

# Kalman filter application to mitigate the errors in the trajectory simulations due to the lunar gravitational model uncertainty

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**Abstract.** This paper aims to simulate part of the orbital trajectory of Lunar Prospector mission to analyze the relevance of using a Kalman filter to estimate the trajectory. For this study it is considered the disturbance due to the lunar gravitational potential using one of the most recent models, the LP100K model, which is based on spherical harmonics, and considers the maximum degree and order up to the value 100. In order to simplify the expression of the gravitational potential and, consequently, to reduce the computational effort required in the simulation, in some cases, lower values for degree and order are used. Following this aim, it is made an analysis of the inserted error in the simulations when using such values of degree and order to propagate the spacecraft trajectory and control. This analysis was done using the standard deviation that characterizes the uncertainty for each one of the values of the degree and order used in LP100K model for the satellite orbit. With knowledge of the uncertainty of the gravity model adopted, lunar orbital trajectory simulations may be accomplished considering these values of uncertainty. Furthermore, it was also used a Kalman filter, where is considered the sensor's uncertainty that defines the satellite position at each step of the simulation and the uncertainty of the model, by means of the characteristic variance of the truncated gravity model. Thus, this procedure represents an effort to approximate the results obtained using lower values for the degree and order of the spherical harmonics, to the results that would be attained if the maximum accuracy of the model LP100K were adopted. Also a comparison is made between the error in the satellite position in the situation in which the Kalman filter is used and the situation in which the filter is not used. The data for the comparison were obtained from the standard deviation in the velocity increment of the space vehicle.

## 1. Introduction

An artificial satellite around the Moon's surface is disturbed mainly by the non-homogeneity of the lunar gravitational field. This perturbation tends to cause deviations in the satellite trajectory, and, consequently, variations in the orbital elements that characterize the satellite orbit. It is common to use spherical harmonics to expand the expression of the lunar gravitational potential, as done in LP100K model, presented by [1]. The expansion of spherical harmonics, up to degree and order 100, was done to obtain the maximum precision of the model. However, the accuracy of the gravitational disturbing model can be varied according to the degree and order adopted. Studies and analysis of the influence of the accuracy of the model, as well as simulations considering the perturbation due to the lunar gravitational potential in the orbit of an artificial satellite can be found in [2, 3].



The LP100K model and the Kalman filter equations have been implemented in a simulator called Spacecraft Trajectory Simulator - STRS [4]. The orbital trajectory is calculated through the Kepler's equation solution for each simulation step, defined as simulator input parameter. This simulator uses continuous propulsion and trajectory control in closed loop to perform correction and transfer maneuvers, so that deviations and errors in the state variables are minimized. A more detailed description of the orbital trajectory controller can be found in [5].

The Kalman filter is recursive, thus it is an estimator with real-time characteristics, which provides estimates for the moment when the measure is processed. Therefore it is not necessary to have all measures to analyze them later with the estimator [6]. To do this, an estimate of the satellite position is made trying to approximate the results obtained using lower values for the degree and the order of the spherical harmonics, to the results that would be obtained if the maximum accuracy of the model LP100K were adopted.

## 2. Equations and models

The gravitational potential of the Moon is expressed by the coefficients of the normalized spherical harmonics, given by Equation (1) [1, 7]:

$$U(r, \lambda, \phi) = \frac{\mu}{r} + \frac{\mu}{r} \sum_{n=2}^{\infty} \sum_{m=0}^n \left(\frac{a_e}{r}\right)^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin \phi) \quad (1)$$

where  $n$  is the degree,  $m$  is the order,  $\mu$  is the gravitational constant and  $r$  is the orbit lunar equatorial radius.  $\bar{P}_{nm}$  are the fully normalized associated Legendre polynomials;  $a_e$  is the reference radius of the Moon,  $\phi$  is the latitude, and  $\lambda$  is the longitude.

