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Influence of the external torques in the angle between the spin axis and the Sun direction for spin stabilized satellite

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Abstract: The goal of this paper is the study of the influence of the environmental torques in the angle between the spin axis and the Sun direction (solar aspect angle) for spin stabilized satellite. The theory uses a cylindrical satellite in an illumined orbit, considering the gravity gradient, aerodynamic, solar radiation, residual magnetic and eddy current torques. The mathematic model for each torque is shown. The dynamic equations are represented in a reference system fixed in the satellite and described by spin velocity and the right ascension and declination angles of the spin axis. An analytical solution for the spin velocity and the attitude angles is used to study the behavior of the solar aspect angle. The theory is applied for the real data of the Brazilian Satellite of Data Collection - SCD1 and SCD2. Two approaches are presented. The results agree with the real satellite behavior for specific time simulation. Then the theory has consistency and can be applied to predict the behavior of the solar aspect angle.

1 Introduction

The goal of this paper is to analyze the influence of the some external torques in the solar aspect angle, considering a cylindrical spin stabilized satellite in an illumined orbit. The Earth's shadow is not considered. The adopted mathematical model for the gravity gradient, aerodynamic, solar radiation, residual magnetic and eddy current torques can be found in [3,4,6].

The gravity gradient torque (GGT) is created by the difference of the Earth gravity force direction and intensity acting on each satellite mass element and is inversely proportional to the cube of the satellite geocentric distance. Therefore it decreases when the altitude increases.

The aerodynamic torque (AT) is created by the interactions of rarefied air particles with the satellite surface and is predominant in satellites with low altitude, because it depends on the quantity of air molecules in the Earth atmosphere. Calculation of aerodynamic torques for realistic spacecraft is not very accurate because of existing uncertainties in the atmospheric density and in drag coefficient. In this paper TD-88 model is used to describe the atmospheric density.

The solar radiation pressure (SRT) is created by the continuous photons collisions with the satellite surface, which can be able to absorb or reflect on this flow. The total change of the momentum of all the incident photons on the satellite surface originates from the solar radiation force and a torque can be produce.

Magnetic disturbance torques result from the interaction between the spacecraft's residual magnetic field and the Earth´s magnetic field. The residual magnetic torque (RMT) results from the interaction between the spacecraft's residual magnetic moment and the Earth magnetic field and its main effect are to produce a spin axis orientation drift. The torque induced by eddy currents (ECT) is caused by the spacecraft spinning motion and produces a reduction in the satellite spin rate with time.

A spherical coordinate system fixed in the satellite (*body system*) is used to locate the spin axis of the satellite in relation to the terrestrial equatorial system, with the axis z being along the direction of the spin velocity vector. The directions of the spin axis are specified by the right ascension (α) and the declination (δ) as represented in the Fig. 1.

The dynamic equations are represented in a body system and described by spin velocity and the right ascension and declination of the spin axis. These equations depend on the torques components in the body system. The averages of each torque components are determined over on orbital period and are substituted in the equations of motion. Due to the cylindrical form of the satellite and others simplifications, only the gravity gradient and eddy currents torques have a non-null z-components.

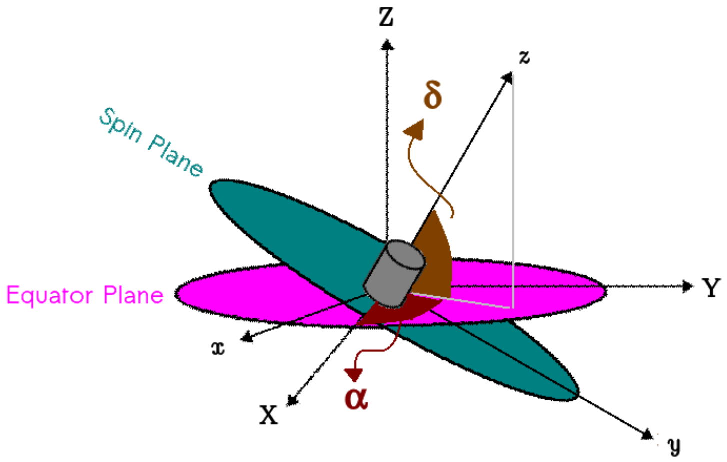


Figure 1. Body System (x,y,z) and Equatorial System (X,Y,Z): right ascension (α) and declination (δ) of the spin axis (z).

