

## Estimation of droplet charge forming out of an electrified ligament in the presence of a uniform electric field

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**Abstract.** The charge on a liquid droplet is a critical parameter that needs to be determined to accurately predict the behaviour of the droplet in many electrostatic applications, for example, electrostatic painting and ink-jet printing. The charge depends on many factors, such as the liquid conductivity, droplet and ligament radii, ligament length, droplet shape, electric field intensity, space charge, the presence of adjacent ligaments and previously formed droplets. In this paper, a 2D axisymmetric model is presented which can be used to predict the electric charge on a conductive spherical droplet ejected from a single ligament directly supplied with high voltage. It was found that the droplet charging levels for the case of isolated electrified ligaments are as much as 60 times higher than that in the case of ligaments connected to a planar high voltage electrode. It is suggested that practical atomization systems lie somewhere between these two extremes and that a better model was achieved by developing a 3D approximation of a linear array of ligaments connected to an electrode having variable width. The effect on droplet charge and its radius was estimated for several cases of different boundary conditions.

### 1. Introduction

Much prior research has been completed to investigate the stability and disintegration of a liquid jet into a stream of discrete droplets in a uniform electric field [1]-[7], but relatively little work has been done on exploring the correlation between the droplet charge and droplet radius. Toljic et al. [8] investigated the charge to radius dependency for conductive particles atomized in an electric field between planar electrodes. The results showed that the ligament length has a strong effect on the particle charge to radius dependency. They found that the radius exponent is equal to two when the particle is in direct contact with a planar electrode, and decreases rapidly as the ligament length increases approaching a limit of 1.1. Osman et al. [9] studied numerically the effect of the presence of previously formed droplets, the presence of adjacent ligaments as well as the ligament radius and length on the predicted droplet charging levels for ligaments formed from a planar high voltage electrode. The effect of the ligament radius on the charging level was found to be significant with larger charge resulting when the ligament radius is smaller than the droplet radius. This is important because in practice a narrow necking normally occurs prior to the droplet ejection. They also investigated the dynamic modeling of droplet formation at different inlet velocities. The results showed that the droplet shape prior to detachment is actually a prolate ellipsoid and that the spherical model underestimates the actual charge. Distortion of the droplet shape agrees with previous studies of droplet deformation in an external electric field [10].

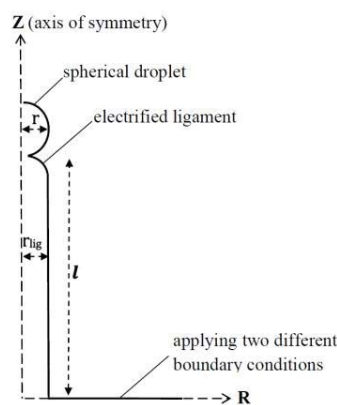
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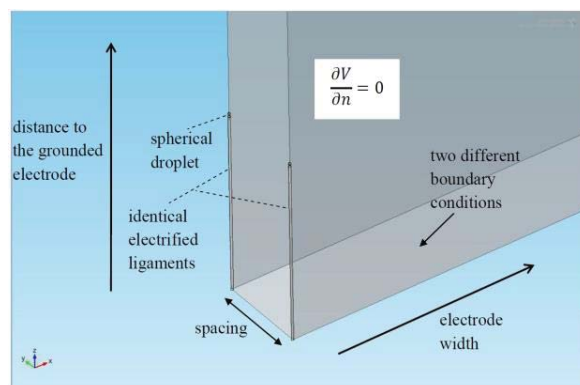
In this paper, a 2D axisymmetric numerical model was created using COMSOL, a Finite Element commercial software, to calculate the droplet charge formed from the end of a variable length ligament. Two cases have been compared: a ligament connected to a planar high voltage electrode and an isolated ligament supplied with a high voltage. In practice, the ligaments in paint sprayers are formed off the lip of a rotating cup, where the edge thickness is much larger (up to 10 times) than the ligament radius. This situation was simulated by expanding the model into a 3D linear array of ligaments forming from an electrode of variable width. With this, the droplet charge could be determined both for the single ligaments and for cases of an array of adjacent electrified ligaments.

## 2. Numerical model

The 2D axisymmetric stationary model consisted of a grounded electrode located 25 cm from the high voltage electrode, a cylindrical conductive liquid ligament of variable length and ejected spherical droplets with different radii. Two boundary conditions were considered. First, the ligament was assumed to be electrified through contact with a high voltage planar electrode. Then, the planar electrode was removed and the voltage applied directly to the ligament (Figure 1). The model was then extended to 3D (Figure 2) to enable the investigation of the effect of a linear array of adjacent ligaments on the droplet charge levels by applying the symmetry boundary conditions along the walls of the computational domain. In this latter model, the width of the lower electrode was also varied.



