Searching for less perturbed elliptical orbits around Europa

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Abstract. Space missions intending to visit Europa, one of the famous Galilean's moons of Jupiter, are among the most important topics in space activities today. There is an increasing interest in the scientific community to send spacecrafts to be inserted into Europa's orbit, with goals like mapping its surface and gravitational field. From the quality of the observations until the orbital maneuvers, the required aspects for the success of the mission will depend on the orbits used by the spacecraft. The present work searches for less perturbed elliptical orbits around Europa, because they are very important, since these orbits are expected to be more stable to place the spacecraft. The development of the study is based on the net effects of the perturbing forces over the time, evaluated by the integral of those forces with respect to the time. The value of this integral depends both on the dynamical model and the orbit of the spacecraft. Jupiter's third-body perturbation and the J_2 and J_3 terms of the gravitational potential of Europa are the perturbing forces considered. The results presented here are obtained by performing numerical integrations of the perturbing forces, and they show the locations of the less perturbed orbits.

1. Introduction

Currently, there exist plans to send spacecrafts to study the planetary moons of our Solar System. Europa is among the group of bodies with greater potential to receive scientific missions in the next decade[1]. The possibilities to explore the inner oceans and the search for organic molecules to better understand the complexity of the existence of life in the universe are among the most important objectives. Nowadays, the most advanced project under development for this purpose is the Jupiter Icy Moon Explorer (JUICE). This one and other similar types of missions require near-circular, near-polar orbits [1].

The present paper has the objective of mapping elliptical orbits around Europa, in order to search for the less perturbed ones, which can be used in some types of missions. Less perturbed orbits have smaller variations of Keplerian elements. In the majority of the cases, they demand less station keeping maneuvers for these types of space missions. The search is based on the integral of the perturbing forces over the time, because this quantity can express the net velocity variation that the spacecraft received from the perturbation forces. Several previous works in the literature used this technique to study problems related to orbital maneuvers, fuel consumption, maneuvering time, and so forth [2–10].

[1] NASA's plans to Europa: http://www.jpl.nasa.gov/missions/europa-mission/

For the present study, the third-body perturbation due to Jupiter and the terms J_2 and J_3 of the gravitational potential of Europa are considered. Jupiter is assumed to be in an orbit coplanar with the Europa's equatorial plane. This will be the reference plane. We did not find information in the literature regarding the influence of the other Galilean moons and the Sun in the dynamics of a spacecraft around Europa. Therefore, we performed computations of the direct maximum perturbations from these bodies and the minimum direct perturbation due to Jupiter. The results obtained shows that the perturbation due to the Sun has maximum order of magnitude of $8.8 \times 10^{-6}\%$ of the perturbation due to Jupiter. Little influences, with the order of magnitude of 0.033%, 0.025%, 0.002% were inferred for Io, Ganymede and Callisto, respectively. The magnitude of these forces were obtained from direct evaluation using the Newton's law of gravitation.

Numerical simulations are performed to map the effects of these perturbations over the spacecraft orbits around Europa. Such mapping is shown as function of the orbital inclination and the semi-major axis. Several characteristics of this system are shown in the present study, in particular orbits with minimum perturbation effects. Low-altitude, near-polar, almost frozen (AF) orbits are very desirable for scientific missions, and they can be found using those minimum values as guidance. This study can be applied to search for orbits of spacecrafts around other celestial bodies. Thus, this paper is structured as follows. In Section 2 we present the theoretical principles used for the numerical simulations. Section 3 is devoted to the analysis of the numerical results, which present near-circular orbits. The theory developed here is used to study the behavior of a spacecraft orbiting the planetary moon Europa, where Jupiter is the disturbing body. We present plots showing the Keplerian elements of the simulated orbits. Final comments are presented in Section 4.

2. Perturbation Integral, force functions and disturbing potentials
The main effect of the forces involved in the system is to change the velocity V of the spacecraft, according to the physical law of impulse:

$$\int_0^T F dt = PI \tag{1}$$

where $PI = \Delta T$ is the variation of kinetic energy per unit of mass.

