Horizontal scale calibration of theodolites and total station using a gauge index table.

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Abstract. This paper shows a methodology to calibrate the horizontal scale of theodolites and total station using a high accuracy index table. The calibration pursued the method of circular scales and precision polygons (also called Rosette Method [1] or multistep). This method consists in the angle comparison of two circular divisions in all relative positions possibilities. Index table errors and theodolite horizontal scale errors were obtained using the method of least squares which is used to process the data from Rosette Method. An experimental setup was used to evaluate this methodology and the details of the mechanical assembly are also described in this paper. Several theodolites and total stations were calibrated using the proposed system and the results infer that the method is suitable to calibrate the different models available in the market. The system showed good stability over time with measurements uncertainties around 1" (one second) depending on instrument features.

1. Introduction

Theodolite is an instrument primarily used in vertical and horizontal angle measurements. The measurements are carry out by a rotational telescope that moves on both axis. In order to measure the displacement of the telescope, each axis contains a graduated scale as shown in figure 1.

The total station consists in the same principle of theodolite, but some changes can be found such as the incorporation of an electronic distance meter (EDM) and encoders instead of a circular graduated scale. The modern total stations can be operated by remote control or automatically measurements even with the target in movement [2].

These instruments have application in different areas, for example, road construction and aircraft turbines alignment. In the industrial measurement, high accuracy electronic theodolites have been used in automobile manufacturing, shipbuilding and railways constructions, the nuclear industry, the oil industry and robotics.

Generally, the technical standards present methodologies of qualifying and classification of these instruments. However, the calibration concept demands traceability evidences by higher accuracy metrological standards (national or primary standards). Many other calibration methods of theodolites horizontal scales and total stations can be performed based on circular scale and encoder calibration technique [3]. This paper presents a technique that was developed in the National Institute of Metrology, Quality and Technology (Inmetro) to calibrate the horizontal scales of theodolites and total stations. In particular, the reference [2] presents a vertical scales calibration method using a coordinate measuring machine (CMM).

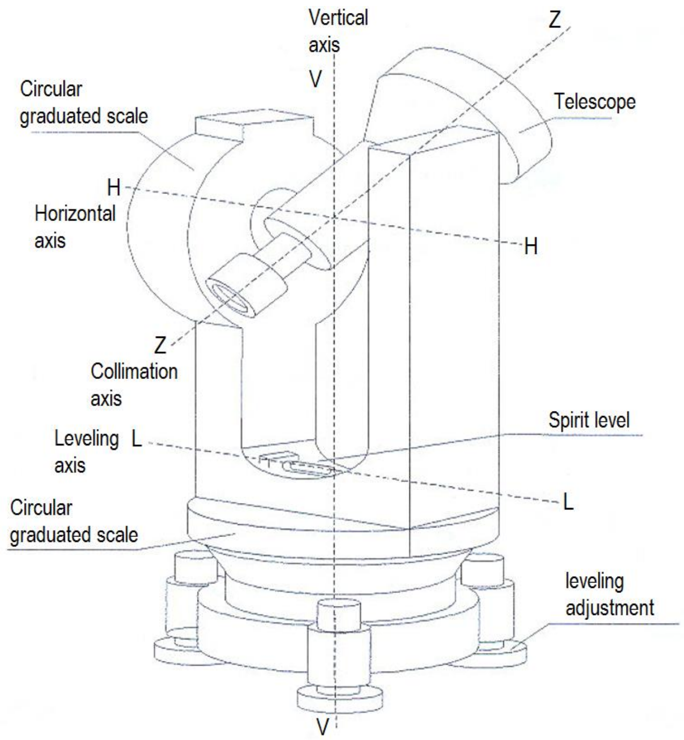

Figure 1. Theodolite schematic drawing [4].

2. Objective

This paper presents a methodology to calibrate the horizontal scales of theodolites and total with a gauge index table applying a method which is commonly used to perform the angle unit by the comparison of two circular divisions, the Rosette Method.

