

Out-of-plane orbital maneuvers using swing-bys with the Moon

J B S Neto¹, A F B A Prado¹ and J K S Formiga¹

¹National Institute for Space Research –INPE, Av dos Astronautas 1758, SJC-SP, Brazil.

E-mail: neto.jbs91@gmail.com.br

Abstract. This paper has the goal of showing some cases of plane change maneuvers using a swing-by with the Moon to decrease the magnitude of the impulses used, when compared to a classical Hohmann maneuver. The analytical model is based on the "patched-conics" approach, where a series of "two-body" problems is considered to build the whole maneuver. A study of the effects of the semi-major axes of the transfer orbits were made, to complete some previous studies made in the literature. The results show that, for some final inclinations, the use of the swing-by in the Moon is really advantageous.

1. Introduction

During the design phase of artificial satellite missions, one the most important factors is the cost. Optimization methods are usually applied to search for lower cost solutions from the conception to the end of the useful life of the equipment. More recently, studies are also made considering the later disposal of the spacecraft. Looking for solutions to reduce the magnitude of the impulses required in orbital maneuvers, several methods have been developed and used, such as the classical transfers of Hohmann and Bi-elliptical [1], both of them considering coplanar orbits. Now, if the objective is to make a plane change, it is necessary to apply some kind of propulsion with a component perpendicular to the orbital plane. This type of maneuver are usually very expensive in terms of fuel consumption and different options have been considered to reduce the fuel consumption. An alternative solution, already considered in the literature, is to use a gravity assisted maneuver (swing-by) with the Moon to make the out-of-plane part of the maneuver, as shown in [2].

The swing-by is a type of maneuver that does not use propulsion based on engines, but takes advantage of the gravitational influence of a celestial body when the spacecraft passes through its zone of influence [3-6]. The smaller body that is passing by the larger one is accelerated or braked by the gravity field of the celestial body and so its trajectory is modified. Successful missions, such as Voyager 1 and 2 [7], showed the efficiency of the swing-by in saving fuel. An example of the use of this maneuver to change the orbital inclination of a spacecraft is the Ulysses mission, presented in [8], where the spacecraft was placed in a near polar orbit around the Sun, in order to observe the poles of the Sun.

This paper aims to show that, for some desired final inclinations, a swing-by with the Moon (in the Earth-Moon system) can be more economical then using plane change classical maneuvers. The



contribution of the present paper is to analyze the influence of the semi-major axis of the transfer orbit of the spacecraft, which was a point not considered in previous studies [2].

2. Methodology

The complete maneuver is divided in three parts. In the first phase an impulse is used to put the satellite in a close approach trajectory with the moon. In the second stage the three-dimensional swing-by is made and, in the last phase, two impulses are applied to take the satellite back to an orbit with the same semi-major axis and eccentricity of the initial orbit, but now with the final desired inclination.

The analytical model used is presented in [9], and it considers a system consisting of three bodies: a primary (M_1 , the Earth), a secondary (M_2 , the Moon) and a third body of negligible mass with its motion governed by the other two previous cited bodies (M_3 , the artificial satellite). The three phases of the problem is dominated by the celestial mechanics of the two bodies, so it is possible to use the patched-conics approach. Initially, it is considered that the satellite is in an orbit around the Earth that is coplanar with the orbit of the Moon. For the calculations, it was assumed a canonical system of units, where: (i) the gravitational constant is equal to one; (ii) the unit of distances is the Earth-Moon distance (384400.05 km); (iii) the angular velocity of the primaries is equal to one; (iv) the mass of the moon is given by $\mu_L = m_2/(m_1+m_2)$ (where m_1 and m_2 are the masses of the Earth and Moon, respectively), and the mass of the Earth (μ_T) is given by $\mu_T = 1 - \mu_L$, so the total mass of the system is equal to one; (v) the time unit is set such that the period of the primaries is 2π .

2.1. Initial impulse

The impulse used in the first step to generate the transfer orbit (ΔV_1) and the eccentricity of this orbit (e_1) are given by:

$$\Delta V_1 = \left(\frac{2\mu_T}{a_0(1-e_0)} - \frac{\mu_T}{a_1} \right)^{1/2} - \left(\frac{2\mu_T}{a_0(1-e_0)} - \frac{\mu_T}{a_0} \right)^{1/2} \quad (1)$$

$$e_1 = 1 - \frac{a_0(1-e_0)}{a_1} \quad (2)$$

where a_1 is the semi-major axis of the transfer orbit and its minimum value is given by $a_1 = (1 + a_0)/2$, where a_0 is the semi-major axis of the initial orbit of the satellite around the Earth and e_0 is the eccentricity of this orbit.

