

Confined acoustic phonon-mediated spin relaxation in a two-dimensional quantum dot in the presence of perpendicular magnetic field

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Abstract. The spin-relaxation time due to the electron-acoustic phonon scattering in GaAs quantum dots is studied after the exact diagonalization of the electron Hamiltonian with the spin-orbit coupling. It has been shown that in comparison with flexural phonons, the electron coupling with the dilatational phonons causes 3 orders faster spin relaxation. We have found that the relaxation rate of the spin-flip is an order of magnitude smaller than that of the spin-conserving.

1. Introduction

Manipulation and readout of spins in nanostructures is a promising way not only for developing spin based quantum logic gates, spin memory devices for various quantum information processing applications, but also because they have greatly improved our understanding of the role of different spin-dependent interaction mechanisms [1,2]. The spin-orbit coupling, which is one of the main mechanisms of spin relaxation, is a relevant intrinsic interaction in nonmagnetic semiconductor nanostructures. In this paper we have considered spin-relaxation in a two dimensional parabolically confined quantum dot embedded inside a free-standing semiconductor slab. The relaxation occurs via acoustic phonon-mediated scattering processes in the presence of both perpendicular magnetic field and spin-orbit coupling, represented by the Rashba and Dresselhaus terms. Our study has been motivated by a recent experiment, where the long spin relaxation times (30ns) have been measured in GaAs quantum dots [3].

2. Theory

We consider a QD system, where the QD is confined by a parabolic potential with frequency ω_0 in the quantum well of width a . A magnetic field \mathbf{B} is applied perpendicularly to the surface of the quantum well. The total Hamiltonian is given by

$$H = H_0 + \alpha(\sigma_x P_y - \sigma_y P_x) + \beta(-\sigma_x P_x + \sigma_y P_y) + H_{ph} + H_{e-ph} \quad (1)$$

where $H_0 = \mathbf{P}^2/2m^* + m^*\omega_0^2(x^2 + y^2)/2 + g^*\mu_B B\sigma_z/2$ is electron Hamiltonian without the spin-orbit coupling, second and third terms represent the Rashba and Dresselhaus spin-orbit couplings, respectively, $\mathbf{P} = -i\hbar\nabla + (e/c)\mathbf{A}$ with $\mathbf{A} = (B/2)(-y, x, 0)$, m^* is the electron effective mass, $g^*\mu_B B\sigma_z/2$ is the Zeeman energy, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ represents the Pauli matrices.



Recently, the freestanding quantum well structures have been developed using advanced nanofabrication techniques, which have provided the possibility of observation of effect of confinement of acoustic phonon modes in a quantum plane. In elastic continuum model the confined acoustic phonon modes in freestanding quantum wells have been classified, according to their spatial symmetries, as shear, dilatational, and flexural modes. The acoustic phonon Hamiltonian in free standing quantum well (FSQW), (H_{ph} in Eq. (1)), has the standard form $H_{ph} = \sum_q \hbar \omega_q b_q^\dagger b_q$, where the quantum number $q = (\beta, n, \mathbf{q})$ includes the phonon symmetry β (β takes values from the set dilatational, flexural, shear), the mode number n , and the in plane wave vector \mathbf{q} . The most important mechanism of the interaction between electrons and acoustic vibrations in semiconductors is via the deformation potential coupling. In a FSQW structure, the electron-confined acoustic phonon interaction Hamiltonian is given by

$$H_{el-ph} = \sum_{\beta, n, \mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{r}} \Gamma_\beta(\mathbf{q}, n, z) [b_{\beta, n, \mathbf{q}} + b_{\beta, n, -\mathbf{q}}^\dagger].$$

The functions $\Gamma_d(\mathbf{q}, n, z)$ and $\Gamma_f(\mathbf{q}, n, z)$, describing the intensity of the electron interactions with dilatational and flexural phonon modes (shear waves don't interact with electrons through the deformation potential) given in [4].

The wave functions, $\Psi_{n,l,\sigma}$, and the energy spectrum, $E_{n,l,\sigma}$, of the Hamiltonian H_0 can be written as:

$$\Psi_{v,l,\sigma} = \sqrt{v!/\pi R^2(v+|l|)!} \zeta^{|l|} e^{-\zeta^2/2} e^{il\theta} L_v^{|l|}(\zeta^2) \chi_\sigma,$$

$$E_{v,l,\sigma} = \hbar \Omega(2v + |l| + 1) - \hbar \omega_c l + g^* \mu_B B \sigma_z / 2,$$

where χ_σ represents the eigenfunction of σ_z . $v = 0, 1, 2, \dots$, $l = 0, \pm 1, \pm 2, \dots$ are quantum numbers, $\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$, $\omega_c = |e|B/m^*c$, ζ denotes the scaled polar coordinate ($\zeta = \rho/R$, with $R = \sqrt{\hbar/m^*\Omega}$), $L_n^{|l|}(x)$ is the generalized Laguerre polynomial. The spin-orbit coupling can be treated as a small perturbation to the discrete orbital energy level spectrum in the QD. As a result, the electron spin states are admixtures of spin and orbital states. Thus, in first-order perturbation theory, we get $\varepsilon = E_{n,l,\sigma}$, and