3. Methodology

The calibration method consists in a comparison between two circular divisions. In this case, one circular division is the theodolite horizontal scale and the other one is the index table. The theodolite angles are compared with the index table angle in all possible relative positions and the method is based on circle closing where the sum of partial angles errors is equal to zero. This methodology is self-consistent and can be realized without other standards as reference, allowing the simultaneous calibration of the theodolite scale and the index table.

The experimental setup was assembled on a rigid bench, as shown in figure 2. A rotary table was used to provide the relative position changing in the comparison system. The gauge index table was assembled on the rotary table and the theodolite was mounted onto this setup. Between the theodolite and the gauge index table was installed a levelling platform, which supported fixation and levelling. A collimator was mounted on the rigid bench and aligned relative to the theodolite telescope. This collimator emulates large distances and has a reticle which is used as a reference (target).

Figure 2. Measurement system assemble

As the system is composed by several manual mechanisms, it is necessary to be careful with instrument aligning and all other system components. The calibration started with index table rotation to an angle counterclockwise. After that, the theodolite telescope was returned clockwise to the same angle until the collimator reticle coincides with the theodolite reticle. The indicated value in the theodolite horizontal scale represents the index table error subtracted by the theodolite scale error and the alignment error between the two circular divisions ($d = \alpha - \beta - \gamma$). The procedure continued covering all possible relative positions until all data were obtained.

This comparative method generates a system of n^2 equations with $2(n - 1) + n = 3n - 2$ unknowns, where n is the number of measurement points of the instrument scale. Therefore, with more equations than unknowns, it is impossible to find exact solutions. The errors of the scale and the index table are estimated using the least squares method [5, 6].

The measured values (d_{ij}) are represented by the equation:

$$d_{ij} = \alpha_i - \beta_j - \gamma_k \tag{1}$$

With *i, j* e *k,* ranging from 0 to (n-1).

$$k = \begin{cases} i - j; & \text{if} \quad i - j \geq 0 \\ i - j - n; & \text{if} \quad i - j < 0 \end{cases} \tag{2}$$

Where:

d_{ij} = measured values;
α_i = error of the angles of indexing table;
β_i = errors of the angles of the horizontal scale;
γ_k = alignment error between the origins of the indexing table angles and horizontal scale.

The solution to α_i, β_j e γ_k are as follows:

$$\hat{\alpha}_i = n^{-1}.(\sum_{j=0}^{n-1} d_{ij} - \sum_{j-0}^{n-1} d_{0j}) \qquad \text{i = 1 a } (n\text{ - 1}) \tag{3}$$

$$\hat{\beta}_j = -n^{-1}.(\sum_{i=0}^{n-1} d_{ij} - \sum_{i=0}^{n-1} d_{i0}) \qquad \text{j = 1 a } (n\text{ - 1}) \tag{4}$$

$$\hat{\gamma}_k = -n^{-1}\sum_i d_{i,(i-k)} + 2n^{-2}\sum_i\sum_j d_{ij} - n^{-1}\sum_i d_{i0} - n^{-1}\sum_j d_{0j} \qquad k = 0 \text{ to } (n\text{ - 1}) \tag{5}$$

The estimative of standard deviation (S) of index table and horizontal scale accumulated angles errors of the theodolite is given by:

$$S = S_d.\sqrt{2n^{-1}} \tag{6}$$

S_d is given by:

$$S_d^{\ 2} = \frac{\sum_{i=0}\sum_{j=0}(\hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k - d_{ij})^2}{(n-1)(n-2)} \tag{7}$$

Where *(n – 1)(n - 2)* is the degrees of freedom number, in other words, the number of measurements (n^2) subtracted by the number of unknowns *(3n – 2)*.

The expanded uncertainty of the proposed method is given by the following equation:

$$U = k \cdot \sqrt{u_s^{\ 2} + u_r^{\ 2} + u_o^{\ 2}} \tag{8}$$

Where:

u_s= uncertainty due to the standard deviation;
u_r= uncertainty due to resolution of the instrument;
u_o= uncertainty due to operator error.

4 Results and discussion

Two theodolite Wild, T3 model with reading 0.1"; two total stations Berger, CST-202 models and CST-205 both with reading 1" were calibrated. The figure 3 shows the means of the theodolite Wild T3 indication errors, calibrated in 2008.