An analytical solution was gotten for the spin velocity and the attitude angles for one orbit period [3]. These analytical solutions are used to study the behavior of the solar aspect angle, which is gotten by the dot product of the Sun direction and the spin axis, and depend on the declination and the right ascension of the spin axis and declination and the right ascension of the Sun direction [4].

The theory is applied for the real data of the Brazilian Satellite of Data Collection - SCD1 and SCD2, which are quite appropriated for verification and comparison of the data generated and processed by the Satellite Control Center (SCC) of the Brazil National Research Institute (INPE). Two approaches were presented. In the first one the attitude and orbital data are daily updated with real attitude data supplied by INPE. In the second approach the attitude and orbital data are not updated.

Others analysis of the solar aspect angles [1, 4] have been developed but they haven't considered the all torques acting together. Then the present theory could be more realistic than the other ones.

2. Analytical Solution for the Equations of Rotational Motion

The variations of the spin velocity, the declination and the right ascension of the spin axis for spin stabilized artificial satellites are given by the Euler equations in spherical coordinates [10], when the mean components of the external torque are included.

$$\frac{dW}{dt} = \frac{N_{gzm}}{I_z} + \frac{N_{izm} W}{I_z}, \tag{1}$$

$$\frac{d\delta}{dt} = \frac{N_{tym}}{I_z W} + \frac{N_{iym}}{I_z}, \tag{2}$$

$$\frac{d\alpha}{dt} = \frac{N_{txm}}{I_z W \cos\delta} + \frac{N_{ixm}}{I_z \cos\delta}, \tag{3}$$

where I_z is the moment of inertia along the spin axis, N_{ixm}, N_{iym}, N_{izm} are the mean components of eddy currents torques and $N_{tym} = N_{Ay} + N_{sy} + N_{gym} + N_{rym}$ and $N_{txm} = N_{Ax} + N_{sxm} + N_{gxm} + N_{rxm}$, being N_{Axm}, N_{Aym}, N_{Azm}, N_{sxm}, N_{sym}, N_{szm}, N_{gxm}, N_{gym}, N_{gzm}, N_{rxm}, N_{rym}, N_{rzm} the mean components of the aerodynamic, solar radiation, gravity gradient and residual magnetic torques, respectively. The details of each mean external torque can be observed in [1,2,3,9,10].

The differential equations of Eqs. (1) - (3) can be integrated assuming that mean components of each torque is constant over one orbital period. Then for one orbit period the solution of the differential equation of spin velocity is given by Eq. (4).

$$\text{W}= \left(W_0 + \frac{N_{gzm}}{N_{izm}}\right) e^{\frac{N_{izm}}{I_z}t} - \frac{N_{gzm}}{N_{izn}}. \tag{4}$$

In order to get the solution of the others equations, it is necessary to substitute the solution given by Eq. (4) in the Eqs. (2) and (3) for declination and right ascension of spin axis, respectively, and after some algebraic manipulations the analytical solution can simply be expressed by Eqs. (5) and (6).

$$\delta = \frac{t}{I_z}\left(N_{iym} - \frac{N_{tym}\,N_{izm}}{N_{gzm}}\right) + \frac{N_{tym}}{N_{gzm}} \ln\left(\frac{W}{W_0}\right) + \delta_0 \,, \tag{5}$$

$$\alpha = \frac{t}{I_z \cos\bar{\delta}}\left(N_{ixm} - \frac{N_{txm}\,N_{izm}}{N_{gzm}}\right) + \frac{N_{txm}}{N_{gzm}\,\cos\bar{\delta}} \ln\left(\frac{W}{W_0}\right) + \alpha_0 \,, \tag{6}$$

with: $\bar{\delta} = \frac{\delta - \delta_0}{2}$, δ is the computed declination, W_0, δ_0 and α_0 are the initial values for spin velocity, declination and right ascension of the spin axis.

These solutions are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth oblateness. With this approach, the analytical theory will be close to the real attitude behavior of the satellite.