**Figure 1.** 2D model for a single ligament with two BCs: a) applied high voltage (90 kV) and b)  $dV/dn=0$  for mirror plane.



**Figure 2.** 3D model for an array of electrified ligaments with two BCs: a) applied high voltage (90 kV) and b)  $dV/dn=0$  for mirror plane.

## 3. Numerical results and discussion

### 3.1. Droplet charge for a single ligament

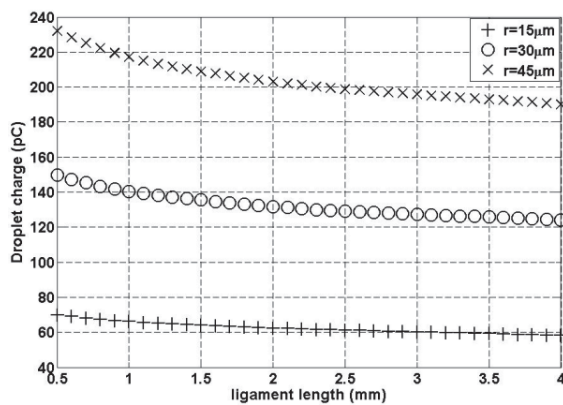
Osman et al. [9] previously estimated the electric charge on the surface of a spherical droplet, which is attached to a single ligament connected to a planar high voltage electrode in the 2D axisymmetric model. The simulation results were calculated for different droplet radii ( $r=15, 30$  and  $45 \mu\text{m}$ ) and ligament lengths (from 0 to 4 mm), and it was initially assumed that in all cases the droplet and ligament radii are equal. Their results demonstrated that the charging level increases with the droplet size and increases monotonically, as the ligament length increases. When a single spherical droplet is in direct contact with the surface of the planar electrode, it was confirmed that the charge magnitude coefficient and the radius exponent agree well with Félici's [11] predicted value, confirming the validity of the model.

In this section, the planar high voltage electrode was removed so that this case represents a single ligament-droplet system directly energized with high voltage. The results in this case reveal that the droplet charge also increases with radius, but in the order of 55 to 60 times higher than reported earlier [9] for the case, where the ligament was connected to a planar electrode. Also, the results show an opposite

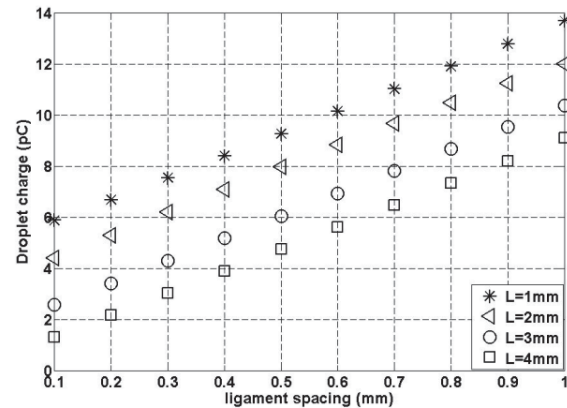
trend, where the charge slightly decreases, as the ligament length increases (Figure 3). These results are somewhat surprising as they show such large differences in the level of charge and the influence of ligament length. However, it appears that the effect of the planar electrode is to partially screen the electric field at the end of the ligament, and this influence decreases as the ligament gets longer. On the other hand, for the case where the ligament stands alone, the field strength is much stronger at the end of the ligament and it decreases slightly with length, as more electric flux is attracted to the surface of the ligament.

### 3.2. Linear array of electrified ligaments

This section presents a study to examine the effect of a linear array of ligaments on the level of charging over the surface of a spherical droplet of radius equal to  $15\ \mu\text{m}$  in absence of the planar electrode. A 3D model described in [9] was modified such that a condition of zero normal partial derivative was applied to the lower electrode and walls of the computational domain, which is equivalent to the symmetry boundary conditions. This assumption creates a model, where a single ligament is accompanied on both sides by an infinite row of identical electrified ligaments. The droplet and ligament radii are assumed to be equal. The charge magnitudes over the droplet surface have been calculated for different values of the spacing between adjacent ligaments and for different ligament lengths (Figure 4).

