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Study of the decay time of a CubeSat type satellite considering perturbations due to the Earth's oblateness and atmospheric drag

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Abstract: In this work the time required for the occurrence re-entry of a CubeSat satellite type is studied. Perturbations due to the Earth's oblateness and atmospheric drag are considered. The main objective is to study the initial conditions to determine the maximum length of stay in orbit of the CubeSat satellite type given to its mass and its altitude using numerical simulations of the motion equations and an analytical solution was proposed.

1. Introduction

The first CubeSat satellites were developed at the end of the nineties of the 20th century at the California Polytechnic State University in partnership with the Stanford University's Space Systems Development Laboratory. The CubeSat satellites are based on cubic modules with 100 mm edges, known as unit, or simply U. These satellites have been widely used to sending and receiving data.

In this work, it is performed an numerical study about the effects of atmospheric drag and of the Earth's oblateness on the orbit of CubeSat satellites type aiming predict the time required for the occurrence of its re-entry.

The protection of the Earth's space environment is producing a revival of the interest in solutions to the artificial satellite problem and new analytical and semi-analytical tools for the analysis and design of end-of-life disposal strategies have been proposed [1], [2], [3], [4]. One of the main problems of these theories is that the improvements of the analytical modelling for the atmospheric drag endanger analytical solutions for the equations involved in.

The equations of motion used here are written in Cartesian coordinates, taking into account perturbations due to harmonics factored by J_2 and to the atmospheric drag [5]. The numerical integrations in the present paper were made using a Legendre-Gauss-Radau – LGR [6] integrator of fourth order and step size control, with a numerical accuracy of 10^{-8} and were considered two models for the atmospheric density: the TD-88 model [7] and NASA model [8]. The TD-88 analytical termospheric density model is easy to be implemented and agree quite well with some spread used atmospheric model [9]. For the perturbations due to the oblateness, Brouwer theory was used [10]. Initial conditions were considered depending on some usual physical and orbital characteristics (weight of the satellite, semi-major axis, eccentricity and inclination of the orbit), for a CubeSat remain in orbit for a period not longer than 25 years.

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2. The TD-88 and NASA atmospheric density models

In what follows the atmospheric density models TD -88 [7] and NASA [8] are briefly described below:

2.1. TD-88

Following [7], the density can be expressed as

$$\rho = f_\chi f_0 k_0 + \sum_{n=1}^{7} h_n\, g_n \tag{1}$$

where,

$$f_x = 1 + a_1(F_x - F_b), \qquad g_1 = 1,$$
$$f_0 = a_2 + F_n, \qquad g_2 = \frac{F_n}{2+a_4},$$
$$k_0 = 1 + a_3(k_p - 3), \qquad g_3 = sen(d - p_3)sen\varphi,$$
$$f_n = \frac{(F_b - 60)}{160}, \qquad g_4 = (a_5 f_n + 1)sen(d - p_4),$$
$$h_n = h_{n,0} + \sum_{j=0}^{3} k_{n,j}\, exp^{\frac{(120-h)}{29j}}, \qquad g_5 = (a_6 f_n + 1)sen2(d - p_5),$$
$$g_6 = (a_7 f_n + 1)sen(t - p_6)cos\varphi,$$
$$g_7 = (a_8 f_n + 1)sen2(t - p_7)cos^2\varphi$$

where: F_x is the solar flux measured on 10.7 cm/day; F_b is mean solar flux; h is the altitude; a_i (i=1,2,…8) and p_i (i=3,…7) are numerical constants; k_p is the magnetic index; d is the day count of the year; t is the local time and φ is the latitude.