The LP100K model uses data obtained by the Lunar Prospector mission (1998-1999), which is the third mission of the Discovery NASA's exploration program. Its output provides the components  $x$ ,  $y$  and  $z$  of the lunar gravitational acceleration, considering an inertial reference system centered on the Moon, at each instant of time along the simulation of the orbit of an artificial satellite, from the Equation 1. The gravity acceleration provided by LP100K model is compared with the gravity acceleration from a central field to obtain a disturber velocity increment applied in the satellite. Through the inverse problem it is possible to obtain the Keplerian elements that characterize the orbit of the artificial satellite.

Because it is a non-linear model, we used the extended Kalman filter. The extended filter updates the reference trajectory around the most current available estimate, wherein the filtering process consists of two stages: propagation phase and update phase.

The Equation 2 shows the propagation phase of the state and covariance, respectively.

$$\begin{aligned} \dot{\bar{w}} &= f(\bar{w}) \\ \dot{\bar{P}} &= F\bar{P} + \bar{P}F^T + GQG^T \end{aligned} \quad (2)$$

where  $F$  is the jacobian matrix of  $f$  with respect to  $w$ , and  $w$  represents one of the position coordinates of the satellite uma das coordenadas cartesianas da posição do satélite the satellite position;  $P$  is the covariance matrix and  $\bar{w}$  is the propagated state vector.

The Equation 3 shows the update phase.

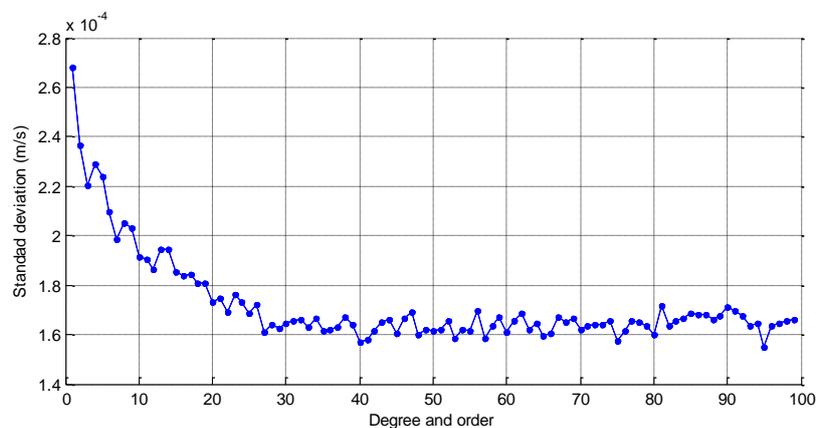
$$\begin{aligned} K &= \bar{P}H^T(H\bar{P}H^T + R)^{-1} \\ \hat{P} &= (1 - KH)\bar{P} \\ \hat{w} &= \bar{w} + K[y - h(\bar{w})] \end{aligned} \quad (3)$$

where  $K$  is the Kalman gain,  $H$  is the jacobian matrix of  $h(w)$  with respect to  $w$  and  $\hat{w}$  is the estimated state vector [6, 8].

### 3. Simulations and results

One of the objectives of the work is to vary the relation between the sensor and the uncertainty of the model, to thereby, choose a sensor which the uncertainty of the measures allows to generate an estimative that approximates the maximum precision of the LP100K model, when considered the use of the Kalman filter. In this way, the work begins with a study of the error in the trajectory of a lunar artificial satellite when one adopted values to degree and order less than 100. The Figure 1 shows this error through the sum of the standard deviation related to the values of the degree and order of the spherical harmonics. That is, we want to know the error that would be committed considering, for example, up to degree and order 10 (all terms added 1 to 10).

This analysis is made for all values of degree and order from 1 to 99, always compared with the value 100. This value is used as a reference basis, for being the maximum accuracy of the model, and for being considered the best way of describing the non-uniform distribution of Moon's mass. Thus, in this work, the value 100 to degree and order was regarded as a value without error, and therefore with a null standard deviation. This study was limited to the initial conditions of the last phase in Lunar Prospector mission: periapsis 17 km, apoapsis 43 km, inclination 90° and eccentricity 0.00735.



