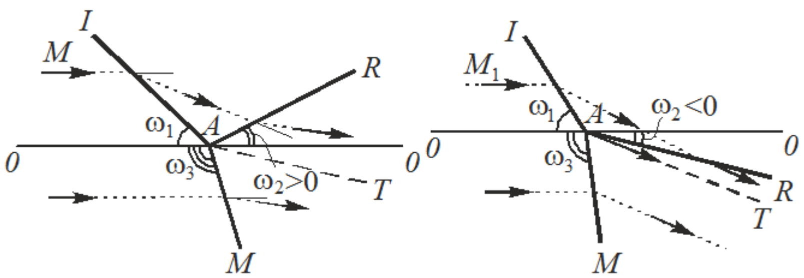


Figure 2. Scheme of three-shock configuration: Left at $\omega_2 > 0$, right at $\omega_2 < 0$.

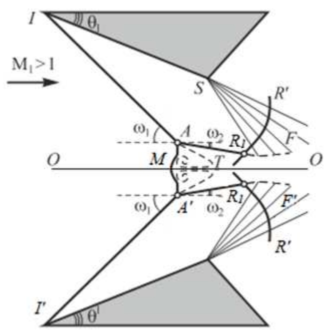


Figure 3. Unsteady double Mach reflection with negative reflection angle. IA—incident shock, AR_1R'—reflected shock, AM—Mach wave, AT—slip stream, OO—line of symmetry. R_1—second triple point.

be located below the direction of inflow stream ($\omega_2 < 0$). This constitutes a new configuration with a negative angle of reflection (figure 2 right). It has been also shown, that this configuration could be unstable and its appearance in the problem of flow around a system of two wedges leads a radical change in the entire flow pattern. This conclusion has been made after numerical study of the problem. It were shown that in the case then the parameters near a triple point ω_1, M_1, γ those ω_2 is negative then a new configuration appears—double Mach reflection with negative angle of reflection (figure 3). This configuration is absolutely unstable. The triple point moves upstream.

Waves pattern arising in the problem of gas outflow from the supersonic nozzles, is similar in many respects to the problem of air intake [8]. Thus, we can suggest, that if in the nozzle flow the parameters M_1, γ and ω_1 near the triple point, were such that the negative reflected angle appears, then a new form of reflection—the double Mach reflection should result. For example, lets consider the case in figure 4. If the reflected wave, which is going at the negative reflection angle ω_2, were to intersect the line of symmetry OO, then the gas would start to accumulate in the closed area ARM. This would lead to the choking the flow and to the movement of the Mach stem upstream. So, there would be a radical change in the entire flow pattern.

The main goal of the present paper is to develop a numerical method for the investigation of the jet flow at different initial conditions to predict emergency situations at rocket engine operation. In this paper, in the framework of numerical simulation, the structure of the jet issuing from the tapered (constant angle) supersonic nozzle has been investigated for different values of NPR (nozzle pressure ratio—the ratio between the stagnation pressure in the inlet and the ambient pressure) and different values of γ.

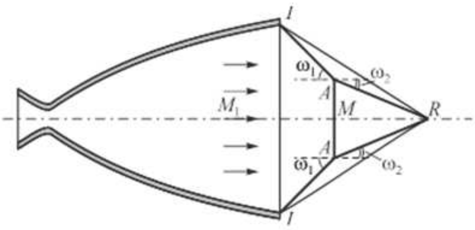


Figure 4. Jet flow. Triple shock configuration with negative reflected angle

2. Geometry and mesh model

As the object of study a tapered supersonic nozzle is considered. The geometric parameters are presented in table 1. The software for solid modeling SOLIDWORKS is used to construct a solid model of the nozzle and the surrounding area. The computational domain is chosen from the condition that the distance from the nozzle exit to the exit boundary equals to not less than 10 diameters of the output section. Figure 5 presents a solid model of a part of the computational domain.