2.2. NASA model

The NASA atmospheric density is given [8],

$$\rho = \frac{P}{(0.2869 * (T + 27301))} \tag{2}$$

Where the temperature and pressure are given by,

$$T = -131.21 + 0.00299h \qquad P = 2.488\left(\frac{T+273.1}{216.6}\right)^{-11.388}$$

2.3. Comparison of TD-88 model and the NASA model for given initial conditions

Figure 1 shows values for the atmospheric density varying with the altitudes from 200 km to 700 km considering: the TD-88 and NASA models. The conditions used in the TD-88 model were d = 80 current day, f_x = f_0 = 150 W, k_0 = 4 (dimensionless), t = 3 h and φ = 0 °. Note that for h = 200 km the TD-88 model has the maximum density value ρ = 4.50 x 10^{-10} kg/m^3 while for the NASA model ρ = 9.85 x 10^{-9} kg/m^3. The minimum density occurs at h = 700 km: ρ = 6.94 x 10^{-14} kg/m^3 for the TD-88 model and ρ = 1.11 x 10^{-14} kg/m^3 for the NASA model.

3. Decay of a satellite type CubeSat

The equations of motion were implemented in a FORTRAN code program using the numerical integrator Radau [6] to calculate the orbital decay. The decay time was computed using values of position and velocity obtained from the numerical integration and substituting these values in the equation for the semi-major axis updated at the time step of the numerical integration which was 10^{-8}. The used program stopping criterion was when the orbital radius of the CubeSat is less than or equal to the orbital radius of the Earth.

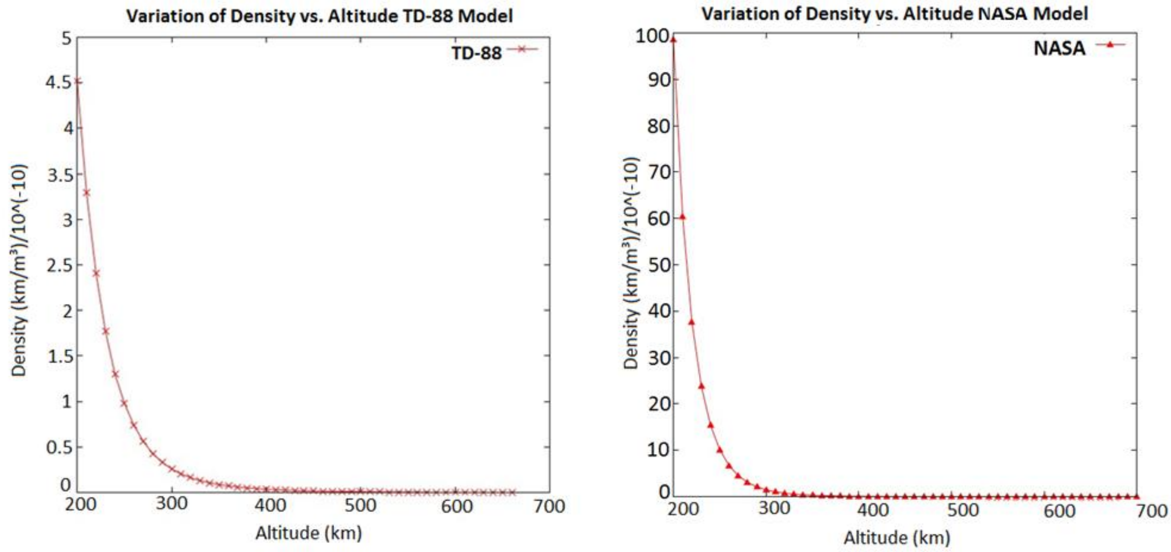

Figure 1. Atmospheric density with altitude considering: the model TD-88 and the NASA model.