**Figure 1.** Standard deviation average for the values of degree and order from 1 until 99

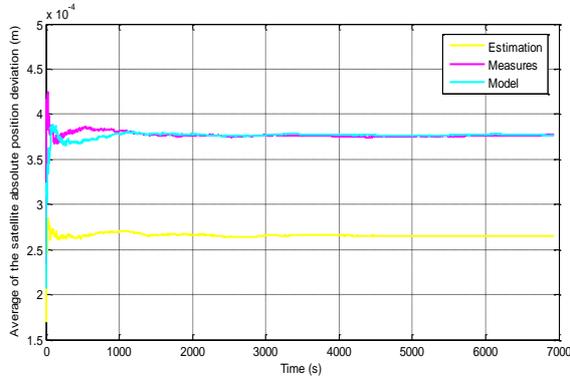
In [3, 9] it was analyzed and quantified the influence of each term of the lunar gravitational potential between the values 1 to 100 to degree and order. An expected prevalence of the first term of the gravitational potential (degree and order 2) was presented. It has been shown the existence of a significant contribution of the first 10 terms, however it was noted that the other terms are also relevant.

The results obtained in this work are consistent with previous published studies. Since it is a summation of values, the error decreases as higher values for degree and order are adopted. This decrease does not show a monotonic behavior due to the variability of the lunar gravitational potential. We cannot say that the latter term exerts less influence than the previous term, as shown in [3, 9]. Other studies have been conducted with the objective of evaluate errors in the gravitational potential due to the omission of higher terms of spherical harmonics, according to the covariance, especially for the geopotential case, as found in [10], [11], [12], [13] and [14].

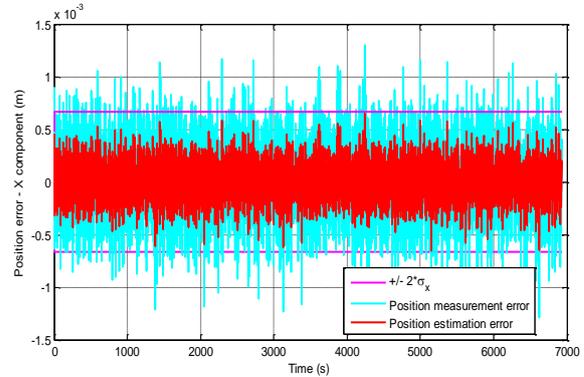
The Figures 2, 4 and 6 show the average absolute deviation in the satellite position and the Figures 3, 5 and 7 show a study of the error in the component  $x$  of the satellite position vector to the value 2 of degree and order, value commonly used in simulations of lunar satellites trajectories. The results showed that the standard deviation for this case was  $2.365 \times 10^{-4}$  m/s.

The study below was made considering the situation with less accuracy as possible, wherein components  $x$ ,  $y$  and  $z$  have the same uncertainty value. The case where the sensor and the model have the same value for the uncertainty (case 1), the case where the sensor uncertainty is 10 times

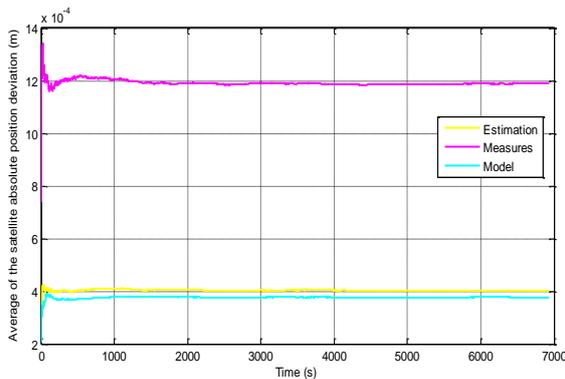
greater than the model uncertainty (case 2), and, finally, the case where the sensor uncertainty is 100 times greater than the model uncertainty (case 3) were analyzed.