2.2. The Swing-by maneuver

In the second step the calculation of the swing-by starts when the satellite approaches the Moon and its motion is governed by this body.

Figure 1 shows the geometry of the swing-by, where: r_{ap} is the distance of the closest approach of the satellite with the moon (perilune) and V_p is the velocity at this point; the angles α and β specify the periapsis position in the three-dimensional space and λ is the angle formed between the vector \vec{V}_p and the horizontal plane passing by the perilune. Figure 2 shows the sum of the velocity vectors during the swing-by, where: \vec{V}_∞^+ and \vec{V}_∞^- are the velocity vectors with respect to the moon before and after the swing-by, respectively, and the angle between them is 2δ , where δ is the so called deflection angle; \vec{V}_i and \vec{V}_o are the satellite velocity vectors before and after the swing-by, respectively, with respect to the Earth; \vec{V}_2 is the velocity vector of the Moon with respect to the Earth; γ is called flight-path-angle, which is the angle between \vec{V}_2 and \vec{V}_i ; ϕ is the angle between \vec{V}_2 and \vec{V}_∞^- .

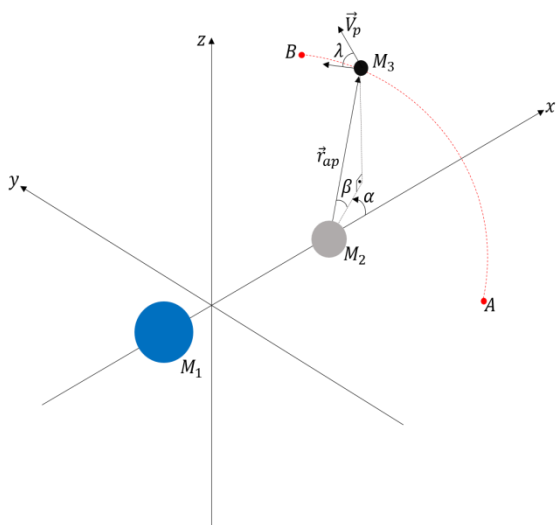


Figure 1. Geometry of the three-dimensional Swingby.

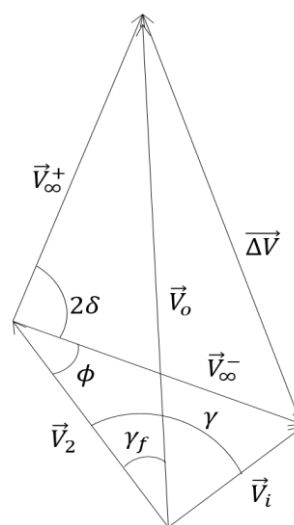


Figure 2. The sum of vectors during the swing-by.

Initially, it is calculated \vec{V}_i and the true anomaly (θ):

$$V_i = \left(\mu_T \left(2 - \frac{1}{a_1} \right) \right)^{1/2} \quad (3)$$

$$\theta = \cos^{-1} \left[\frac{1}{e_1} (a_1(1 - e_1^2) - 1) \right] \quad (4)$$

It is now possible to calculate γ and the magnitude of the velocity of the satellite with respect to the moon (V_∞):

$$\gamma = \tan^{-1} \left[\frac{e_1 \sin \theta}{1 + e_1 \cos \theta} \right] \quad (5)$$

$$V_{\infty} = (V_i^2 + V_2^2 - 2V_i V_2 \cos \gamma)^{1/2} \quad (6)$$

Here ϕ and δ are calculated by:

$$\phi = \cos^{-1} \left[-\frac{V_i^2 - V_2^2 - V_\infty^2}{2V_2 V_\infty} \right] \quad (7)$$

$$\delta = \sin^{-1} \left[\left(1 + \frac{r_{ap} V_{\infty}^2}{\mu_L} \right)^{-1} \right] \quad (8)$$

