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Influence of plasma self-radiation model on thermonuclear burning simulation results

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Abstract. The one-dimensional problem on two-sided irradiation by proton beams of the plane layer of condensed DT mixture with density $\rho_0 = 100\rho_s$, where ρ_s is the fuel solid-state density at atmospheric pressure, is considered. The effect of the plasma self-radiation on the thermonuclear burn-up factor as well as the role of the inverse Compton effect are studied. It is shown that the inverse Compton effect decreases the burn-up factor down to about 3% while the plasma self-radiation in whole decreases it down to about 9%.

1. Introduction

For realization of inertial confinement fusion (ICF), it is necessary to create not only hot, but also high-density plasma. As a most promising approach at present time, the fast ignition of a target is considered [1], where the fuel is at first compressed by one driver up to require density, and then its small part is heated as fast as possible by another igniting driver.

The object of our previous works [2–5] was to study peculiarities of hydrodynamic flows with plane wave of thermonuclear burn and detonation, including its reflection from the symmetry plane, in the case of one dimension with reference to problems of fast ignition of thermonuclear targets.

In the present paper, we consider the model of plasma self-radiation from [3–5] in more details and study the effect of inaccuracy in spectral distribution of radiation on the burn-up factor of condensed deuterium–tritium (DT) mixture, for the initial density $\rho_0 = 100\rho_s$, where $\rho_s \approx 0.22$ g/cm^3 is the fuel solid-state density at atmospheric pressure, as well as the role of the cooling of electrons by the inverse Compton effect.

2. Problem statement and numerical method

At initial point of time $t = 0$, a motionless plane layer of the mixture of equal amount of D and T with atomic weight $A = 2.5$ is situated in a region $0 \le x \le H$. At point $x = H$, initially, a free boundary with the atmospheric pressure takes place. On this boundary, an igniting proton beam acts with constant intensity $J_b = 10^{19}$ W/cm^2 during $\Delta t = 50$ ps, kinetic energy of a proton is 1 MeV. At point $x = 0$, the symmetry condition is stated, which is equivalent to the simultaneous action of the identical driver on the symmetrically-situated layer of the mixture.

The primary reaction of nuclear DT fusion is taken into account, because of which the α-particle and neutron appear. The latter particle is supposed as flying out of the fuel without interaction with it.

A mathematical model is based on the equations [6]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\frac{\partial u}{\partial x}, \quad \rho\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial p}{\partial x},$$

$$\rho\frac{\mathrm{d}\varepsilon_e}{\mathrm{d}t} = -p_e\frac{\partial u}{\partial x} + \frac{\partial}{\partial x}\kappa_e\frac{\partial T_e}{\partial x} + \frac{3}{2}n_i k\frac{T_i - T_e}{\tau_T} + D_e + W_e + R,$$

$$\rho\frac{\mathrm{d}\varepsilon_i}{\mathrm{d}t} = -p_i\frac{\partial u}{\partial x} + \frac{\partial}{\partial x}\kappa_i\frac{\partial T_i}{\partial x} + \frac{3}{2}n_i k\frac{T_e - T_i}{\tau_T} + D_i + W_i,$$

where u is the mass velocity, $\mathrm{d}/\mathrm{d}t = \partial/\partial t + u\partial/\partial x$ is the Lagrangian derivative with respect to time, p_e and p_i are the pressures of electrons and ions, $p = p_e + p_i$, ε_e and ε_i are the specific internal energies of electrons and ions, T_e and T_i are the temperatures of electrons and ions, κ_e and κ_i are the coefficients of electronic and ionic thermal conductivity. An equation of state for hydrogen based on semiempirical model [7] is used to determine the functions $p_e = p_e(\rho, T_e)$, $p_i = p_i(\rho, T_i)$, $\varepsilon_e = \varepsilon_e(\rho, T_e)$ and $\varepsilon_i = \varepsilon_i(\rho, T_i)$. Third items in the right-hand side of the two last equations determine the exchange of energy between electrons and ions, $n_i = \rho(Am_u)^{-1}$ is the number density of ions, m_u is the atomic mass unit, k is the Boltzmann constant, τ_T is the temperature relaxation time. The rest items in the two last equations determine the heating of electrons and ions by the proton beam (D_e and D_i) and α-particles (W_e and W_i), as well as the energy exchange between electrons and self-radiation of plasma (R).

Number of events of DT reaction in unit volume per unit time is equal

$$F = n_\mathrm{D} n_\mathrm{T} \langle\sigma v\rangle_\mathrm{DT},$$

where n_D and n_T are the number density of deuterium and tritium nuclei, $\langle\sigma v\rangle_{DT}$ is the reaction rate averaged over Maxwellian distribution of ions as function of the temperature T_i according to [8]. Burn-up of the mixture nuclei is described by equation

$$\frac{\mathrm{d}n_j}{\mathrm{d}t} = -n_j\frac{\partial u}{\partial x} - F,$$

where the subscripts $j = \mathrm{D}$ and T are for the case of deuterium and tritium, respectively.

