Journal of Physics: Conference Series **641** (2015) 012028 doi:10.1088/1742-6596/641/1/012028

Analysis of the influence of orbital disturbances applied to an artificial lunar satellite

L D Gonçalves[1], E M Rocco[1] and R V de Moraes[1, 2]

[1]Instituto Nacional de Pesquisas Espaciais, INPE, São José dos Campos, Brazil
[2]Universidade Federal de São Paulo, UNIFESP,São José dos Campos, Brazil

E-mail: lianadgon@gmail.com

Abstract. This paper analyzes the influence of the orbital disturbance forces in the trajectory of lunar satellites. The following gravitational and non-gravitational orbital disturbances are considered: the non-homogeneity of the lunar gravitational field; the gravitational attraction due to the third body, considering the Earth and the Sun; the lunar albedo; the solar radiation pressure. Numerical models were developed and implemented in an orbital trajectory simulator aiming to understand the dynamics of the orbital motion of an artificial satellite in lunar orbit when considering the simultaneous effect of all disturbances. Different orbits were simulated in order to characterize the major and the minor influence of each disturbing force as function of the inclination and the right ascension of the ascending node. This study can be very useful in the space mission analysis and in the selection of orbits less affected by environmental disturbances.

1. Introduction

The existence of environmental disturbing forces acting on an artificial satellite tends to cause variations in the elements that characterize the vehicle's orbit. To achieve the mission objectives it is necessary to analyze and study such disturbance forces previously, which can contribute to the efficiency with respect to important variables, such as the mission time and the fuel consumption.

To study the main disturbances acting on the orbital motion of a lunar artificial satellite, the disturbances models due to the lunar gravitational potential, lunar albedo, gravitational attraction of the Earth and Sun and solar radiation pressure were inserted into the Spacecraft Trajectory Simulator (STRS),[1], to be made the analysis of the magnitude of such forces applied in predefined orbits.

The model of the lunar gravitational field is based on spherical harmonics, according to the model presented by Konopliv [2] allowing to consider the spherical harmonics up to degree and order 100. The gravitational attraction of the Sun and Earth was modeled from the Newton’s law of universal gravitational attraction, making use of the movement model for all bodies involved (Sun, Earth, Moon and Satellite). The lunar albedo model is based on the reflectivity of the Moon surface; it is possible to divide the lunar surface up to 51840 cells, to analyze the behavior of light reflected by each cell. The solar radiation pressure is modeled from the lunar albedo model, but considering the solar radiation shining directly into the satellite surface. Different orbits are simulated in order to characterize the major and minor influence of each disturbing force.

Journal of Physics: Conference Series **641** (2015) 012028 doi:10.1088/1742-6596/641/1/012028

2. Methodology

Considering that the objective of the study is to analyze the magnitude of the main disturbing forces acting on the motion of an artificial satellite around the lunar surface, models of lunar orbital perturbations were developed and inserted in simulation environment STRS. The STRS was developed by Rocco [1] and works with some specific features: the trajectory can be controlled using a closed loop control system, seeking to minimize the error in the vehicle's path and the deviation of the final state; uses a continuous propulsion system making it possible to assume that the magnitude of the thrust is finite and is applied during a time interval different of zero. A simplified block diagram of the orbital trajectory control system, used by the STRS, is shown in Figure 1.

The process starts with the determination of a reference state (Xref) which may vary from each mission objectives. Such reference state should be a good estimative of the trajectory to be followed by the satellite. The current state of the satellite (Xdet) is measured by sensors and compared with the reference state. The difference resulting from this comparison is an error signal that acts as controller input parameter. The control signal generated by the controller, which uses the techniques PID (proportional, integral and derivative), is sent to the actuators, which are able to define the magnitude and direction of corrections to be applied. The actuator output signal is added to external disturbances, and inserted to the dynamics of the orbital movement to be determined the current position and velocity of the artificial satellite. Then the current position of the satellite (Xdet) is determined, by using the sensors, from the information obtained by adding the signal of orbital dynamics with the sign of the disturbances.(Xactual). Finally the current state is compared to the reference position (Xref), generating an error signal and restarting the cycle of the control system.