The resulting geometrical model is imported into the software package STAR-CCM+, in which the mesh model was established and further calculations were carried out. To split the geometry on the finite volumes the unstructured polyhedral mesh has been used with smoothing at the solid walls. The advantages of polyhedral cell type over tetra or even to structured hexagonal cells are described in detail [9]. Note that the polyhedral cells allow for the same amount of cells to describe better the gradients than tetra and structured hexa-cells. This is particularly important in the study of supersonic gas flow, which often has to deal with large gradients (shock waves, rarefaction wave, etc.). It should also be noted that the use of polyhedral cells reduces the time for the calculations due to the more rapid achievement of the convergence

The turbulent flow is modeled in the present study. In the numerical study of turbulent flows, it is important to create on the surface of wall not only small but also quite uniform grid with a little stretching in height. This is called the creating the prismatic layer of cells. The size of the first layer is selected basing on the used turbulence model. In this case, a model SST was chosen and y + for this model should not exceed 5. Figure 6 presents the detail of the prismatic layer of the wall.

Table 1. Parameters of the nozzle.

Parameter	Value
Throat radius, D_t	16 mm
Area ratio of the exit section of nozzle to the throat, A_e/A_t	8
Angle of the nozzle	10°
Length of the computational domain	2400 mm
Height of the computational domain	1600 mm
Radius of curvature of the convergent part	24 mm

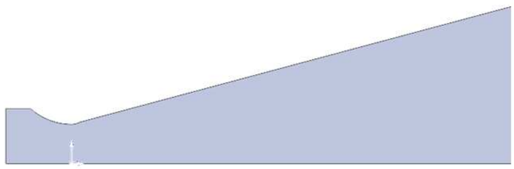


Figure 5. Geometry model of a fragment of the nozzle

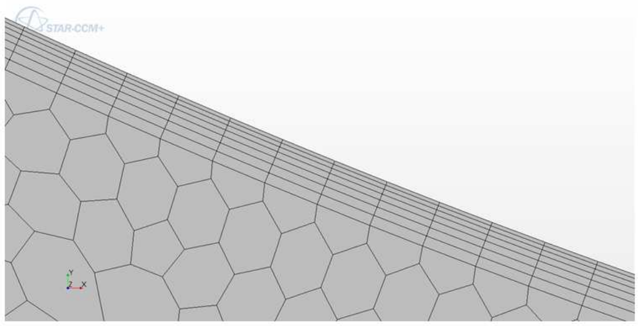


Figure 6. Fragment of the mesh model near a wall

3. Numerical model

A system of Navier-Stokes equations, Reynolds averaged, and the energy equation is used to describe the dynamics of a viscous turbulent gas flow in the nozzle. This system has the following form [10]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial \rho \bar{u}_i}{\partial t} + \frac{\partial \rho \overline{u_i u_j}}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \rho \overline{u_i' u_j'}\right), \tag{2}$$

$$\frac{\partial \rho c_p T}{\partial t} + \frac{\partial \rho u_i c_p T}{\partial x_j} = \frac{\partial q_i}{\partial x_i} + \frac{\partial \tau_{ij} u_j}{\partial x_j}. \tag{3}$$

Where ρ—density, T—temperature, u_i—the i-th component of the velocity, i = 1,2,3, x_i—coordinates, c_p—specific heat at constant pressure, τ_{ij}—viscous stress tensor, q_i—heat flux due to the thermal conductivity λ. Expression $(\rho \overline{u_i' u_j'})$ is called the Reynolds stress tensor, which is

Table 2. Parameters of the nozzle.

Parameter	Value
At the nozzle inlet—stagnation parameters	$P_* = 50$ kPa, $T^* = 200$ K
At the Symmetry Plane	$u_n = 0$, $u_\tau = 0$
At the exit—Static Pressure	$P = \text{const}$
On the wall—no slip	$\vec{u} = 0$

used to close the turbulence model SST, the equation for which are given by [11]:

$$\frac{\partial \rho k}{\partial t} + \bar{u}_j \frac{\partial \rho k}{\partial x_j} = \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right], \tag{4}$$

$$\frac{\partial \rho \omega}{\partial t} + \bar{u}_j \frac{\partial \rho \omega}{\partial x_j} = \gamma \frac{\omega}{k} \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta^* \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \rho \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}. \tag{5}$$

