

Does the Finite Size of Electrons Affect Quantum Noise in Electronic Devices?

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Abstract. Quantum transport is commonly studied with the use of quasi-particle infinite-extended states. This leads to a powerful formalism, the scattering-states theory, able to capture in compact formulas quantities of interest, such as average current, noise, etc.. However, when investigating the spatial size-dependence of quasi-particle wave packets in quantum noise with exchange and tunneling, unexpected new terms appear in the quantum noise expression. For this purpose, the two particle transmission and reflection probabilities for two initial one-particle wave packets (with opposite central momentums) spatially localized at each side of a potential barrier are studied. After the interaction, each wave packet splits into a transmitted and a reflected component. It can be shown that the probability of detecting two (identically injected) electrons at the same side of the barrier is different from zero in very common (single or double barrier) scenarios. This originates an increase of quantum noise which cannot be obtained through the scattering states formalism.

1. Introduction

The scattering theory [1] is a useful formalism for quantum transport, allowing to obtain important quantities in a compact form (current, noise...). This theory uses as initial state for the electron quasi-particle infinite-extended states. Regarding this last feature, many reasonable questions appear: *Is it completely justified to choose as the initial state of an electron an infinite-extended state? Or at least, is it always mandatory? Does this initial state provide always the correct results? Can a finite spatially-extended initial state be chosen?* In fact, some relevant scattering experiments (Hong-Ou-Mandel kind) [2, 3] exhibit an increase of quantum noise compared to the expected results from the scattering-states theory. Here, we give theoretical support to these experiments, explaining its origin from the finite size of electrons.

With this motivation, we have recently discussed in Ref.[4] the size-dependence scattering probabilities when using as initial state a quasi-particle finite wave packet. We analyze scattering events when two quasi-particle wave packets impinge on a potential barrier from each side simultaneously (Fig.1) with the same energy but opposite momentum. We introduce the exchange interaction using as wave function the antisymmetric two-particle Slater determinant. Commonly, one would expect that after the interaction with the barrier (when the wave packet has split in a reflected and a transmitted part), only possibilities of finding both electrons at each side are possible $\mathcal{P}_{\mathcal{LR}}$ (Figs.1 (a) and (b)). The reason is that Pauli principle forbids two fermions being at the same position with the same state [5], and this is effectively the case when considering mono-energetic initial scattering states. However, if the initial states are not infinitely-extended, the reflected and transmitted states may not be equal and therefore,



the cases where both electrons are found at the same place $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ and $\mathcal{P}_{\mathcal{R}\mathcal{R}}$ (Fig.1 (c) and (d) respectively) become now possible.

In this conference, we go beyond the results of Ref.[4] and we explore which are the consequences of these new possibilities when analyzing quantum noise.

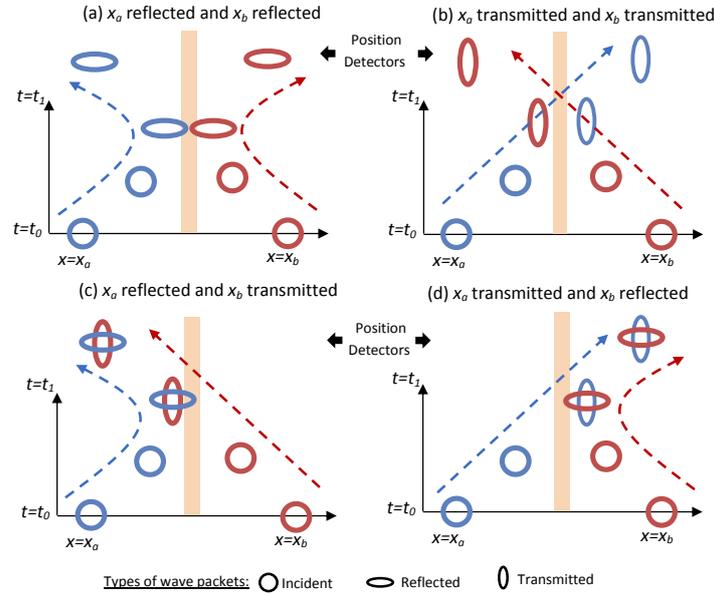


Figure 1. Two identically injected wave packets from the left x_a and from the right x_b of a scattering barrier. Solid regions represent the barrier region and shaded regions represent the particle detectors. (a) and (b) each particle is detected on a different side of the barrier at final time t_1 when the interaction with the barrier has almost finished. (c) and (d) both particles are detected on the same side of the barrier.