The approach angle α and the angle λ are given below, respectively, by:

$$\alpha = \pi + \phi + \delta \quad (9)$$

$$\lambda = \sin^{-1}[-\tan(\delta)\tan(\beta)] \quad (10)$$

The vectors \vec{V}_i and \vec{V}_o are given by [9]:

$$\begin{aligned} \vec{V}_i = \vec{V}_\infty^- + \vec{V}_2 = V_\infty \sin \delta (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta) + \\ + V_\infty \cos \delta (-\sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha, -\sin \gamma \sin \beta \sin \alpha + \\ + \cos \gamma \cos \alpha, \cos \beta \sin \gamma) + (0, V_2, 0) \end{aligned} \quad (11)$$

$$\begin{aligned} \vec{V}_o = \vec{V}_\infty^+ + \vec{V}_2 = -V_\infty \sin \delta (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta) + \\ + V_\infty \cos \delta (-\sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha, -\sin \gamma \sin \beta \sin \alpha + \\ + \cos \gamma \cos \alpha, \cos \beta \sin \gamma) + (0, V_2, 0) \end{aligned} \quad (12)$$

\vec{V}_i and \vec{V}_o allows the calculation of the impulse generated by the swing-by ($\Delta \vec{V}$), given by [9]:

$$\Delta \vec{V} = \vec{V}_o - \vec{V}_i = -2V_\infty \sin \delta (\cos \alpha \cos \beta, \cos \beta \sin \alpha, \sin \beta) \quad (13)$$

$$\Delta V = |\Delta \vec{V}| = 2V_\infty \sin \delta \quad (14)$$

The calculation of the energy change due to the swing-by is given by:

$$\Delta E = \frac{1}{2}(V_o^2 - V_i^2) = -2V_2 V_\infty \cos \beta \sin \alpha \sin \delta \quad (15)$$

The angular momentum vectors before (\vec{C}_i) and after the swing-by (\vec{C}_o) are given by [9]:

$$\begin{aligned} \vec{C}_i = \vec{R} \times \vec{V}_i = d_{TL} V_\infty (0, -\sin \beta \sin \delta + \cos \beta \cos \delta \sin \lambda, \frac{V_2}{V_\infty} + \cos \alpha \cos \delta \cos \lambda + \\ + \cos \beta \sin \alpha \sin \delta - \cos \delta \sin \alpha \sin \beta \sin \lambda) \end{aligned} \quad (16)$$

$$\begin{aligned} \vec{C}_o = \vec{R} \times \vec{V}_o = d_{TL} V_\infty (0, \sin \beta \sin \delta - \cos \beta \cos \delta \sin \lambda, \frac{V_2}{V_\infty} + \cos \alpha \cos \delta \cos \lambda - \\ - \cos \beta \sin \alpha \sin \delta - \cos \delta \sin \alpha \sin \beta \sin \lambda) \end{aligned} \quad (17)$$

where $\vec{R} = (d_{TL}, 0, 0)$ is the position vector of the Moon with respect to the Earth, and d_{TL} is the Earth-Moon distance. Then, the magnitude of the angular momentum change (ΔC) is given by [9]:

$$\begin{aligned} \Delta C = |\vec{C}_o - \vec{C}_i| = |2d_{TL} V_\infty \sin \delta (0, \sin \beta, -\cos \beta \sin \alpha)| \\ = 2dV_\infty \sin \delta (\cos^2 \beta \sin^2 \alpha + \sin^2 \beta)^{1/2} \end{aligned} \quad (18)$$

After the swing-by maneuver is completed it is possible to calculate the final inclination (i_o) by [9]:

$$\cos(i_o) = \frac{C_{oz}}{|\vec{C}_o|} = \left(1 + \left(\frac{\sin \beta \sin \delta - \cos \beta \cos \delta \sin \lambda}{\frac{V_2}{V_\infty} + \cos \alpha \cos \delta \cos \lambda - \cos \beta \sin \alpha \sin \delta - \cos \delta \sin \alpha \sin \beta \sin \lambda} \right)^2 \right)^{-1/2} \quad (19)$$