The goal of this paper is to analyze the solar aspect angle η, which is the angle between the spin axis $\hat{k}$ and the Sun direction $\hat{u}$, which can be computed by the dot product of the Sun direction $\hat{u}$ and the spin axis $\hat{k}$, and depend on the declination (δ) and the right ascension (α) angles of the satellite spin direction and the declination (δ_S) and the right ascension (α_S) angles of the Sun direction. They are represented in the Fig.2.

Then solar aspect angle can be computed by Eq.(7).

$$\cos\eta = \hat{u} \bullet \hat{k} = M \quad \text{and} \quad \eta = acos\, M \quad \text{with } 0^o < \eta < 180^o \,. \tag{7}$$

For the mission of the satellite SCD1 and SCD2 there is a restriction for the solar aspect angle: $60^o < \eta < 90^o$ for SCD1 and $80^o < \eta < 100^o$ for SCD2 [4,6].

3. Applications

The theory developed has been applied to the spin stabilized Brazilian Satellite SCD1 and SCD2 for verification and comparison of the theory against data generated by the SCC of INPE. Operationally, SCC attitude determination comprises: sensors data pre-processing, preliminary attitude determination and fine attitude determination. The pre-processing is applied to each set of data of the attitude sensors collected from every satellite that pass over the ground station. Afterwards, from the whole pre-processed data, the preliminary attitude determination produces estimative to the spin velocity vector from every satellite that pass over a given ground station. The fine attitude determination takes (one week) a set of spin velocity vector and estimates dynamical parameters (spin velocity vector, residual magnetic moment and Foucault parameter). Those parameters are further used in the attitude propagation to predict the need of attitude corrections.

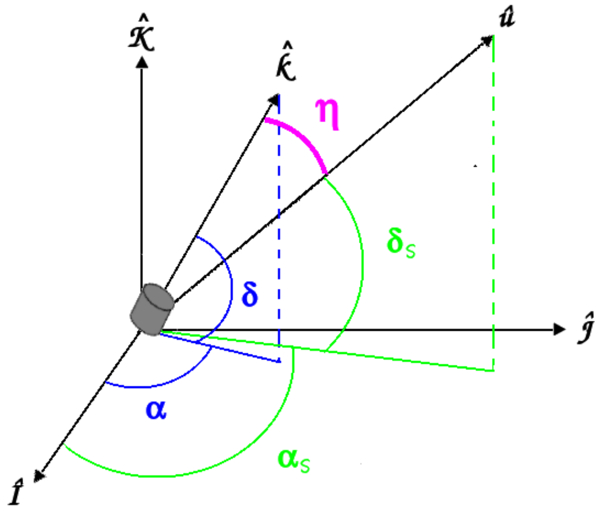


Figure 2. Solar angle η.

Two approaches are assumed to examine the influence of the considered external torques actuating during the evolution of rotational motion of the satellite. In the first approach the attitude and orbit data are updated every 24 hours with the data generated by SCC of INPE. In the second approach the computed attitude and orbit data aren't updated in order to determine the validate period of the analytical solution. In all numerical simulation the orbital elements are updated taking into account the main influence of the Earth oblateness.

3.1. Results for SCD1 satellite

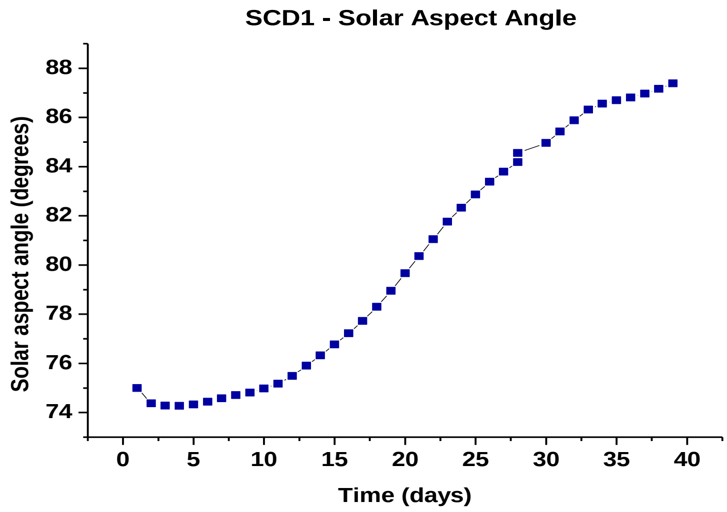
The initial conditions of attitude had been taken for date of July, 24th, 1993 at 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC) for 40 days and can been found in [3].