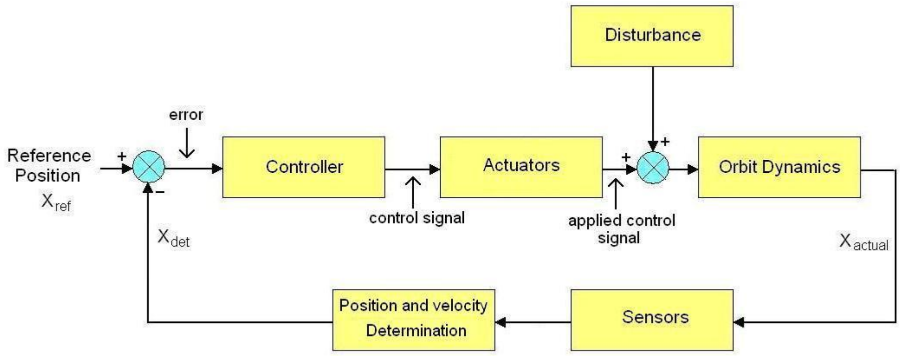

Figure 1. Trajectory closed loop control

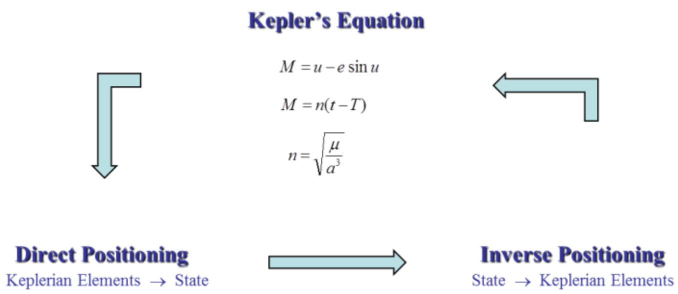

Figure 2. Dynamics of orbital motion

Solving the Kepler's equation ($M = u - e \sin u$ [3]) to each step defined as an input parameter in the simulator (STRS) the orbital motion can be determined, as shown in Figure 2. The Keplerian elements of the orbit can be determined from an initial state, considering a time interval, from the application of the inverse problem [3]. The propagated elements are obtained from the solution of Kepler's equation for the considered time interval. From the new Keplerian elements, the new state can be obtained solving the direct problem of satellite positioning [3].

3. Model for Perturbative Forces

The main disturbances forces capable of altering the orbit of an artificial lunar satellite are the lunar gravitational potential, the gravitational attraction of the Earth and Sun, the lunar albedo and the solar radiation pressure [3]. The models developed and used to study of the magnitude of these forces in predefined lunar orbits of an artificial satellite were based on theoretical study, briefly presented below.

3.1. Lunar Gravitational Potential

The Moon's gravitational potential is expressed by the expansion of normalized spherical harmonic coefficients, given by Equation (1) [2, 4]:

$$U(r,\lambda,\phi) = \frac{\mu}{r} + \frac{\mu}{r}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n (\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\,\mathrm{sen}\, m\lambda)\bar{P}_{nm}(\mathrm{sen}\,\phi) \tag{1}$$

where n is the degree, m is the order, μ is the gravitational constant and r is the lunar equatorial radius. $\bar{P}_{nm}$ are the fully normalized associated Legendre polynomials. a_e is the reference Moon radius, ϕ is the latitude, and λ is the longitude.

3.2. Gravitational Attraction of the Earth and the Sun

Considering the presence of a third body the function of the gravitational potential is given by Equation (2) [3], where μ' is the gravitational constant multiplied by the mass of the third body, r' is the module of the position vector of the third body; ψ is the angle between the position vector of the satellite relative to the Moon and the position vector of the satellite relative to the third body; r is the module of the satellite position vector with respect to Moon.

$$F' = \left(\frac{\mu'}{r'}\right)\left[1 + \sum_{n=2}^{\infty}\left(\frac{r}{r'}\right)^n P_n \cos\psi\right] \tag{2}$$

The general problem of three bodies presented by [5], and used by [6], provides the Equations (3) to calculate the disturbing accelerations due to the gravitational attraction of the bodies, where: $\vec{r}_1$, $\vec{r}_2$ e $\vec{r}_3$ are the positions of bodies, m_1, m_2 e m_3 are the masses of bodies and G is the gravitational constant.

$$\begin{aligned}
\ddot{\vec{r}}_1 &= -Gm_2\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} + Gm_3\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|^3} \\
\ddot{\vec{r}}_2 &= -Gm_3\frac{\vec{r}_2 - \vec{r}_3}{|\vec{r}_2 - \vec{r}_3|^3} + Gm_1\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} \\
\ddot{\vec{r}}_3 &= -Gm_1\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|^3} + Gm_2\frac{\vec{r}_2 - \vec{r}_3}{|\vec{r}_2 - \vec{r}_3|^3}
\end{aligned} \tag{3}$$

3.3. Lunar Albedo

The lunar albedo is defined as the fraction of solar energy reflected diffusely from the lunar surface into space, Equation (4), measured from the reflectivity of the Moon surface [1, 7].

$$albedo = \frac{radiation\ reflected\ back\ to\ space}{incident\ radiation} \tag{4}$$

The lunar albedo model considers the Moon's reflectivity splitting the lunar surface up to 51840 cells. This innovative model was adapted from the Earth albedo model [8]. A detail description of the model and an analysis of the effect of the number of cells in the accuracy of results can be found in [9].