3.1 Decay of a CubeSat satellite considering the atmospheric drag and Earth' oblateness perturbations

Considering some scenarios taking into account different values for the initial conditions - day of the year, local time, solar flux, magnetic index, and latitudes - some information should be placed highlighted, as follows:

- Worst case: the longer decay time. The conditions for the worst case were: day of the year, d = 172, local time, t = 3h, solar flux measured in 10.7 cm/day, f_x= 150 W, solar flux obtained after three rotations, f_0 = 150 W, geomagnetic index location, k_0 = 4 (dimensionless) and latitude, φ = 0º.
- Best case: the lowest decay time. The original terms of the best case were: day of the year, d = 80, local time, t = 3 h, solar flux measured in 10.7 cm/day, f_x = 150 W, solar flux obtained after three rotations, f_0 = 150 W, geomagnetic index location, k_0 = 4 (dimensionless) and latitude, φ = 0 º.

3.2. Atmospheric drag with density model TG-88, and Earth's oblateness

Simulations, considering the worst-case scenario for the decay period of a CubeSat type satellite considering the Earth's oblateness and atmospheric drag perturbation with atmospheric density model TD-88, are show in table 1. The results are presented according to the type of orbit, the mass of satellite and the altitude. The orbital decay period is given in years. For a satellite with a mass of 5 kg, the only altitudes that respect the European Code of mitigation are 500 km and 550 km for circular orbits, and having 7.44 and 15.81 years to decay, respectively. For a satellite with a mass of 10 kg, the only altitude respecting the European Code of mitigation is 500 km, where the decay period is 14.87 years. For all other initial conditions addressed, the decay period is longer than the 25 years stipulated in the European Code of mitigation. The following were the initial conditions used for this situation: day of the year, d = 172, local time, t =3 h, solar flux measured in 10.7 cm/day, f_x = 150 W, solar flux obtained after three rotations, f_0 = 150 W, geomagnetic index site, k_0 = 4 (dimensionless) and latitude φ = 0 º.

In table 2 contains the results considering the best setting with density model TD-88. For satellites of 5 kg, all altitudes in the range from 500 to 650 km will comply with the European Code of spatial mitigation, because none of these conditions exceeds the time limit in orbit around the Earth that is 25 years. For satellites of 10 kg, the only altitudes respecting the European Code of mitigation are between 500 and 550 km, where the decay periods are 6.13 and 12.70 years, respectively.

Table 1. Worst-case scenario for the decay period considering density model TD-88.

Orbit	Mass (kg)	Altitude (km)	Time to decay (years)
Circular and inclination of the 30 º	5	500	7.44
		550	15.81
		600	31.75
		650	61.38
	10	500	14.87
		550	31.63
		600	63.49
		650	122.76

Table 2. Best initial conditions setting for the orbital decay period considering density model TD-88.

Orbit	Mass (kg)	Altitude (km)	Time to decay (years)
Circular and inclination of the 30 º	5	500	3.07
		550	6.35
		600	12.58
		650	24.15
	10	500	6.13
		550	12.70
		600	25.15
		650	48.29

The initial conditions used in this situation were: day of the year, d = 80, local time, t = 3 h, solar flux measured in 10.7cm/day, f_x = 150 W, solar flux obtained after three rotations, f_0 = 150 W, geomagnetic index site, k_0 = 4 (dimensionless) and latitude φ = 0 º.

After several numerical simulations it could be observed the effects of the inclusion of the perturbation due to the Earth's oblateness in the equations of motion are negligible for the decay time.

3.3. Atmospheric drag and Earth' oblateness perturbations and atmospheric density model NASA

In table 3 is showed of the orbital decay time of a satellite type CubeSat considering the Earth's oblateness and atmospheric drag perturbations and atmospheric density model NASA. The results are presented according to the type of orbit, the satellite mass, the altitude at which it is. The orbital decay period is given in years. For a mass of 5 kg satellite, only the first two initial conditions satisfy the European Code of spatial mitigation in this case the altitudes of 500 and 550 km altitude, where the time for orbital decay is 3.06 and 9.83 years, respectively. For a 10 kg mass satellite, the only altitudes that respect the European Code of mitigation are 500 and 550 km, where the decay period is of 6.12 and 19.66 years, respectively.