Systematic errors were found in relation to the eccentricity between the horizontal scale and the vertical axis. The measurement reproducibility was good and the differences between the two calibrations did not exceed 0.6". The calibrations were obtained in a time lapse of one month and showed no incompatibility results, as can be seen in table 1. The measurement uncertainties of these two calibrations were 0.9"and 0.7" for the first and second calibration in 2008, respectively.

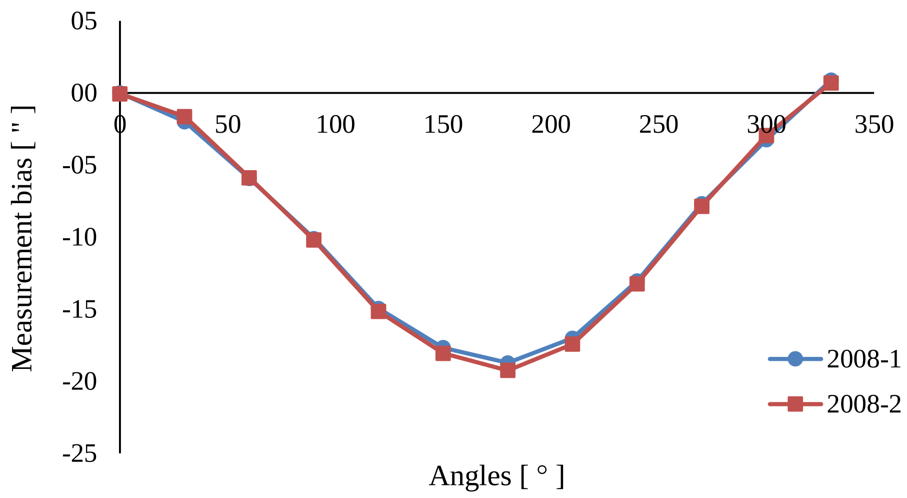

Figure 3. Calibration results of theodolite Wild T3 in 2008

Table 1. E_n results for Wild T3 calibration.

Angles (º)	E_n
0	0.0
30	0.1
60	0.3
90	0.0
120	0.1
150	0.2
180	0.3
210	0.5
240	0.4
270	0.2
300	0.1
330	0.3

The compatibility of calibration results were evaluated by the comparison of measurement errors and expanded uncertainty, pursuing the ISO/IEC 17043 [7]. The Normalized Error (E_n) was calculated using the following equation:

$$E_n = \left| \frac{V_{test} - V_{ref}}{\sqrt{U_{test}{}^2 + U_{ref}{}^2}} \right| \tag{9}$$

Where:

V_{test}= results of object;
V_{ref}= results of reference standard;

U_{test}= expanded uncertainty of object;
U_{ref}= expanded uncertainty of reference standard.

Two values were considered compatible when the $E_n \leq 1$.

Another theodolite Wild T3, was calibrated in 2010 and 2013. The results are shown in figure 4. Only the 60° point revealed an E_n out of the limits as can be seen in table 2. This incompatibility can be justified considering that theodolites are instruments used in the field and submitted to large variations in environmental conditions and collisions. The uncertainties obtained were 0.9" and 0.7" for calibrations done in 2010 and 2013, respectively.