3.4. Solar Radiation Pressure

The solar radiation pressure is a non-gravitational force able to disturb the translational motion of a lunar artificial satellite due to the momentum of the photons that strike the satellite surface, causing a variation in the orbital elements.

In this work the model of the disturbance due to solar radiation pressure was developed based on the albedo model. Considering that is required to model the Sun movement, as well as its emitted radiation, and the Moon and the satellite's movement, for the albedo model, these same subroutines are also used to calculate the solar radiation pressure. However, it should be considered that the solar radiation directly reaches the satellite surface, instead of considering that solar radiation was reflected by the surface of the Moon before reaching the satellite, as in the albedo case.

4. Simulations and Results

With the objective to analyze the magnitude of each disturbance force were simulated some orbits, varying the following three orbital parameters: satellite altitude (15 km, 115 km, 215 km, 315 km, 415 km and 515 km); right ascension of the ascending node (0°, 90°, 180° and 270°); and inclination (0°, 90°, 180° and 270°).

The results for the integral of the perturbative force, for each disturbance adopted (lunar gravitational potential, attraction of the Earth and the Sun, solar radiation pressure, or lunar albedo), during one orbit around the Moon, are shown in Figures 3 to 7. Each figure presents a study for each orbital disturbance. The Figure 3 shows the study in the case of disturbance caused by the lunar gravitational potential, the Figure 4 for the Earth gravitational attraction, the Figure 5 for the Sun gravitational attraction, the Figure 6 for the solar radiation pressure and the Figure 7 for the lunar albedo.

In each situation were varied the values of altitude, inclination and right ascension of the ascending node. The colors change according to the variation of the values of the right ascension of ascending node and, consequently, the orbital plane. Blue for 0^o, red for 90^o, green for 180^o and yellow for 270^o. The altitudes are differentiated by lines connecting the dots in the graphs. For the cases of disturbance due to the gravitational potential and lunar albedo the top line shows the value of lower altitude (15 km) and the last line below shows the value of higher altitude (515 km). For cases of the Earth and Sun gravitational attraction and the solar radiation pressure, the top line shows the value of higher altitude (515 km) and the last line below shows the value of lower altitude (15 km). In the Figures 3 to 7, the altitudes are specified by numbers. Number 1 for altitude of 15 km, 2 for 115 km, 3 for 215 km, 4 for 315 km, 5 for 415 km and 6 for 515 km.

It is worth noting that for the cases studied separately for each perturbation, all other perturbations are also considered. If they were ignored, we would be considering the satellite at a different point in its actual position, which does not represent the real influence of disturbances. It can be observed an expected prevalence of lunar gravitational potential, as well as a significant variation of the disturbance forces integral as a function of the right ascension of the ascending node and the inclination. A detailed analysis of the gravitational potential influence in the orbit of an artificial lunar satellite, which were not considered other disturbances, and the performance of a control system to minimize such perturbative effects, can be found in [10, 11].

In view of the predominance of disturbance due to the lunar gravitational potential, was taken, for the case of inclination and right ascension of the ascending node, a quantitative analysis of all the perturbative forces, excluding the lunar gravitational potential. The results are shown in Table 1.

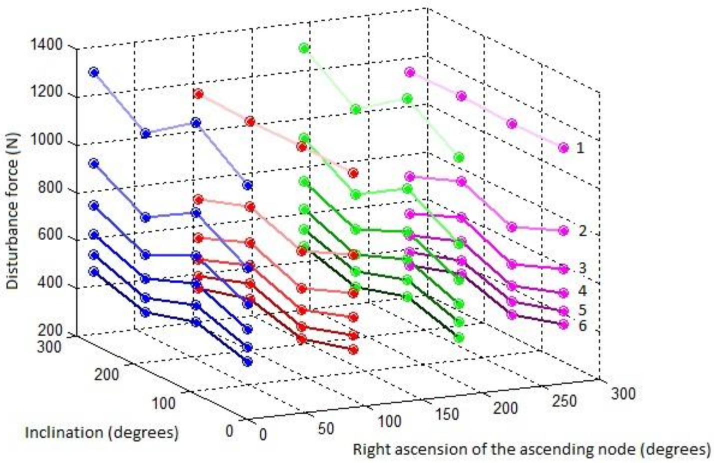

Figure 3. Integral of the disturbance force due to the lunar gravitational potential.