4. Analytical solution

Using the Delaunay variables L_i and l_i (i= 1, 2, 3), setting P_i and Q_i as the components of the atmospheric drag, the secular perturbations in the orbital elements of an artificial Earth satellite taking into account the influence of the atmospheric drag, can be obtained solving the system [11]:

$$\dot{L}_i = \langle P_i \rangle \, ; \; \dot{l}_i = -\langle {}^{\partial H}/_{\partial L_i} \rangle - \langle Q_i \rangle, \quad (i = 1,2,3) \tag{3}$$

where H is the Hamiltonian of the drag free problem. The symbol $\langle f \rangle$ means the non-periodic part of f and the ballistic coefficient is considered constant.

Table 3. The orbital decay time considering the Earth's oblateness and atmospheric drag perturbations with atmospheric density model NASA.

Orbit	Mass (kg)	Altitude (km)	Time to decay (year)
Circular with 30 º	5	500	3.06
		550	9.83
		600	28.74
		650	77.55
	10	500	6.12
		550	19.66
		600	57.48
		650	155.09

After some algebraic manipulation, the TD-88 atmospheric model can be expressed in terms of the mean eccentric anomaly and we get [11]:

$$\rho = f_\chi f_0 k_0 \sum_{n=1}^{7} [k_{n,0} \; + \sum_{j=1}^{3} A_j \, exp^{[c_j \cos(\omega+v)]} exp^{\,[\beta_j \cos E]}] g_n \tag{4}$$

where

$$c_j = \frac{R \; \varepsilon sin^2 i}{2.29/_j}, \qquad \beta_j = \frac{ae}{2.29/_j}, \qquad \text{and} \quad A_j = exp\left[\frac{\left(120+R\left(1-\frac{\varepsilon}{2sin^2 i}\right)-a\right)29}{j}\right]$$

Here, R is the mean Earth's equatorial radius, ε is flattening of the Earth, a, e, i, ω, ν, and E are the semi major axis, eccentricity, inclination, argument of perigee, true anomaly and eccentric anomaly, respectively.

Using the Cayley tables to express the eccentric anomaly in terms of the mean anomaly, the equations of motion can be easily solved and the decay of the semi major axis per revolution is

$$\Delta a = -2\pi a^2 \left(\frac{SC_D}{m}\right) f_\chi f_0 k_0 \sum_{n=1}^{7} [G_{n,0} \; \{k_{n,0} + \sum_{j=1}^{7} k_{n,j} A_j \; [\left(1 + \frac{c_j^2}{4}\right) + \frac{c_j^2}{4} A_{0,j}]\} \tag{5}$$

where: SC_D is the ballistic coefficient. Explicit analytical expressions for $G_{n,0}$, $k_{n,0}$ and $A_{0,j}$ as given in [11] were replaced in the above equation.

5. Conclusion

Two models namely, TD-88 and NASA, were used here to compute the atmospheric density. An analytical expression was exhibited and here adapted to be used for the first time to compute the decay of a CubeSat type satellite. According to the limitation of 25 years imposed by the European Code of spatial mitigation, considering the TD-88 model and a satellite in a circular orbit with inclination of 30º and mass of 5 kg, the maximum altitude at which this satellite can be allocated is around 550 km, where the decay time is about 15.81 years. For the same satellite considering the model of NASA, the maximum altitude is also around 550 km, but the decay period is even higher, 22.95 years. For higher

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altitudes the length of stay in orbit around the Earth goes beyond the stipulated by the European Code of spatial mitigation. For satellites in circular orbit with an inclination of 30 º and mass of 10 kg for both models the maximum altitude to a minimum time for decay is about 500 km, but there is great variation in the orbital decay period in this case: 14.87 years for TD-88 model and 6.12 years for the NASA model. It is worthwhile to recognize that since the TD-88 model was inserted in the proposed analytical solution, this solution agrees better with the numerical integration of the full equations using this density model.

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