2. Two-particle probabilities

In order to study the size-dependence of quasi-particle wave packets in quantum noise with exchange and tunneling, we compute the probability of detecting two (quasi-)electrons at the same side of the barrier ($\mathcal{P}_{\mathcal{L}\mathcal{L}}$ and $\mathcal{P}_{\mathcal{R}\mathcal{R}}$) from the antisymmetric (Slater determinant) wave function (Φ) solution of the time-dependent Schrödinger equation in the configuration space. Contrarily to what is normally accepted [1], we notice (the explicit derivation can be seen in [4]) that in general, these probabilities ($\mathcal{P}_{\mathcal{L}\mathcal{L}}$ and $\mathcal{P}_{\mathcal{R}\mathcal{R}}$) are not zero:

$$\mathcal{P}_{\mathcal{L}\mathcal{L}} = \int_{-\infty}^0 dx_1 \int_{-\infty}^0 dx_2 |\Phi|^2 = R_a T_b - |I_{a,b}^{r,t}|^2, \quad (1)$$

where R_a and T_b are the a -wave packet reflection and b -wave packet transmission coefficients respectively. The last term $|I_{a,b}^{r,t}|^2$ accounts for the overlapping among the different wave packets. An important feature is that depending on this term, which in turn depends on the wave packet size, we can obtain two particular limits: I) the results for infinite-extended scattering-states if the overlapping is maximum $\mathcal{P}_{\mathcal{L}\mathcal{L}} = 0$ and II) the results for classical distinguishable particles if there is no overlapping $\mathcal{P}_{\mathcal{L}\mathcal{L}} = R_a T_b$ (orthogonal states) [4].

As an example, we present here the case of a single barrier using two Gaussian wave packets as the quasi-particle states, more cases are exposed in [4]. As we can see in Fig.2, in general the probability fluctuates, as said above, between $\mathcal{P}_{\mathcal{L}\mathcal{L}} = 0$ and $\mathcal{P}_{\mathcal{L}\mathcal{L}} = R_a T_b$.

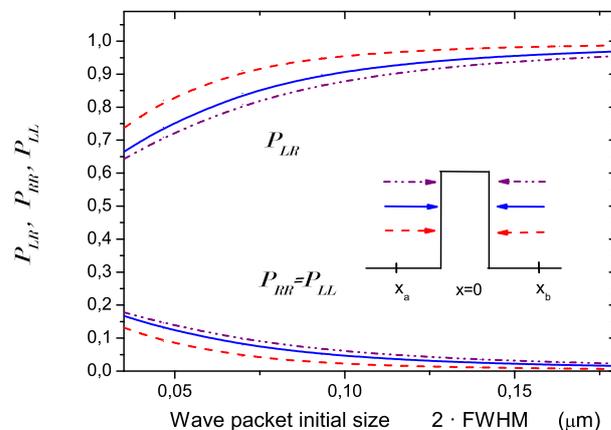


Figure 2. Probability of detecting two (quasi-)electrons at the same side of the barrier ($\mathcal{P}_{\mathcal{L}\mathcal{L}}$ and $\mathcal{P}_{\mathcal{R}\mathcal{R}}$) and one at each side ($\mathcal{P}_{\mathcal{L}\mathcal{R}}$) for three energies, $E_{T=1/2} = 45\text{meV}$ (blue solid line), $E_1 = 55\text{meV}$ and $E_3 = 35\text{meV}$, depending on the wave packet initial size, expressed through its Full Width at Half Maximum (FWHM). We observe that $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ or $\mathcal{P}_{\mathcal{R}\mathcal{R}}$ are maximum for small wave packets and zero for large ones, while $\mathcal{P}_{\mathcal{L}\mathcal{R}}$ reaches the maximum values for large wave packets.