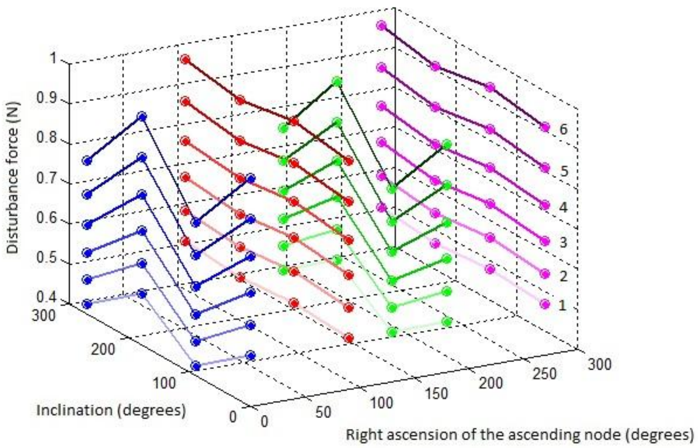

Figure 4. Integral of the disturbance force due Earth gravitational attraction.

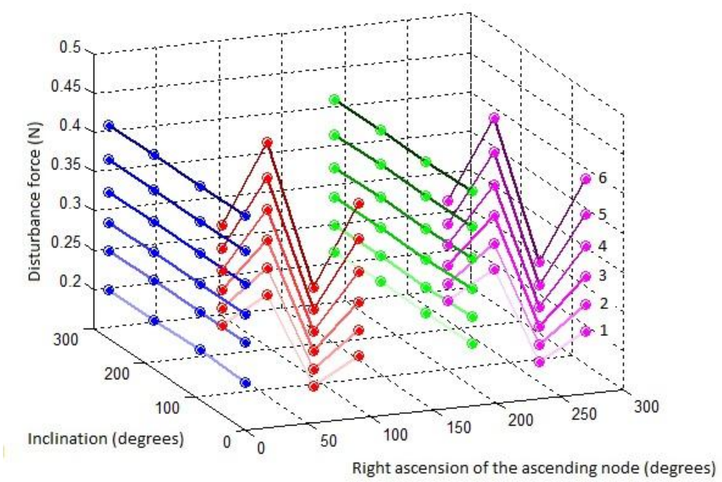

Figure 5. Sum of the perturbation force due Sun gravitational attraction.

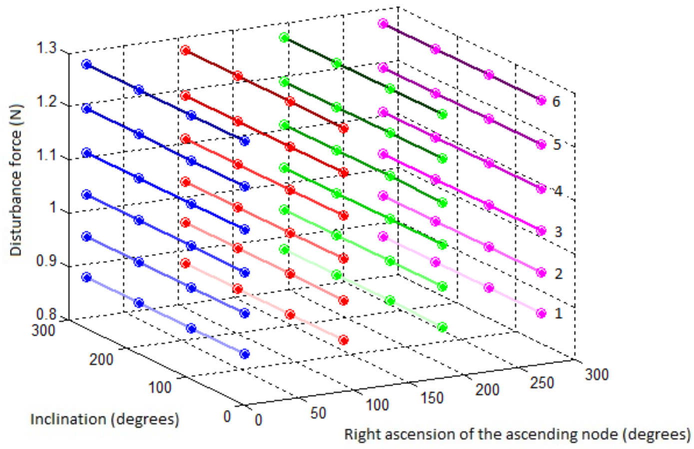

Figure 6. Sum of the perturbation force due to solar radiation pressure.

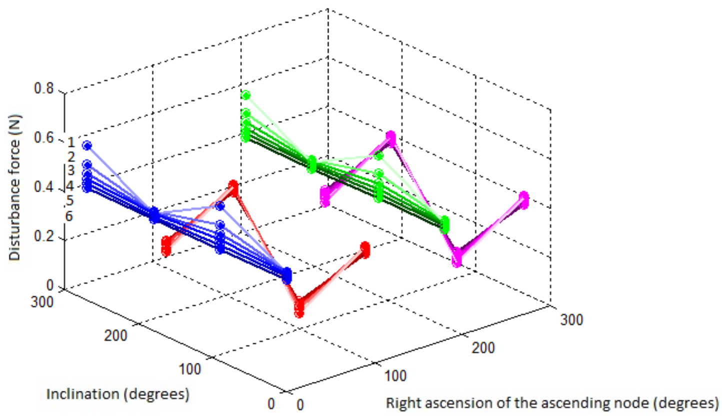

Figure 7. Integral of the disturbance force due to the lunar albedo.