2.3. Final impulses

In the third step, after the swing-by is completed, two impulses are used to return the satellite to its initial orbit (with a_0 and e_0), but keeping the modified inclination. First, it is calculated the semi-major axis of the satellite after the swing-by (a_2):

$$a_2 = \frac{\mu_T}{2\mu_T - V_o^2} \quad (20)$$

Next, it is calculated the semi-major axis of the transfer orbit (a_3) and the first impulse (ΔV_2):

$$a_3 = \frac{a_2(1+e_2) + a_0(1-e_0)}{2} \quad (21)$$

$$\Delta V_2 = \left(\frac{2\mu_T}{a_2(1+e_2)} - \frac{\mu_T}{a_2} \right)^{1/2} - \left(\frac{2\mu_T}{a_2(1+e_2)} - \frac{\mu_T}{a_3} \right)^{1/2} \quad (22)$$

When the satellite reaches the perigee of the transfer orbit one last impulse is used to circularize the orbit (ΔV_3):

$$\Delta V_3 = \left(\frac{2\mu_T}{a_0(1+e_0)} - \frac{\mu_T}{a_3} \right)^{1/2} - \left(\frac{2\mu_T}{a_0(1+e_0)} - \frac{\mu_T}{a_0} \right)^{1/2} \quad (23)$$

After the complete maneuver, all impulses are added:

$$\Delta V_t = \Delta V_1 + \Delta V_2 + \Delta V_3 \quad (24)$$

2.4. Classical Maneuver

To test the swing-by as an alternative to the more conventional types of plane change maneuver, this technique is compared to a mono-impulsive maneuver, given by [10]:

$$\Delta V_m = 2 \left(\frac{2\mu_T}{a_0(1+e_0)} - \frac{\mu_T}{a_0} \right)^{1/2} \sin \frac{i}{2} \quad (25)$$

3. Results

According to the initial objectives of the present paper and using the proposed technique, two simulations are made for each value of the semi-major axis of the transfer orbit: a_1 (minimum semi-major axis) and $5a_1$. For both simulations, the initial eccentricity was $e_0 = 0$, but in the first simulation (simulation 1) the initial semi-major axis was $a_0 = 0.017$ and the periapsis distance $r_{ap} = 0.0046$ [2]. In the second simulation (simulation 2) the initial value of the semi-major axis was $a_0 = 0.069$ and for the periapsis distance it was used again $r_{ap} = 0.0046$. The initial semi-major axis and the periapsis distance are shown in canonical units (the Earth-Moon distance). The simulations are made for β ranging from 0 to π and the results are shown in figures 3 to 10, displaying the final inclination (i_o) in the vertical axis as a function of β in the horizontal axis, both in radians. Some graphics show $\Delta V_t - \Delta V_m$, the difference between the total impulse used by the maneuver that included the swing-by and the magnitude of the mono-impulsive maneuver, given in canonical units of velocity (1 canonical unit of velocity is approximately 1.023 km/s), also as a function of β in the horizontal axis.

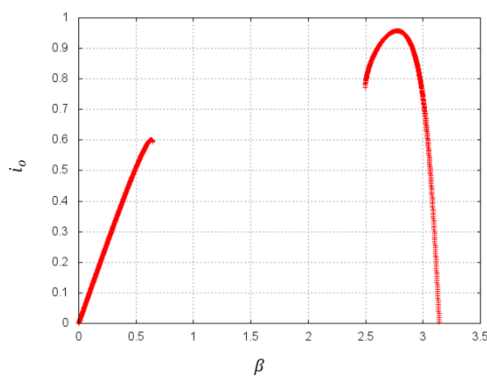


Figure 3. i_o vs. β , for simulation 1 considering the semi-major axis a_1 .

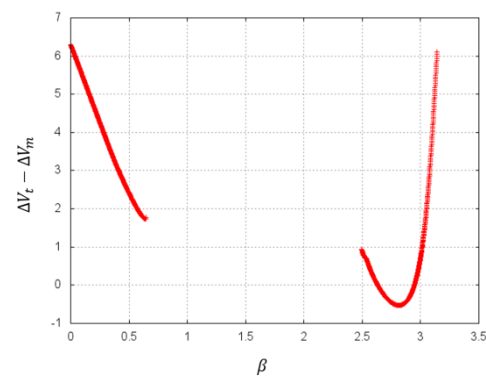


Figure 4. $\Delta V_t - \Delta V_m$ vs. β , for simulation 1 considering the semi-major axis a_1 .