3. Quantum noise in one- or two-particle processes

This simple analysis that we have developed has a quite surprising and far reaching consequence on the quantum noise. At high temperatures, due to the Fermi distribution (f_i , where i is the a or b reservoir), the processes which account for noise are mainly due to single or double scattering processes as the ones showed previously. The non-zero probability $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ leads to a modification of the conventional power spectral density $\langle S \rangle$ in these quasi-particles processes [1]. The (ensemble) power spectral density of the current fluctuations at zero frequency, $\langle S \rangle$ can be defined from:

$$\langle S \rangle = \lim_{t_d \rightarrow \infty} 2q^2 \frac{\langle N^2 \rangle_{t_d} - \langle N \rangle_{t_d}^2}{t_d}, \quad (2)$$

with

$$\langle N^2 \rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N, t_d) N^2, \quad \langle N \rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N, t_d) N, \quad (3)$$

where we define $P(N, t_d)$ as the probability that N particles have been transmitted through the barrier during the time t_d . We consider positive N when the transmission is from left to right and negative otherwise. No net transmission of particles means $N = 0$.

Finally, from Eq.(2) and table 1, we achieve straightforwardly the quantum noise expression:

$$\langle S \rangle = \frac{4q^2}{h} \int_0^\infty dE \{T[f_a(1-f_a) + f_b(1-f_b)] + T(1-T)(f_a - f_b)^2 + 2\mathcal{P}_{\mathcal{L}\mathcal{L}}f_a f_b\}. \quad (4)$$

The great difference between expression (4) and the well-known result of Büttiker [6] for a two-terminal mesoscopic device is the last term $2\mathcal{P}_{\mathcal{L}\mathcal{L}}f_a f_b$. As commented before, $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ is zero for infinite-extended wave packets (i.e. scattering-states). In this way, the Büttiker expression can be straightforwardly obtained. However, the soundness and importance of our expression relies in the fact that, in general, quantum noise is increased due to the new scattering possibilities reported in Fig.1.

a		Injection		No injection
		Transmitted	Reflected	
Injection	Transmitted	$\frac{P_{LR}}{2} f_a f_b$	$\mathcal{P}_{LL} f_a f_b$	$T(1 - f_a) f_b$
		0	-1	-1
Reflected	Reflected	$\mathcal{P}_{RR} f_a f_b$	$\frac{P_{LR}}{2} f_a f_b$	$R(1 - f_a) f_b$
		1	0	0
No injection		$T f_a (1 - f_b)$	$R f_a (1 - f_b)$	$(1 - f_a)(1 - f_b)$
		1	0	0

Table 1. Probability $P(N, t_d)$ (upper) that N (lower) electrons with energy E have effectively been transmitted from left to right reservoir during the time interval t_d .

4. Quantum noise in a multi-particle scenario

Regarding Eq.(2) one could call into question if the noise expression accomplishes the fluctuation-dissipation theorem, which states that at zero temperature noise is zero [7, 8]. At this temperature, due to the Fermi distribution, all possible states are occupied ($f_i = 1$). At first sight, our expression does not fulfil the theorem. Nevertheless, we must remember that at this temperature all the configuration space is fulfilled (below the Fermi energy) and then we cannot consider our one- or two-particle processes, we must consider a multi-particle scenario. As we show in [4], in the multi-particle scenario the new terms tend to zero as the configuration space is fulfilled, because overlapping among different wave packets is maximum and thus the fluctuation-dissipation theorem is perfectly accomplished.

5. Conclusions

In this conference we present the consequences on quantum noise of considering reasonable localized initial states and not the usual infinite-extended plane waves. For this purpose, we study the scattering possibilities (seen in Fig.1 and previously discussed in [4]) in a two-particle potential barrier scenario. It is shown that, contrarily to what it is assumed, if the transmitted and reflected components are not equal, both electrons can be found at the same side of the barrier (see Fig.2).

This fact carries an important consequence because there are new additional sources of noise. This feature is reflected in the quantum noise expression (2) with an additional term. We emphasize that within this expression, the well-known Büttiker expression is obtained as a particular limit when the wave packet size tends to infinite. This work give theoretical support to relevant and surprising experiments which were non well understood until now [2, 3].

Acknowledgments

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