The results for the first approach are shown in Fig. 3. Figure 3a presents the results for temporal behavior of the solar aspect angle. Figure 3b represents the deviation between the computed values and real values of this angle and it is important to observe that they are smaller than 0.5^o for the time simulation, which is within the dispersion range of the attitude determination system performance by SCC (0.5^o). Then the mean error deviation for the solar aspect angle was 0.23127^o for the time simulation.

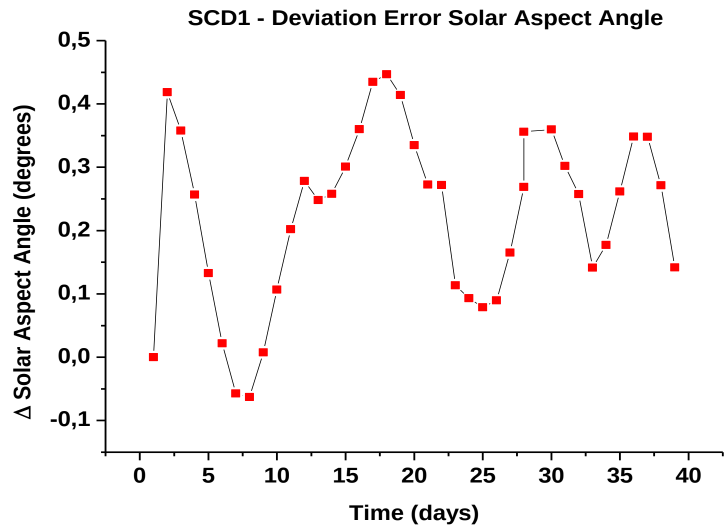
Table 1 presents the results for this approach for 2 days of two different periods. The simulations were interrupted in the 3rd day because the mean deviations errors between the computed values and real values for the spin velocity, right ascension and declination were bigger than INPE's required precision. Other simulations were done for different initial data but in all of them the results were similar, which means that the computed values have a good agreement with the real data only for the 2 days simulations.

Table 1. Deviation between computed and real values without the daily updated data for SCD1.

Day	$\Delta\eta$ (°)	Day	$\Delta\eta$ (°)
07/27/1993	0	08/25/1993	0
07/ 28/1993	0.13293	08/26/1993	0.18526
07/29/1993	0.13945	08/27/1993	0.48049



(a)



(b)

Figure 3. Results for SCD1 with daily updated data a) Temporal behavior of the solar aspect angle η, (b) Difference of computed and real values of the solar angle η .

3.2. Results for SCD2 satellite

The initial conditions of attitude had been taken from February, 1^{st}, 2002 at 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC) for 40 days and can been found in [3]. It is important to observe that when the attitude control is acting, the data are equal to the data supplied by SCC, because the theory doesn't include control torques.

Figure 4 present the results for temporal behavior of the solar aspect angle and the deviation between the computed values and real values of this angle, respectively. It can also be observed that they are smaller than 0.25^{o}, with the mean error deviation for the solar aspect angle is 0.10614^{o} for the time simulation. Note that the differences are zero when the attitude control is acting. Then they are within the dispersion range of the attitude determination system performance by SCC.

Table 2 presents the results for this approach for 2 days of two different periods. The same way of the SCD1, the simulations were interrupted in the 3rd days because the mean deviations errors between the computed values and real values for the others parameters were bigger than INPE's required precision.

Table 2. Deviation between computed and real values without the daily updated data for SCD2.

Day	**$\Delta\eta$ (°)**	**Day**	**$\Delta\eta$ (°)**
02/09/2002	0	03/05/2002	0
02/ 10/2002	-0.24611	03/ 06/2002	0.01108
02/11/2002	-0.24029	03/07/2002	-0.04099

4. Conclusions

In this paper an analytical approach for the spin-stabilized satellite rotational motion was presented taking into account the influence of torques: aerodynamic, gravity gradient, solar radiation, residual magnetic and eddy currents. The goal was to analyze the influence of these torques in the solar aspect angle.