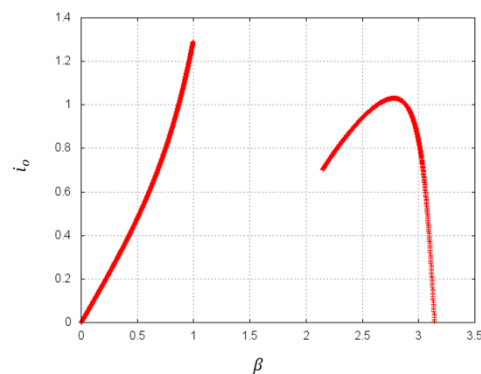


Figure 5. i_o vs. β , for simulation 1 considering the semi-major axis $5a_1$.

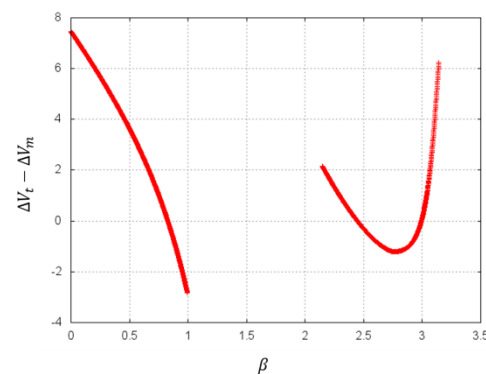


Figure 6. $\Delta V_t - \Delta V_m$ vs. β , for simulation 1 considering the semi-major axis $5a_1$.

The results for the solution 1, considering the semi-major axis a_1 , shown in figures 3 and 4, show that the use of the swing-by is justified only for the final inclinations between 0.91 rad and 0.95 rad, which corresponds to β between 2.63 rad and 2.94 rad. Inside this range the savings in the magnitude of the impulses reaches 0.52 canonical units. However, the results given by Solution 1 considering the semi-major axis $5a_1$, as shown in figures 5 and 6, indicates that the use of the swing-is justified by the first family of solutions for the final inclinations between 0.91 rad and 1.29 rad. For values of β between 0.82 rad and 0.99 rad, the savings in the magnitude of the impulse reaches values up to 2.86.

Now, for the second family of solutions, the use of the swing-by is only justified for values of β between 2.44 rad and 2.99 rad, with final inclinations between 0.91 rad and 1.3 rad, with the savings in the magnitude of the impulses up to 1.2 canonical units.

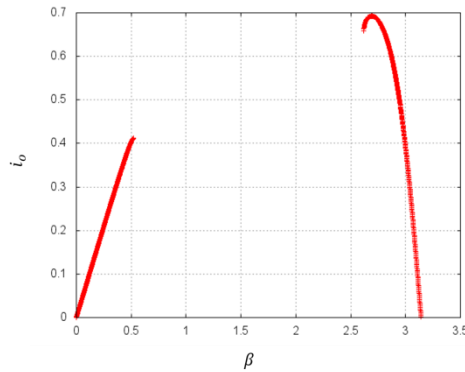


Figure 7. i_0 vs. β , for simulation 2 considering the semi-major axis a_1 .

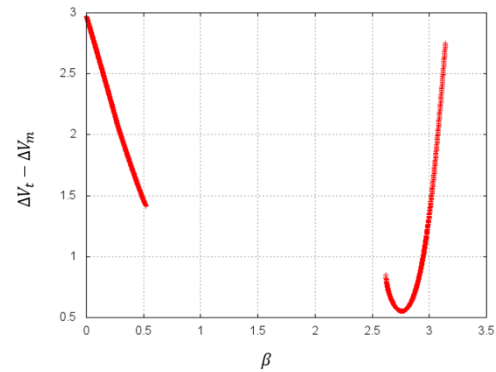


Figure 8. $\Delta V_t - \Delta V_m$ vs. β , for simulation 2 considering the semi-major axis a_1 .

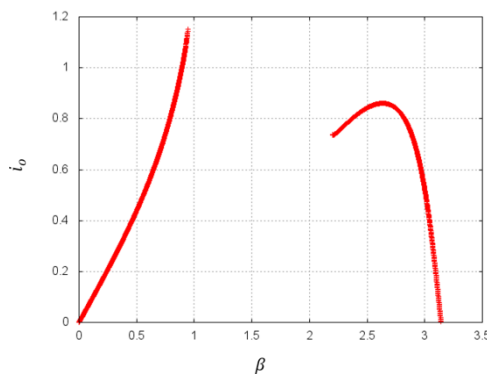


Figure 9. i_0 by β , simulation 2 for $5a_1$.