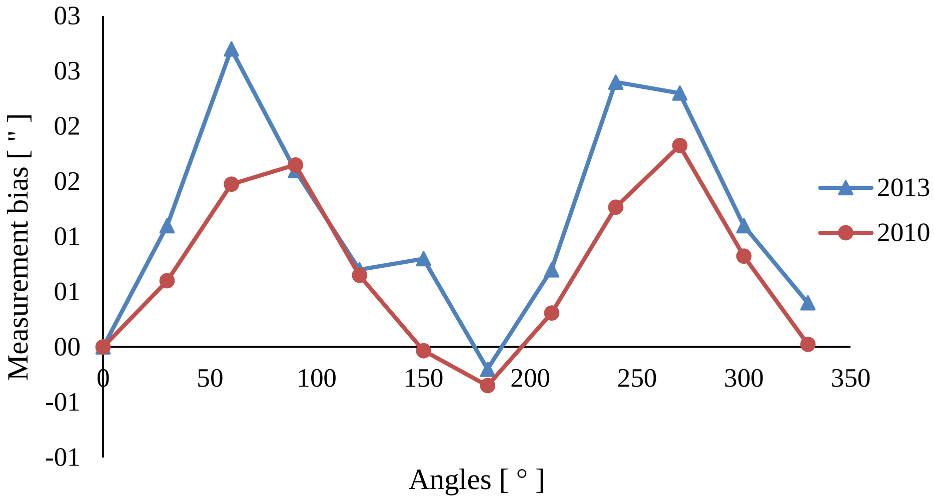

Figure 4. Calibration results of second theodolite Wild T3 in 2010 and 2013

Table 2. E_n results for second Wild T3 calibration.

Angles (°)	**E_n**
0	0.0
30	0.4
60	1.1
90	0.0
120	0.0
150	0.7
180	0.1
210	0.4
240	1.0
270	0.4
300	0.2
330	0.3

The calibrations of the total station CST Berger 202 was performed in three different conditions and the results are in figure 5. All measurements were performed in the same month in 2009, but the measurements 1 and 2 were carried out by different operators and the measurement 3 was performed using the vertical scale in 270°.

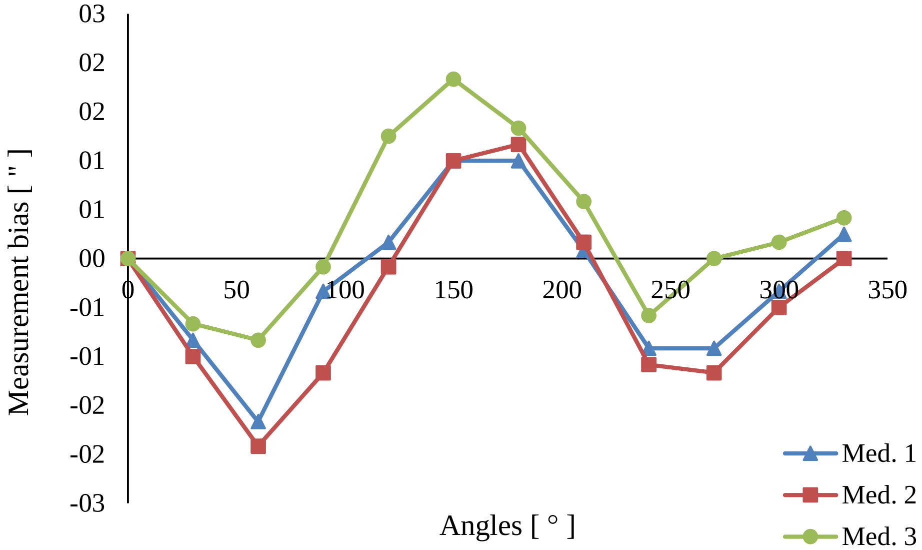

Figure 5. Calibrations results of total station CST Berger 202

Although the calibration has been performed in different conditions as described above, the results revealed good reproducibility considering the uncertainties of the method, which were limited by the instrument resolution (1"). There is no incompatibility between the results in these calibrations, as shown in table 3. The measurement uncertainties obtained for the three calibrations were respectively 1.8"; 1.7" and 1.7".

Table 3. E_n results for Berger CST 202 calibration.

Angles (°)	**E_n 09-1/09-2**	**E_n 9/1/2010**	**E_n 9/2/2010**
0	0.0	0.0	0.0
30	0.2	0.0	0.2
60	0.2	0.3	0.4
90	0.2	0.8	1.0
120	0.2	0.3	0.5
150	0.0	0.2	0.2
180	0.2	0.5	0.3
210	0.6	1.3	0.5
240	0.5	1.6	1.0
270	0.4	1.2	0.7
300	0.2	0.8	0.5
330	0.2	0.4	0.2

Another Berger total station was calibrated in 2009 and 2010, the model CST station 205. The results of the three calibrations are shown in figure 6. The two calibrations in 2009 were performed by different operators and in 2010 by the operator of first calibration in 2009. A comparison between calibrations was performed and the results are shown in table 4.