Table 1. Disturbances Percentages

	Earth	Sun	Radiation	Albedo
15 km	19%	11%	41%	29%
115 km	21%	12%	43%	24%
215 km	23%	12%	43%	21%
315 km	24%	13%	44%	19%
415 km	25%	14%	44%	17%
515 km	26%	14%	44%	15%

It is seen a predominance of disturbance due to solar radiation pressure even when varying the altitude of the satellite. For altitudes closer to the lunar surface there is a higher relevance of albedo. This situation changes as the satellite reaches higher altitudes, where the influence of the Sun and Earth's gravitational attraction become more significant when compared to the lunar albedo. Note that the values of the disturbance changes as function of the altitude of the satellite. The data in Table 1 show the percentages of such disturbance forces.

5. Conclusions

This paper presents the problem of quantize the magnitude of the forces that are able to disturb the orbit of a lunar satellite. For this, the influence of each force was considered and studied separately, but simultaneously with all disturbance forces. In other words, the contributions of each disturbance were obtained and then added to be obtained the total disturbance applied on the satellite. This kind of study can be very useful in the space mission analysis and in the selection of orbits less affected by environmental disturbances. For all the analyzed orbits, the disturbance due to the lunar gravitational potential was higher than the sum of all other disturbances considered. When the disturbance due to the lunar gravitational potential is excluded from the analysis was verified that the magnitude of non-gravitational forces is significant. This leads us to conclude that such disturbances should not be neglected in the studies of the orbital motion of a satellite around the lunar surface if it is necessary highly accurate results in the simulation of the trajectory, as is the case of a mission analysis of an artificial satellite.

6. Acknowledgements

The authors acknowledge the support provided by grants2012/21023-6 from São Paulo Research Foundation (FAPESP); by grants3085922/2014-0 from the National Council for Scientific and Technological Development (CNPq); and the financial support from the National Council for Scientific and Technological Development (CNPq).

References

[1] Rocco E M 2008 Perturbed orbital motion with a PID control system for the trajectory *XIV Colóquio Brasileiro de Dinâmica Orbital*, Águas de Lindóia, Brasil
[2] Konopliv A S, Asmar, S W, Carranza, E, Sjogren W L and Yuan, D N 2001 Recent gravity models as a result of the lunar prospector mission *Icarus* **150** 1-18
[3] Chobotov, V A 1991 *Orbital Mechanics* (Washington, DC: American Institute of Aeronautics and Astronautics) p 365
[4] Kuga H K, Carrara V and Kondapalli R R Artificial 2011 Satellites - Orbital Motion (in Portuguese) INPE Available in: http://urlib.net/8JMKD3MGP7W/3ARJ3NH
[5] Szebehely, V 1967 *Theory of orbits* (New York: Academic Press Inc)
[6] Prado, A F B A, Kuga, H K 2001. Space Technology Fundamentals (in Portuguese) Instituto Nacional de Pesquisas Espaciais, São José dos Campos ISBN: 85-17-00004-8
[7] Rocco E M 2008 Analysis of the deviations of the trajectory due to the terrestrial albedo applied to some scientific missions *International Conference on Mathematical Problems in Engineering, Aerospace and Sciences*, Genova, Italy
[8] Rocco, E M 2010 Evaluation of the Terrestrial Albedo Applied to Some Scientific Missions *Space Science Reviews* **151** 135-147
[9] Gonçalves, L D 2013 Evaluationofthetrajectorydeviationof a spacecraftduetothe lunar albedo *Congresso Brasileiro de Engenharia Mecânica*, Ribeirão Preto, Brasil
[10] Gonçalves L D, Rocco E M and de Moraes R V 2013 Orbital Disturbance Analysis due to the Lunar Gravitational Potential and Deviation Minimization through the Trajectory Control in Closed Loop *Journal of Physics: Conference Series* **465** 012008
[11] Gonçalves, L D, Rocco, E M, de Moraes, R V and Prado, A F B A 2013. Study of the potential effects on Lunar Spacecraft Trajectories (in Portuguese) *XI Conferência Brasileira de Dinâmica, Controle e Aplicações*, Fortaleza, Brasil