$$\Phi_{0,0,\uparrow} = \Psi_{0,0,\uparrow} + \sum_{n,l,\sigma} \frac{\langle \Psi_{n,l,\downarrow} | H_{SO}^{D(R)} | \Psi_{0,0,\uparrow} \rangle}{E_{0,0,\uparrow} - E_{n,l,\sigma}} \Psi_{n,l,\downarrow}, \quad \Phi_{0,0,\downarrow} = \Psi_{0,0,\downarrow} + \sum_{n,l,\sigma} \frac{\langle \Psi_{n,l,\uparrow} | H_{SO}^{D(R)} | \Psi_{0,0,\downarrow} \rangle}{E_{0,0,\downarrow} - E_{n,l,\sigma}} \Psi_{n,l,\uparrow},$$

$$\Phi_{0,1,\uparrow} = \Psi_{0,1,\uparrow} + \sum_{n,l,\sigma} \frac{\langle \Psi_{n,l,\downarrow} | H_{SO}^{D(R)} | \Psi_{0,1,\uparrow} \rangle}{E_{0,1,\uparrow} - E_{n,l,\sigma}} \Psi_{n,l,\downarrow}, \quad \Phi_{0,1,\downarrow} = \Psi_{0,1,\downarrow} + \sum_{n,l,\sigma} \frac{\langle \Psi_{n,l,\uparrow} | H_{SO}^{D(R)} | \Psi_{0,1,\downarrow} \rangle}{E_{0,1,\downarrow} - E_{n,l,\sigma}} \Psi_{n,l,\uparrow}.$$

As spin-orbit couplings mix the spin states with different orientations in the Zeeman sublevels, the spin relaxation is possible via electron-phonon interaction. Using Fermi's golden rule for the transition from the initial state i to the final state f ,

$$W_{i \rightarrow f}^\pm = \frac{2\pi}{\hbar} |\langle f | H_{el-ph} | i \rangle|^2 \delta(E_f - E_i \pm \hbar \omega_n^\beta(\mathbf{q})),$$

and the expressions for the four lowest levels with the Dresselhaus (H_{SO}^D) and Rashba (H_{SO}^R) SO couplings, we get the relaxation rate assisted with the dilatational and flexural acoustic phonon scattering as follows:

$$W_{i \rightarrow f}^{\pm}(\beta, n, \mathbf{q}) = \frac{\pi E_d^2 |F_{\beta, n}|^2}{A \rho \omega_n^{\beta}(\mathbf{q})} \left| \langle \Psi_{n, l, \downarrow} | H_{SO}^{D(R)} | \Psi_{m, p, \uparrow} \rangle \right|^2 \left(N_n^{\beta}(\mathbf{q}) + \frac{1}{2} \pm \frac{1}{2} \right) (q_{t, n}^2 - q^2)^2 (q_{l, n}^2 + q^2)^2 \left(tsc1_{\beta} \left(\frac{q_{t, n} a}{2} \right) \right)^2 \left| \int_0^a \varphi_{s'}(z) tsc2_{\beta}(q_{l, n} z) \varphi_s(z) dz \right|^2 \delta(E_f - E_i \pm \hbar \omega_n^{\beta}(\mathbf{q}))$$

where + (-) is used to denote emission (absorption), β distinguishes the dilatational (d) and flexural (f) confined acoustic modes, the label $n = 1, 2, 3, \dots$ is the index for different branches of confined modes, $N_n^{\beta}(\mathbf{q})$ is the Bose distribution of phonon with mode (β, n, \mathbf{q}) , E_d is the acoustic deformation potential constant, ρ is the density of the material, and A is the area of the FSQW structure. The symbols representing trigonometric functions are $tsc1_{\beta} = \sin$ and $tsc2_{\beta} = \cos$ if $\beta = d$ (dilatational) and $tsc1_{\beta} = \cos$ and $tsc2_{\beta} = \sin$ if $\beta = f$ (flexural). The function $F_{\beta, n}$ is the normalization constant defined in terms of $q_{l, n}$ and $q_{t, n}$ [4], $\varphi_s(z)$ are eigenfunctions of the 1D Schrodinger equation (in calculations the Fang–Howard wavefunction [5] is used). The spin relaxation rate, Γ (time, τ), can be determined by

$$\Gamma = 1/\tau = \sum_i f_0(E_i) \sum_f (1 - f_0(E_f)) \sum_{\pm} \sum_{\beta, n, \mathbf{q}} W_{i \rightarrow f}^{\pm}(\beta, n, \mathbf{q}).$$

3. Numerical results and discussion

In figure 1 we present the spin-flip relaxation rate as a function of magnetic field due to Rashba spin-orbit coupling mediated by the electron interaction with both dilatational and flexural acoustic phonon modes. As we can see, the relaxation rate rises with the increase of the intensity of the magnetic field. This feature is quite opposite to the bulk, two- and one- dimensional cases where the spin relaxation rate always decreases with the magnetic field. This is because in the dot case there are only discrete energy levels and the magnetic field helps to increase the spin-flip scattering.