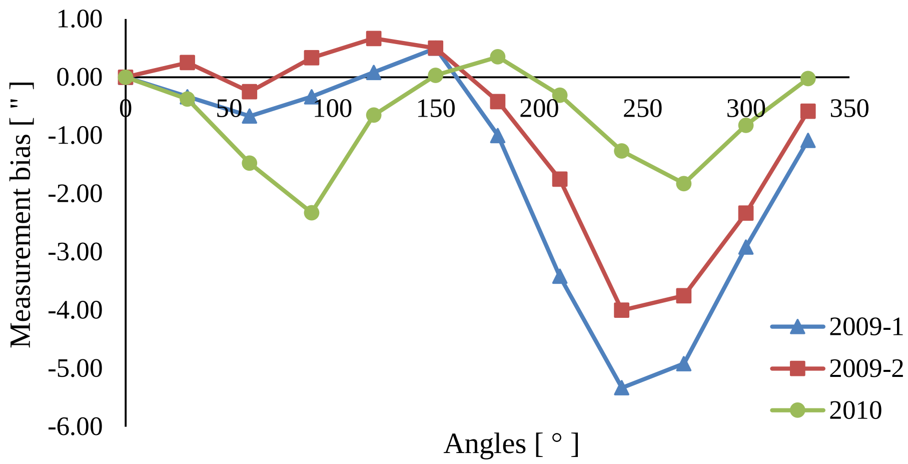

Figure 6. Calibrations results of total station CST Berger 205

Table 4. E_n results for Berger CST 205 calibration.

Angles (°)	E_n 1-2	E_n 1-3	E_n 2-3
0	0.0	0.0	0.0
30	0.1	0.1	0.1
60	0.1	0.3	0.5
90	0.3	0.1	0.5
120	0.1	0.4	0.6
150	0.0	0.3	0.3
180	0.1	0.1	0.1
210	0.0	0.2	0.2
240	0.1	0.1	0.2
270	0.1	0.4	0.5
300	0.1	0.2	0.3
330	0.1	0.1	0.2

In some points there are variation between the results of the 2010 and 2009. However a variation of approximately 4" still within the tolerance specified by the manufacturer [8]

Analyzing the comparison results, some incompatible values can be found, but may have occurred, in this case, an underestimation of measurement uncertainties.

5. Conclusion

The measurement results prove that the proposed method is suitable for performing calibration of theodolites and total stations horizontal scales. The measurement system is easily applicable and is not dependent on the type and size of the different equipment models available on the market.

The measuring uncertainty depends practically of two components: the instrument resolution and the repeatability inherent from data analysis method (least squares method). The operator influence was perceived mainly in the case of analogic theodolites.

References

[1] OIML 1994 Methods of reproduction of plane angle units *OIML Bulletin* **35** 23-29

[2] Filho W L O, Marques A and Vieira L H B 2011 Método Para Calibração Da Escala Angular Vertical De Teodolitos E Estações Totais Através De Medição Por Coordenadas *2º Cong. Int. de Metrologia Mec*

[3] Bručas D, Giniotis V, and Petroškevičius P 2006 The construction of the test bench for calibration of geodetic instruments Geodezija ir kartografija **32** 66–70

[4] Branco D F 1998 Calibração de Teodolitos *master's thesis from Pontifícia Universidade Católica do Rio de Janeiro*

[5] Vieira L H B 1996 Metrologia dimensional: Realização das unidades de ângulo plano *master's thesis from Universidade Federal Fluminense*

[6] Cook A H 1954 The calibration of circular scales and precision polygons *Br. J Appl. Phy* **5** 367-371

[7] Associação Brasileira de Normas Técnicas (ABNT) 2011 Avaliação de conformidade - Requisitos gerais para ensaios de proficiência. *ABNT NBR ISO/IEC* **17043**

[8] CST/Berger Electronic Total Station User's Guide n.d. Retrieved Jul 06, 2014 from http://www.southern-tool.com/store/media/cst/total-station/56-CST202.pdf