Goldstone Mode Induced by Skyrmions and Collective Modes in Disordered Quantum Hall Crystals

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Abstract. We have proposed the effective Lagrangian density for the skyrmion-like solitons around the filling factor $\nu \sim 1$, and have introduced the massless gauge field (Goldstone mode) induced by the hedgehog-like solitons. We have discussed the skyrmion glassy behaviour.

1. Introduction

From field theory method [1,2], a new type of topological excitation, Skyrmion, is predicted to occur around filling factor, $1/m$, with odd integer m. Charged excitations in the quantum Hall ferromagnet are skyrmions. A skyrmion is a spin texture, rotating several spins simultaneously, and electric charge $\pm e/m$. A fractional charge excitation is possible, because it is a coherent excitation. A skyrmion is a topological soliton spread over two energy levels. Skyrmion excitations can be detected experimentally by measuring their spin. An experimental measurement [3] of the spin polarization has been made around $\nu = 1$. The spin polarization is observed both for $\nu \geq 1$ and for $\nu \leq 1$. It is a clear signal of skyrmion excitations. Recently the present author has proposed that in quasi-$2+1$ dimensional quantum antiferromagnet the doped hole induces magnetic-disordered hedgehog-like soliton, which is composed of the doped hole and the cloud of SU(2) Yang-Mills fields with the spin disorder around the hole, by using the gauge-invariant effective Lagrangian with spontaneous symmetry breaking [4-7]. In this study, extending the theoretical formula, we introduce the gauge-invariant effective Lagrangian density for the skyrmions around the filling factor $= 1$, and present the massless gauge field (Goldstone mode) induced by the Skyrmion. The massless gauge field introduces the long-range interaction, which will play an important role in the Skyrme crystal [8]. Taking into account the correlation effect among hedgehog-like solitons, we will discuss the mechanism of the long relaxation of the skyrmion-like soliton dynamics. In addition, we suggest the presence of the skyrmion glass.

2. A model system

The size of the hedgehog-like soliton is determined by a competition between Coulomb energy and the Zeeman energy. That is, when charge is spread over a wider domain of radius $\sim R_C$, the energy becomes smaller. On the other hand, the number of reversed spins is decreased, as increased the Zeeman energy. Taking account of that a hedgehog-like soliton in planar geometry is similar to the O(3) nonlinear sigma model (the SU(2) excitation) [1], we think that the perturbing gauge fields A_μ^a introduced by the hedgehog-like soliton have a local SU(2) symmetry. Furthermore, it is assumed that SU(2) gauge fields A_μ^a are spontaneously broken through the Anderson-Higgs mechanism in a way similar to the breaking of the quantum hall ferromagnetic symmetry around the hedgehog-like soliton. We set the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0,0,\mu\rangle$ of the Bose field ϕ_a in the Lagrangian density as follows,

$$\begin{aligned} L = & \frac{1}{2}\left(\partial_i S^j - g_1 \varepsilon_{i\hat{j}k} A_i^a S^k\right)^2 \\ & -\frac{1}{4}\left(\partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_3 \varepsilon_{abc} A_\mu^b A_\nu^c\right)^2 \\ & +\frac{1}{2}\left(\partial_\mu \phi_a - g_4 \varepsilon_{abc} A_\mu^b \phi_c\right)^2 - \lambda^2\left(\phi_a\phi_a - \mu^2\right)^2 . \end{aligned} \tag{1}$$

In order to introduce the hedgehog-like soliton with a size, R_C, we add the fourth term in Eq.(1), which depends on the Zeeman energy and Coulomb energy. $\hat{j}$ corresponds to the direction of the external magnetic field. When the soliton is formed, we set the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0,0,\mu\rangle$. On the other hand, when the anti-soliton is formed, we set the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0,0,-\mu\rangle$. Then, we can obtain the effective Lagrangian density, $\mathcal{L}_{eff}$, at small introducing of hedgehog-like solitons around the filling factor $\nu = 1$. The value, $\mu = \langle 0|\phi_3|0\rangle$, of the symmetry breaking depends strongly on the Zeeman energy and the Coulomb energy. That is, when the Zeeman energy becomes large, the value of μ becomes larger. On the other hand, when the Coulomb energy become larger, the value of μ becomes smaller.