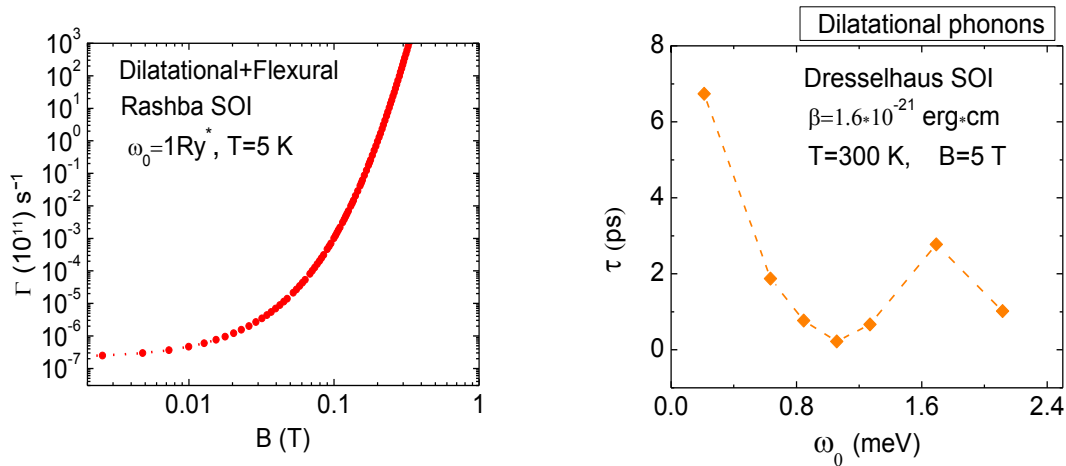


Figure 1. Spin relaxation rate as a function of a magnetic field.

Figure 2. Spin relaxation time as a function of the parabolic confinement potential frequency.

The calculated spin-flip relaxation time from parabolic confinement potential frequency is plotted in figure 2. Dots represent calculated values. Only a few values of the relaxation time have been calculated, which don't allow us to determine the frequency behavior of the spin relaxation. The spin

relaxation time as a function of temperature for the spin flip transitions caused by Dresselhaus spin-orbit interaction is presented in figure 3 by considering the electron-dilatational acoustic phonon (figure 3a) and the electron-flexural acoustic phonon (figure 3b) interactions. It is clearly seen that in comparison with flexural phonons, the electron coupling with the dilatational phonons causes 3 orders faster spin relaxation.

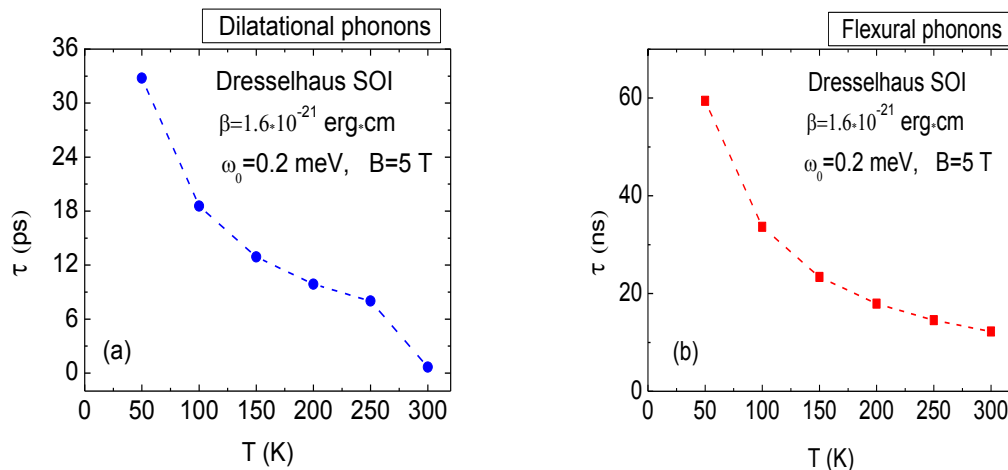


Figure 3. Spin relaxation time as a function of temperature for the spin flip transitions caused by Dresselhaus spin-orbit interaction: (a) via electron-dilatational acoustic phonon scattering, (b) via electron-flexural acoustic phonon scattering.

One can also notice from the figures 3a and 3b that with the increase of temperature, the spin relaxation time decreases. These features can be understood as follows: with the increase of the temperature, the average phonon number gets larger. This enhances the electron-phonon scattering and leads to the larger transition probability. We have found that the spin-flip relaxation rate is one order of magnitude smaller than that of the spin-conserving scattering.

Acknowledgements

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