Where k—kinetic energy of turbulent fluctuations, ω—specific dissipation rate, β, β^*, σ_k, σ_ω—turbulence constants. Turbulent viscosity and Reynolds stresses are determined similarly as in the k-ω model, namely:

$$\mu_t = \rho \frac{k}{\omega}, \tag{6}$$

$$\tau_{ij} = -\rho \overline{u_i' u_j'}. \tag{7}$$

To close the above system of equations it is necessary to determine the condition of uniqueness. In this problem, the following boundary conditions are used (table 2). Equations (1)–(7), together with the boundary conditions form a closed system which can be solved by numerical methods.

4. Results

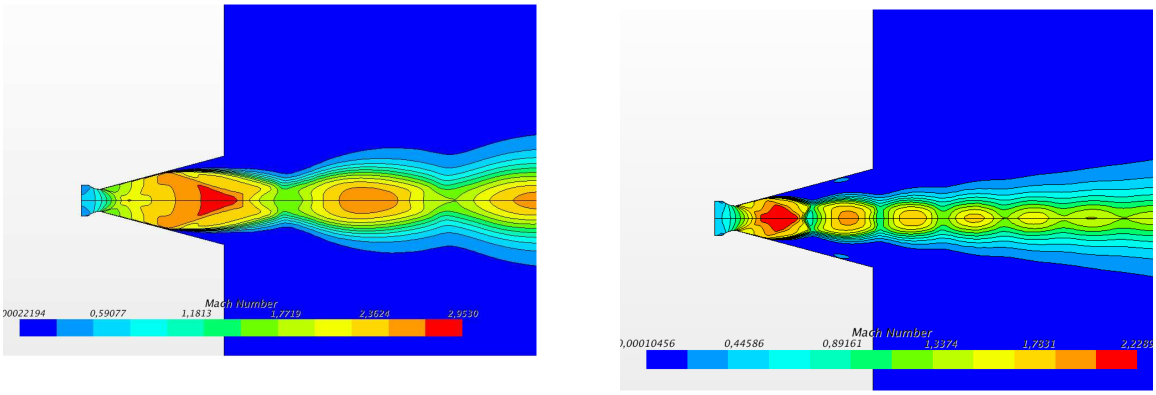
The perfect gas is used for the modeling the working fluid with the equation of state:

$$P = \rho R T. \tag{8}$$

In this case, to assess the impact on the structure of the adiabatic index the studiers have been made for two values of γ—1.4 and 1.2.

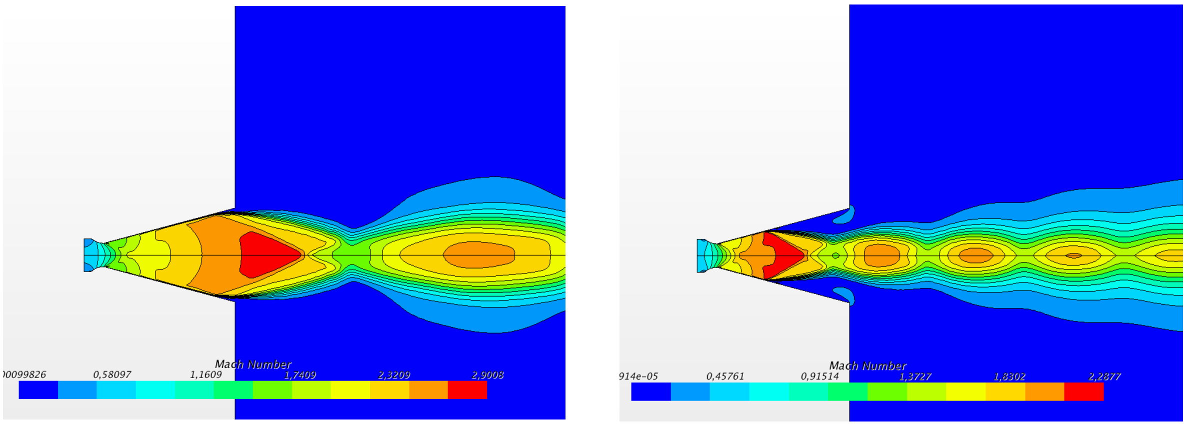
Figures 7 (a-b) show the calculated jet with $\gamma = 1.4$. At NPR = 20 (not shown) this structure is characteristic of regular reflection. The point of intersection is disposed at certain distance from the outlet of the nozzle. When NPR reducing to 10 the structure is resembled the regular reflection, however, the point of intersection moves inside the nozzle, because there is a small flow separation. With further decrease of the number of NPR to 5, the numbers of "diamonds" inside the nozzle increase. The subsonic region separating the incident wave from the "diamonds" appears. As NPR reduces, the subsonic region is close to the point of reflection. The separation zone is greatly increased. This mode is preceded the formation of Mach reflection. Note that usually in ideal nozzles Mach reflection should appear after regular reflection before the flow separation. The observed inverse sequence is typical for the tapered nozzles [12].