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{1}{2}\left(\partial_i S^j - g_1 \varepsilon_{i\hat{j}k} A_i^a S^k\right)^2 \\ & -\frac{1}{4}\left(\partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_3 \varepsilon_{abc} A_\mu^b A_\nu^c\right)^2 \\ & +\frac{1}{2}\left(\partial_\mu \phi_a - g_4 \varepsilon_{abc} A_\mu^b \phi_c\right)^2 \\ & +\frac{1}{2} m_1^2 \left[(A_\mu^1)^2 + (A_\mu^2)^2\right] + m_1\left[A_\mu^1 \partial_\mu \phi_2 - A_\mu^2 \partial_\mu \phi_1\right] \\ & + g_4 m_1 \left\{\phi_4\left[(A_\mu^1)^2 + (A_\mu^2)^2\right] - A_\mu^3\left[\phi_1 A_\mu^1 + \phi_2 A_\mu^2\right]\right\} \\ & -\frac{m_2^2}{2}(\phi_4)^2 - \frac{m_2^2 g_4}{2m_1}\phi_4(\phi_a)^2 - \frac{m_2^2 g_4^2}{8m_1^2}(\phi_a\phi_a)^2, \end{aligned} \tag{2}$$

where S^j is the spin parameter, $m_1 = \mu \cdot g_4$, and $m_2 = 2(2)^{1/2}\lambda \cdot \mu$. The effective Lagrangian describes two massive vector fields A_μ^1 and A_μ^2, and one massless $U(1)$ gauge field A_μ^3. Because masses of A_μ^1 and A_μ^2 are formed through the Higgs mechanism by introducing the hedgehog-like soliton, the fields A_μ^1 and A_μ^2 exist around the hedgehog-like soliton within the length of $\sim 1/m_2 \equiv R_C$. From the first term in Eq.(2), the spin component parallel to $\hat{j}$, which corresponds to the direction of the external magnetic field, is induced strongly around the hedgehog-like soliton. The density $\rho_{\hat{j}}$ for the spin component parallel to $\hat{j}$ around the hedgehog-like soliton is given approximately by

$$\rho_{\hat{j}}(r) \sim \frac{1}{\pi}\frac{R_C^2}{\left[r^2 + R_C^2\right]^2} \cdot \frac{R_C^2}{\left[r^2 + R_C^2\right]} \tag{3}$$

For the effective gyromagnetic ratio, g, $\ll 1$, the size, $R_C \sim 1/m_1$, of the hedgehog-like soliton is estimated by Sondhi et al. [2] as

$$\left(\frac{R_C}{l}\right)^3 \propto \frac{l}{a}\left(g \mid |\ln g|\right)^{-1} \tag{4}$$

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where l and a are the Landau length and Bohr radius, respectively. Now we can define the topological number q for excited hedgehog-like solitons as follows,

$$q = \frac{1}{2\pi}\int_{\sum} \mathrm{d}S_{\mu\nu}\left(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\right), \tag{5}$$

where $\sum$ is a sphere, whose radius is larger than $R_C \sim 1/m_2$. If a sphere $\sum$ surrounds completely one soliton, whose center position is r_i, the value of q_i is $+1$. If a sphere $\sum$ surrounds completely one anti-soliton, whose center position is r_i, the value of q_i is -1. When the hedgehog-like soliton is located at the position r_i and $|r - r_i| \ll 1/m_2 \sim R_C$ is assumed, the Goldstone gauge field $A^3_\mu(r, r_i)$ at the position r is represented as $A^3_\mu(r, r_i) \propto q_i/|r - r_i|$. Thus we can introduce the interaction, which is mediated by the massless gauge field (Goldstone mode), between the hedgehog-like solitons at positions r_i and r_j as $V_{ij} \propto q_i \cdot q_j/|r_i - r_j|$, which plays an important role in the Skyrme crystal [8]. It has been suggested [9,10] that the effect of disorder is important on the ground state of a two-dimensional electron gas in the quantum Hall regime at filling factors slightly deviating from unity. This suggests strongly that the aggregation of hedgehog-like solitons is mulch related to the long relaxation time of the spin. In addition, stripe-shaped magnetic domains has been observed [11] . The stripe-shape of the domain means that long-range frustration plays an important role on the domain formation [12] . In order to discuss the spin dynamics, we envisage an effective hamiltonian, H, for the hedgehog-like soliton, $O(r_{\tilde{i}})$, which is introduced in eq.(2),