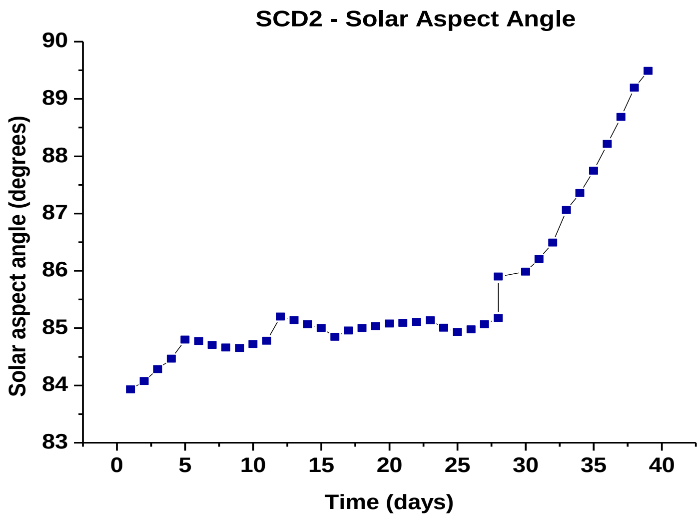
The analytical solution shows that the spin velocity has an exponential variation due to the eddy current torque and a linear variation due to the gravity gradient torque. The combinations of the others torques contribute for a precession and drift on the spin axis, due to the temporal variation on the right ascension and declination angles respectively. The theory was applied to the spin stabilized Brazilian's satellites SCD1 and SCD2. Two approaches were developed.

In the first one the attitude and orbital data are daily updated with real attitude data supplied by INPE. The results showed a good agreement between the computed and real data during 40 days. The mean error deviation for the solar aspect angle was 0.23127° for SCD1 and 0.10614° for SCD2, which are within the dispersion range of the attitude determination system used for this satellite.

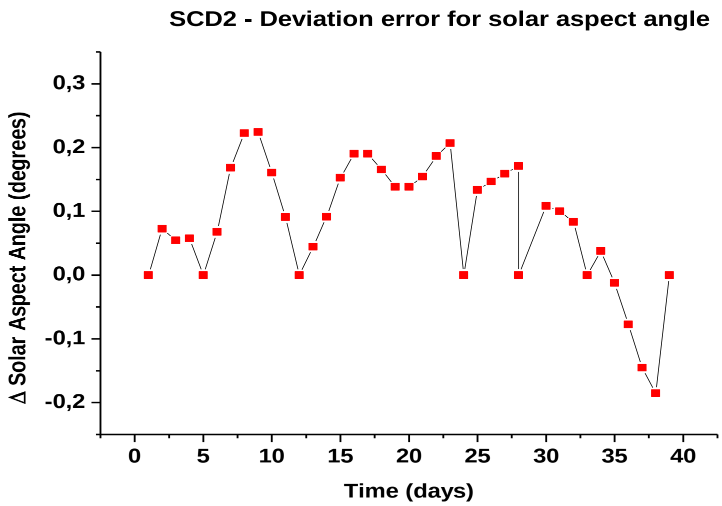
In the second approach the attitude and orbital data are not updated. The results presented a good agreement between the analytical solution and the actual satellite behavior only for two days simulation. For more than 2 days the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required for SCC (0.5°).

For both approaches it is possible to note the influence of the declination of the spin axis in the calculation of the solar aspect angles. Results have shown the agreement between the analytical solution and the real satellite behavior for specific time simulation and two approaches were presented. Then the theory has consistency and can be applied to predict the behavior of the solar aspect angle.

Others analysis of the solar aspect angles [1, 4] have been developed but they haven't considered the all torques acting together. Then the present theory could be more realistic than the other ones.



(a)



(b)

Figure 4. Results for SCD2 with daily updated data: (a) Temporal behavior of the solar aspect angle η, (b) Difference of computed and real values of the solar angle η .

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