Figure 8 show the results of the calculation of the jet with $\gamma = 1.2$. Note that for the same NPR, compared to $\gamma = 1.4$, the reflection point is closer to the nozzle exit as in the case with $\gamma = 1.4$. In addition, the flow separation on the same modes is substantially less (figure 9).



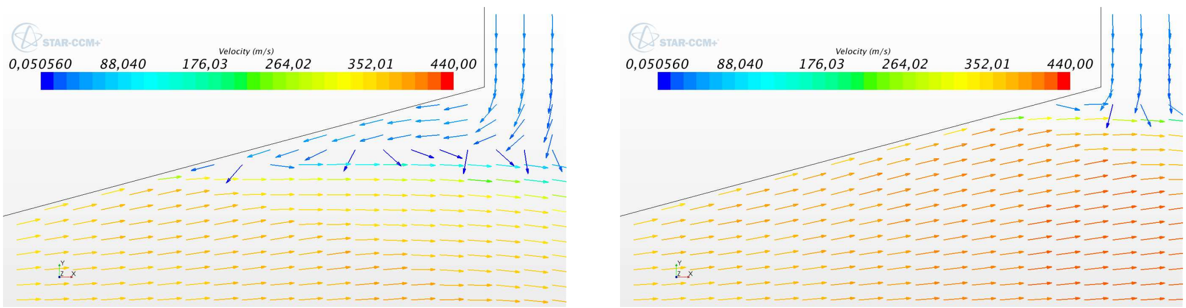
a) b)

Figure 7. Mach number contours at $\gamma = 1.4$ in a tapered nozzle while reducing NPR. a) NPR=10, b) NPR=5



a) b)

Figure 8. Mach number contours at $\gamma = 1.2$ in a tapered nozzle while reducing NPR. a) NPR=10, b) NPR=5



a) b)

Figure 9. Compase of velocity vector fields at NPR=10 for different γ. a) $\gamma = 1.4$, b)$\gamma = 1.2$

5. Conclusion

The solid state mesh model has been created for the numerical simulation of the flow of the gas inside the jet issuing from the nozzle. The calculation have been carried out for the values of specific heats $\gamma = 1.4$ and $\gamma = 1.2$, the value of the parameter NPR being 20, 15, 10, 5. With the parameter NPR = 20 there is a typical structure for regular reflection with the intersection point at the axis located near the exit of nozzle. When reducing NPR to 10 the structure resembling the regular reflection remains, however it is shifted into the nozzle, the flow separation occurs. With further decrease of NPR to 5 the number of consecutive normal shock waves in the nozzle increases. Subsonic separating zones appear. This mode is preceded to the formation of Mach reflection. For the same value of the parameter NPR, but at a smaller value of γ, the intersection point shifts further away from the inlet section of the nozzle. The flow separation became substantially less.

Acknowledgments

The present study is supported by the Russian Federation for Basic Research, grant No. 14-08-01070.

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