For simulation of α-particle deceleration in plasma we use the track method [8, 9] and its modification [4] that is a numerical method for the known Cauchy problem for the kinetic steady-state homogeneous equation in Fokker–Plank approximation [10]. The rate of deceleration (acceleration with negative value) is a given function of thermodynamic parameters of plasma [10]. The quasi-one-dimensional model [2–5] based on the track method is used. We introduce a length parameter R_α and solve the 1D problem taking into account the escape of α-particles from a cylinder of the radius R_α.

The intensity of a monoenergetic proton beam is determined by the proton velocity v as $J = n_p v m_p v^2/2$, where n_p is the proton number density, m_p is the proton mass. The value of n_p is determined by given values of the initial proton velocity ($v_0 \approx 13$ Mm/s) and the boundary beam intensity J_b.

Self-radiation of plasma is described by the steady-state transfer equation in the diffusion approximation by solid angle [11]. Similarly to [12], we take into account the cooling of electrons by the inverse Compton effect using the known approximate formula [13,14]. Resulting equations are as follows,

$$\frac{\partial q_\nu}{\partial x} = \kappa_a\big(B_\nu(T_e) - u_\nu\big), \quad \frac{\partial u_\nu}{\partial x} = -3\kappa_a q_\nu, \tag{1}$$

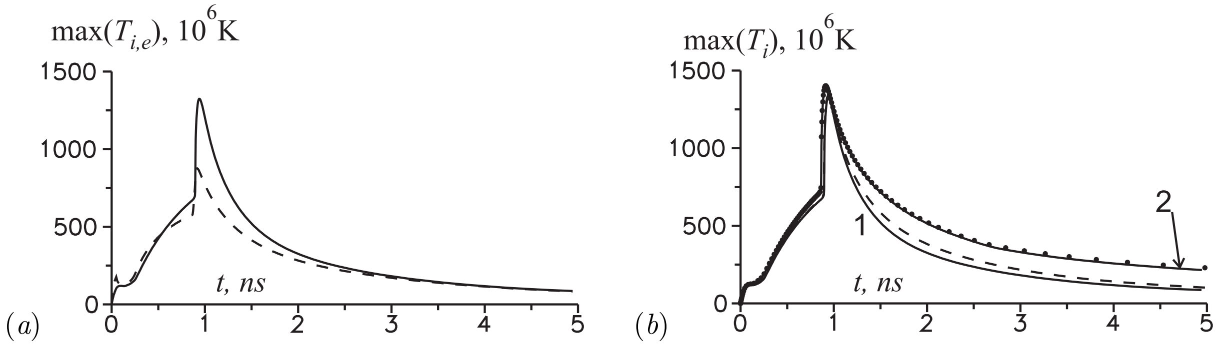

Figure 1. (a) The maximal in x ion (solid line) and electron (dashed line) temperatures versus time: $\nu_{\max} = \nu^{(2)}$. (b) The maximal on x ion temperature versus time: 1—$\nu_{\max} = \nu^{(2)}$ with (solid line) and without (dashed line) taking the inverse Compton effect into account, 2—$\nu_{\max} = \nu^{(3)}$ (solid line) and ignoring the plasma self-radiation (dots).

where ν is the frequency, $B_\nu(T_e)$ is the Planck function, $\kappa_a = \kappa_a(\rho, T_e, \nu)$ is the absorption coefficient with accounting for the induced emission [14]. The term in the right-hand part of the energy equation for electrons has the form

$$R = -\frac{\partial Q}{\partial x} - \frac{4\sigma_{\mathrm{T}} n_e U}{m_e c^2} k_{\mathrm{B}}(T_e - T_r), \quad Q = \int\limits_0^\infty q_\nu \mathrm{d}\nu, \quad U = \int\limits_0^\infty u_\nu \mathrm{d}\nu, \tag{2}$$

where $n_e = zn_i$ is the electron number density, m_e is the electron mass, c is the speed of light, σ_{T} is the Thomson scattering cross-section of photons by free electrons, T_r is the photon temperature determined by the equality

$$\int\limits_0^\infty B_\nu(T_r)\mathrm{d}\nu = U.$$

At calculation of all included coefficients in the model, plasma is supposed to be fully ionized.

The grid on the frequency ν for solving of equation (1) is determined by the following parameters: the maximal frequency $\nu_{\max}$, the integer number N and the set of integer numbers n_i, $i = 1, \ldots, N$. First, the rough grid $\overline{\nu}_i = \nu_{\max} 10^{-i}$, $i = 0, 1, \ldots, N$ is constructed. Within the each interval of the rough grid $\overline{\nu}_i \le \nu \le \overline{\nu}_{i-1}$, $i = 1, \ldots, N$, an additional grid ν_{ij} consisting of n_i intervals, $\log(\nu_{ij}/\overline{\nu}_{i-1}) = -j/n_i$, $j = 0, 1, \ldots, n_i$, is introduced. The integration with respect to the frequency in equation (2) is performed by the trapezium method.