$$\begin{aligned} H &= -J\sum_{<\tilde{i},\tilde{j}>}\frac{S_{\tilde{i}\cdot\tilde{j}}}{|S_{\tilde{i}}||S_{\tilde{j}}|}O(r_{\tilde{i}})\cdot O(r_{\tilde{j}}) \\ &\quad +\frac{1}{2}K\sum_{\tilde{i}\neq\tilde{j}}\frac{O(r_{\tilde{i}})\cdot O(r_{\tilde{j}})}{|r_{\tilde{i}} - r_{\tilde{j}}|} \end{aligned} \tag{6}$$

with $J > K > 0$ and the first sum taken only over nearest neighbor(the distance between each magnetic soliton is $\leq 2R_c$) and the second taken over all pair($\tilde{i} \neq \tilde{j}$ means $|r_{\tilde{i}} - r_{\tilde{j}}| \gg 2R_c$). $S_{\tilde{i}} \equiv \sum_{i\in\pi R_c^2} S_i$. That is, $S_{\tilde{i}}$ is the summation of the ferromagnetic spin, S_i, within $\sim \pi R_c^2(\tilde{i})$ around the hedgehog-like soliton at the site $r_{\tilde{i}}$. $S_{\tilde{i}}$ represents the effective spin of the soliton $O(r_{\tilde{i}})$. The first term corresponds to short-range ferromagnetic ordering interaction and the second corresponds to long-range frustration. If g_3 in eq.(2) is assumed to be equal to $\sqrt{\pi/K}$, where K is the long-range interaction constant in the effective Hamiltonian, eq.(9). Taking into account this model along lines originally suggested by Chayes et al. [13], there are two emergent length scales that are long compared to R_c. The first of these is the correlation length ξ_0 which governs the fluctuations in the absence of frustration. The second length, $L_0 \propto \xi_0^{-1} \cdot K^{-\frac{1}{2}}$, structure is broken up due to frustration, i.e. the frustration-limited domain size [14]. The characteristic length ξ_0 and L_0 are dependence on temperature and number of hedgehog-like solitons. If $O(r_{\tilde{i}})$ is the local order parameter for the ferromagnetic soliton, this means that

$$\begin{aligned} < O(r_{\tilde{i}})\cdot O(r_{\tilde{j}}) > &\propto R_c|r_{\tilde{i}} - r_{\tilde{j}}|^{-1}\exp[-|r_{\tilde{i}} - r_{\tilde{j}}|/\xi_0] \\ &\quad + m^2 \end{aligned}$$

for $\xi_0 \ll |r_{\tilde{i}} - r_{\tilde{j}}| \ll L_0$,

$$< O(r_{\tilde{i}} \cdot O(r_{\tilde{j}}) > \propto L_0 m^2 |r_{\tilde{i}} - r_{\tilde{j}}|^{-1}\exp[-|r_{\tilde{i}} - r_{\tilde{j}}|/L_0]$$

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for $L_0 \ll |r_{\tilde{i}} - r_{\tilde{j}}|$, where m is the expectation value of the local order parameter. Then we write the free energy F of the ferromagnetic domain of characteristic size L as

$$F = \delta L^2 - \phi L^3 + S(L)L^3.$$

The coefficient ϕ is a measure of the free energy density gained by aggregation of the hedgehog-like solitons. The domain-wall surface tension $\delta \propto (\xi_0)^{-2}$ is positive. The strain coefficient $S(L)$ is $\sim S_0 L^2$ for L small compared to the radius of curvature of the ideal space. The relaxation of a local order parameter in a finite system occurs most efficiently through the creation and movement of defect walls. The creation of such a defect wall is proportional to δL^2. In this system, domains are special because reduction in the range of the local order parameter reduces the strain. So we expect the collective activation free energy, $E_{col}(L) \propto \delta L^2 - mS_0 L^5$.

Now, we shall consider the long relaxation time, which can be described in terms of a normalized relaxation function, $f(t)$, for which $f(0) = 1$. The collective behavior can be described in terms of domains with a size distribution $\rho(L)$ and a most probable size, $L = L_0 \gg R_c$. We introduce the relaxation function within each domain as exponential with a decay-time dependent upon an activation free energy $E_{col}(L)$ [15] as follows,

$$f(t) = \int_0^\infty dL L^2 \rho(L) \exp\{-(t/\tau_\infty) \exp[-E_{col}(L)/T]\}, \tag{7}$$

where τ_∞ is taken to be a T-independent parameter.

3. Conclusion

We have introduced the gauge-invariant effective Lagrangian density for the skyrmion-like solitons around the filling factor $\nu \sim 1$ and have introduced the massless gauge field(Goldstone modes). We have discussed the mechanism of the long relaxation of skyrmion-like solitons.

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