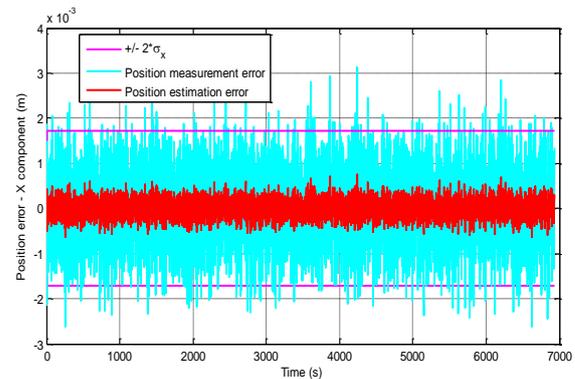
**Figure 2.** Average of the satellite absolute position deviation (case 1).



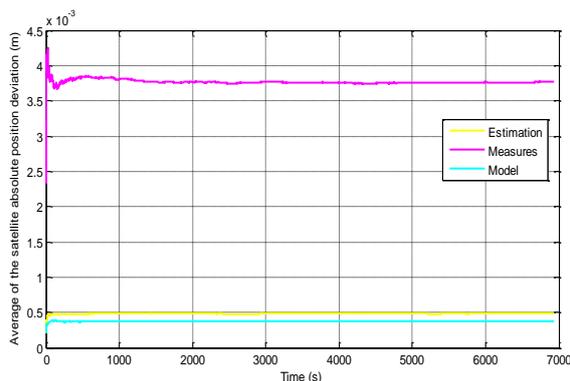
**Figure 3.** Position error (case 1).



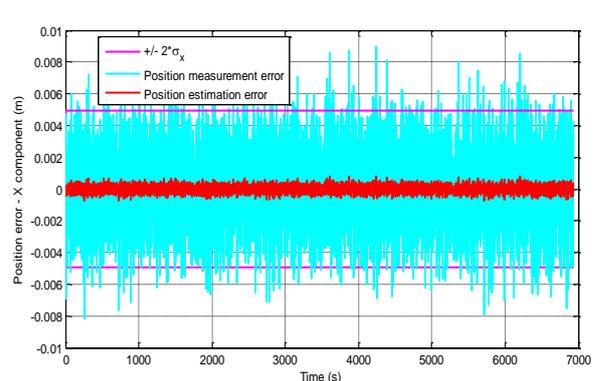
**Figure 4.** Average of the satellite absolute position deviation (case 2).



**Figure 5.** Position error (case 2).



**Figure 6.** Average of the satellite absolute position deviation (case 3).



**Figure 7.** Position error (case 3).

In the Figures 2, 4 and 6 the blue line represents the mean error in the propagated position generated by the model, the pink line represents the mean error of the measured position, and the yellow line represents the mean error of the position estimated by the Kalman filter. In the Figures 3, 5 and 7 the blue line represents the error between the measured position and the propagated position, the

red line represents the error between the propagated position and the position estimated by the Kalman filter and the pink line represents 2 standard deviations.

The results show that increasing the sensor uncertainty causes the model accuracy to become increasingly significant and the uncertainty in the estimated position approaches the estimated uncertainty of the model. Other studies comparing the uncertainty in the component  $x$  of the spacecraft position vector, for different cases, can be found in [15] and [16].

#### 4. Conclusions

The work begins with a study of the inserted error in simulations due to the use of low values to degree and order for the expansion of spherical harmonics. This study was done by determining the standard deviation of the velocity increment, as function of the accuracy of the disturbance model adopted. The Kalman filter model has been successfully implemented and tested, for the use with lower values of degree and order of the expansion of spherical harmonics, allowing an estimate of the satellite position, as good as the position determined for the case when the maximum accuracy of the gravitational model is used. Finally, the study performed in order to find a sensor which the uncertainty in the measures, when using the Kalman filter, generate an estimate similar to the maximum accuracy of LP100K model was successfully completed. The approach adopted in this work to choose the accuracy of the sensor could be used in a real mission analysis to specify the sensor for the satellite.

#### 5. Acknowledgements

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