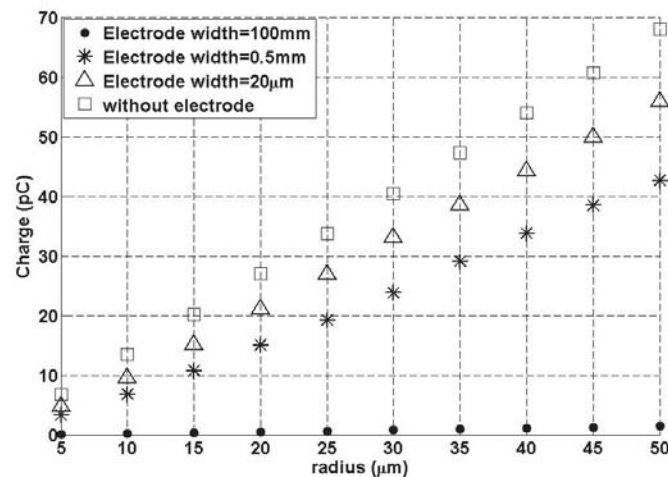


**Figure 3.** The droplet charge for a single ligament in absence of the planar electrode assuming three different droplet sizes.



**Figure 4.** The charge for a  $15\ \mu\text{m}$  droplet formed from an array of ligaments of different lengths in absence of the planar electrode.

Similarly to the previous results presented in [9], it was found that the charging level increases, as the spacing between ligaments of fixed length increases, but for the longer ligaments, the droplet charge decreases compared with the shorter ones at a given spacing (Figure 4). A linear array of ligaments of fixed length of 1 mm and spacing of 0.5 mm was then assumed for ten values of droplet radius and the effect of the electrode width on the droplet charging level determined. The electrode width was varied from one extreme condition that of a planar electrode, approximated as a 100 mm strip, to the case of no electrode. These results are shown in Figure 5. For the case of a planar electrode, the results show that the estimated droplet charge increases monotonically with the droplet radius. As the electrode width shrinks to 0.5 mm, the charge magnitude increases by about a factor of 30 and for an even narrower width of  $20\ \mu\text{m}$ , where the electrode width is in the same order of the droplet radius, charge is approximately 40 times larger. Finally in the extreme case when the electrode is absent and the ligaments become isolated, the charge magnitude was found to increase up to 50 times the case of planar electrode. These results also enable the value of the dependency of charge on radius to be estimated and show an exponent value of about 1.1 which agrees well with the value reported in [9].



**Figure 5.** The droplet charge levels for an array of ligaments of 1 mm length, spaced 0.5 mm apart assuming variable electrode widths

### Conclusions

This study sheds some light on the design parameters that allow the charge level to be predicted for different droplet sizes and ligament lengths. A 2D axisymmetric model was generated for predicting the charge magnitude by assuming different boundary conditions. When the electrode is removed and the ligament itself forms the high voltage electrode, it was found that the droplet charge greatly increases in magnitude by a factor of up to 60 times. Also, a 3D model has been created to examine the effect of the spacing between a linear array of identical ligaments on the droplet charge in the absence of the planar electrode. It was found that the droplet charge increases, as the spacing between adjacent ligaments increases. Also it was demonstrated that the electrode width strongly affects the droplet charge. In particular, as the width of the electrode decreases, the charges on the droplets in the linear array may increase by a factor of up to 50. This large influence of the boundary conditions suggests that the next stage of the modelling, currently underway, is to include the cylindrical geometry of the sprayer as well as the other important parameters particularly the space charge due to previously formed droplets.

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### References

- [1] Macky W A 1931 *Proc. R. Soc. Lond.* **133** 565-87
- [2] Pfeifer R and Hendricks C D 1967 *Phys. Fluids*. **10** 2149-54
- [3] Schweizer J and Hanson D 1971 *J. Colloid Interface Sci.* **35** 417-23
- [4] Schneider J M, Lindblad N R, Hendricks Jr C D and Crowley J M 1967 *J. Appl. Phys.* **38** 2599-605
- [5] Wong M C Y and Shrimpton J S 2004 *IEEE Trans. on Dielectr. and Electr. Insul.* **11** 362-8
- [6] Lopez-Herrera J M and Ganán-Calvo A M 2004 *J. Fluid Mech.* **501** 303-26
- [7] Zhakin A I and Belov P A 2013 *Surface Eng. and Appl. Electrochem.* **49** 205-14
- [8] Toljic N, Castle G S P and Adamiak K 2010 *J. Electrostatics* **68** 57-63
- [9] Osman H, Ghazian O, Adamiak K, Castle G S P, Fan H and Simmer J 2014 *Proceedings of the IEEE/IAS Annual Meeting*, Vancouver, BC, 1-7
- [10] Adamiak K and Floryan J M 2011 *IEEE Trans. on Ind. Appl.* **47** 2374-83
- [11] Félici N J 1966 *Revue générale de l'électricité*. **75** 1145-60