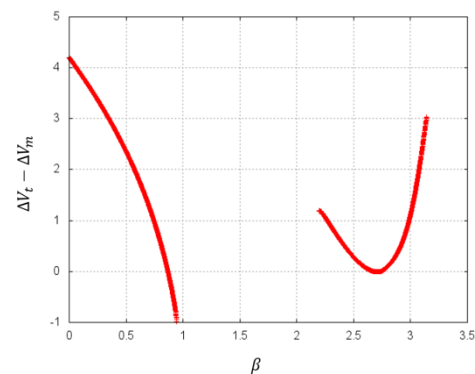


Figure 10. $\Delta V_t - \Delta V_m$ by β , simulation 2 for $5a_1$

Figures 7 and 8 show the results for the solution 2 considering the semi-major axis as a_1 . In this case the use of the swing-by is not justified. The results of the solution 2 for a semi-major axis of $5a_1$ are shown in figures 9 and 10. It indicates that the use of the swing-by is only justified for the first family of solutions and for the final inclinations between 0.93 rad and 1.15 rad, which corresponds to β between 0.86 rad and 0.94 rad. The economy in the magnitude of the impulse can reach up to 0.98 canonical units.

In general, the results are according to the expected for this type of maneuver. There is a cost to send the spacecraft to the Moon and back, even in planar maneuvers. To take advantage of this maneuver, it is necessary to have savings in the out-of-plane maneuver that compensates this extra consumption. This is the physical reasons of the limitations in the region of advantages of the proposed technique. It can be applied when in the situations where the inclination change is larger, so the savings in this part of the maneuver compensates the extra costs involved. Although expected, the results shown in the present paper quantifies the exact locations of the regions where the maneuver has gains over the standard ones and how much are the gains. In particular, the present paper extends studies made before by considering the effects of the semi-major of the transfer orbit in the savings obtained. This is a free important parameter when designing a mission.

4. Conclusion

After analyzing the results, it is concluded that, for some final inclinations desired for the orbits, the use of the swing-by maneuver as an alternative to the classical maneuvers can be a very good option, generating savings even greater than 1 canonical units of velocity. Regarding the analysis of the influence of the semi-major axis of the transfer orbit, the result show that larger semi-major axis allows the existence of new families of solutions, with higher final inclinations. This type of analysis can help mission designers to design missions that can benefit from the technique shown here.

5. References

- [1] Prado A F B A and Rios-Neto A 1993 Um estudo bibliográfico sobre o problema de transferências de órbitas *Revista brasileira de ciências mecânicas* **15** pp 65-78
- [2] Torre K S and Prado A F B A 2006 Changing inclination of earth satellites using the gravity of the moon *Mat. Prob. Eng.* **2006** pp 1-13
- [3] Gomes V M and Prado A F B A 2010 A study of the impact of the initial energy in a close approach of a cloud of particles *WSEAS Trans. Mat.* **09** pp 811-820
- [4] Prado A F B A and Broucke R A 1995 Effects of atmospheric drag in swing-by trajectory *Acta Astr.* **36** pp 285-290
- [5] Gomes V M and Prado A F B A 2008 Swing-by maneuvers for a cloud of particles with planets of the solar system *WSEAS Trans. Ap. Theor. Mec.* **03** pp 869-878
- [6] Prado A F B A and Broucke R A 1995 A classification of swing-by trajectories using the moon *App. Mec. Rev.* **48** pp 138-142
- [7] Kohlhase C E and Penzo P A 1977 Voyager mission description *Sp. Sc. Rev.* **21** pp77-101
- [8] Wenzel K P, Marsden R G, Page D E and Smith E J 1992 The ULYSSES mission *Astr. and Astrop. Suppl.* **92** pp 207-219
- [9] Prado A F B A 2000 An analytical description of the close approach maneuver in three dimensions *Proc. Int. Astr. Congress* (Rio de Janeiro)
- [10] Roy A E 1988 Orbital motion (Bristol)