3. Numerical experiments

We consider the problem with the length parameters $H = 2.5$ mm and $R_\alpha = 0.1$ mm corresponding to the values of the parameters $H\rho_0 \approx 5$ g/cm^2 and $R_\alpha \rho_0 \approx 0.2$ g/cm^2. Numerical results for three grids on the frequency ν differed from each other by the parameter $\nu_{\max}$ that equals to $\nu^{(1)} \approx 1.4 \times 10^{11}$, $\nu^{(2)} \approx 5 \times 10^{10}$ and $\nu^{(3)} \approx 2 \times 10^7$ K will be discussed. Another parameters are the same for all of the grids: $N = 10$, $n_i = 20$ at $i \le 4$, $n_5 = 10$ and $n_i = 5$ at $i \ge 6$.

The main features of the flow are illustrated by figure 1a where the time dependencies of $\max_x(T_i)$ and $\max_x(T_e)$ are presented for the grid parameter $\nu_{\max} = \nu^{(2)}$. The proton beam increases the temperatures up to about 10^8 K. Then the detonation wave arises and increases the both temperatures as it moves. The rapid temperature rise at about 1 ns is connected with

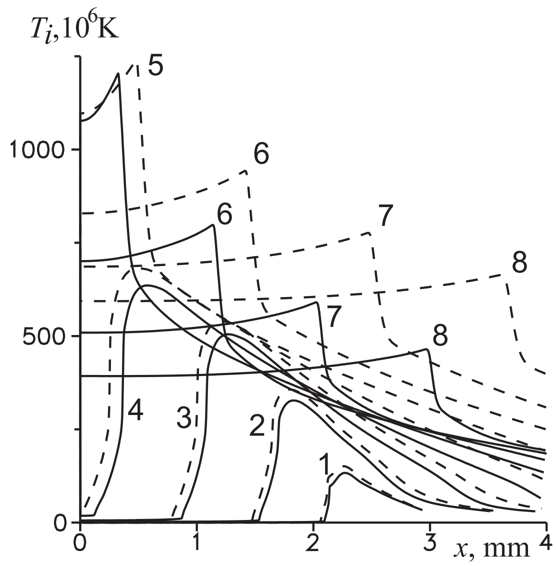

Figure 2. Spatial profiles of the ion temperature at the time points 0.2 (1), 0.4 (2), 0.6 (3), 0.8 (4), 1 (5), 1.2 (6), 1.4 (7) and 1.6 ns (8), corresponding to the incident (1–4) and the reflected (5–8) detonation waves for $\nu_{\max} = \nu^{(2)}$ (solid lines) and $\nu_{\max} = \nu^{(3)}$ (dashed lines).

reflection of the detonation wave from the symmetry plane. Upon the reflection, the flow with the linear velocity profile along x and with a weak dependence of the thermodynamic functions on x occurs.

As one can see in figure 1a, $\max_{x,t}(T_e) \approx 10^9$ K. Since the Plank function reaches its maximum at $\nu \approx 3T_e$, the parameter $\nu_{\max} = \nu^{(2)}$ is most likely enough to describe the tail of the Plank function even for the maximum possible temperature. To make sure that this is the case, the parameter $\nu_{\max} = \nu^{(1)} \approx 3\nu^{(2)}$ is chosen. Numerical results for the both parameters are very close. Numerical results for $\nu_{\max} = \nu^{(2)}$ and $\nu^{(3)} \approx 0.02 \max_{x,t}(T_e)$ are compared below.

The time dependencies of $\max_x(T_i)$ are presented in figure 1b for different computations. After reflecting the detonation wave, the curves for $\nu_{\max} = \nu^{(2)}$ and $\nu^{(3)}$ differ markedly from each other while the last curve close to that obtained without taking the self radiation into account. The inverse Compton effect gives an insignificant decrease in the ion temperature. Reducing the role of the inverse Compton effect by increasing the plasma density was shown previously [12].

Figure 2 shows the ion temperature as a function of x at eight points of time, obtained on grids with the parameter $\nu_{\max} = \nu^{(2)}$ and $\nu^{(3)}$. The last curves close to those obtained ignoring the plasma self-radiation. At four last time points the ion temperatures differ considerably while, qualitatively, the flow pattern is the same.

At $\nu_{\max} = \nu^{(2)}$ (so as at $\nu_{\max} = \nu^{(1)}$) the burn-up factor $B \approx 0.67$, the gain with respect to the energy of neutrons about 2000. Ignoring the inverse Compton effect gives $B \approx 0.69$. At $\nu_{\max} = \nu^{(3)}$ we have $B \approx 0.73$, that differ insignificantly from the value when the plasma self-radiation is ignored, $B \approx 0.74$.

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