This integral will be called *Perturbation Integral PI*, due to its physical meaning, and will be measured in km/s. This concept was presented in [7] and this expression will be used to evaluate the changing on the orbital velocity of the spacecraft. The larger is this value, the more are the variations of the Keplerian elements and, consequently, the larger is the fuel consumption required for the station keeping in the majority of the cases. It makes these orbits less interesting for practical applications. The value of this integral is a characteristic of the dynamical model considered and the orbit of the spacecraft.

2.1. Force Function due to the disturbing body
It is possible to expand the potential due to a third-body in Legendre polynomials. The disturbing potential R_2 expanded up to the second order can be written in the form [3, 11]:

$$R_2 = \frac{1}{2} N^2 r^2 (3 \cos^2(S) - 1) \tag{2}$$

where r is the norm of the position vector of the spacecraft, N is the mean motion of Jupiter and S is the angle between the position vectors of the spacecraft and Jupiter, both orbiting Europa.

According to [12], the relation between the angle S and the true anomaly (f) of the spacecraft is given by $\cos(S) = \alpha \cos(f) + \beta \cos(f)$, where the coefficients α and β are presented in [13].

2.2. Force function due to the non-sphericity of Europa

Analogously to the case of R_2, it is developed now the force functions for the J_2 and J_3 (R_{J2} and R_{J3}, respectively) terms of the potential due to the non-sphericity of Europa. However, these expressions are too large to be presented here in details.

The potential of a massive body at a distance r and the force per unit of mass are related by [14]:

$$F = \frac{\partial R}{\partial r} \tag{3}$$

where $R = R_2 + R_{J2} + R_{J3}$ is the disturbing potential. All the force functions due to the disturbing potentials (R_2, R_{J2} and R_{J3}) considered in the present paper will be obtained using this last expression. So, Equation (1) is numerically integrated using a computer code developed to analyze the influence of each perturbation (R_2, R_{J2} and R_{J3}) on the orbit of the spacecraft around Europa. All the computer codes developed for this work are developed for the software Maple®.

3. Numerical Simulations and Results

The data used in the simulations are presented in Table 1 that shows the mean Keplerian elements of Jupiter, considered in a planar elliptic orbit around Europa, being these Keplerian elements denoted by the subscript *J*. The Table 1 also presents the numerical data used for evaluating the perturbations due to the non-sphericity of Europa and its mean radius, which is a value of reference for the semi-major axes used in the numerical simulations. These values were hundreds of *km* added to such mean radius.

It was developed a computer code, as mentioned previously, in order to evaluate the effect of these force functions in changing the velocity of the orbital trajectory of a spacecraft around Europa (and consequently its orbit). The effects of each *PI* due to the different disturbing potentials were calculated by this program. The results presented in Figures 1 and 2 show several characteristics that depends on the orbit of the spacecraft, like:

1) The role of each individual term of the perturbing forces (Figure 1);

2) As expected, the effects of the Jupiter perturbation increase as the orbital semi-major axis increase. On the other hand, the resulting effects from the perturbations due to J_2 and J_3 increase as the orbital semi-major axis decreases. (Figure 1);

3) The forces have different signs with respect to each other. Depending on the position of the spacecraft, some forces tend to destroy the Keplerian orbit, while others give assistance to the propulsion system to keep the nominal orbit. In such cases, the fuel consumption is reduced (Figure 1);

4) The existence of orbits with minimum value for this index with respect to the semi-major axis and inclination, for fixed values of inclination and semi-major axis, respectively (Figure 2);

5) The orbits having the semi-major axes ranging from 1760.8 up to 1960.8 km present a flat minimum, i.e., regions with inclinations from 75 to 105 deg present approximately constant values for *PI*. In special, the median value of such range ($a = 1860.8$ km) presents the lower

Table 1. Mean Keplerian elements of Jupiter around Europa (see: http://ssd.jpl.nasa.gov/) and harmonics J_2 and J_3 of Europa (see: [15] and http://ssd.jpl.nasa.gov/).

e_J	$a_J(km)$	$\omega_J(deg)$	$\Omega_J(deg)$	$M_J(deg)$
0.0094	671100	88.970	219.106	171.016

J_2	J_3	Europa's Mean radius
4.355×10^{-4}	1.3784×10^{-4}	1560.8 *km*

values for *PI*, which means that it is the semi-major axis presenting less net perturbations. This
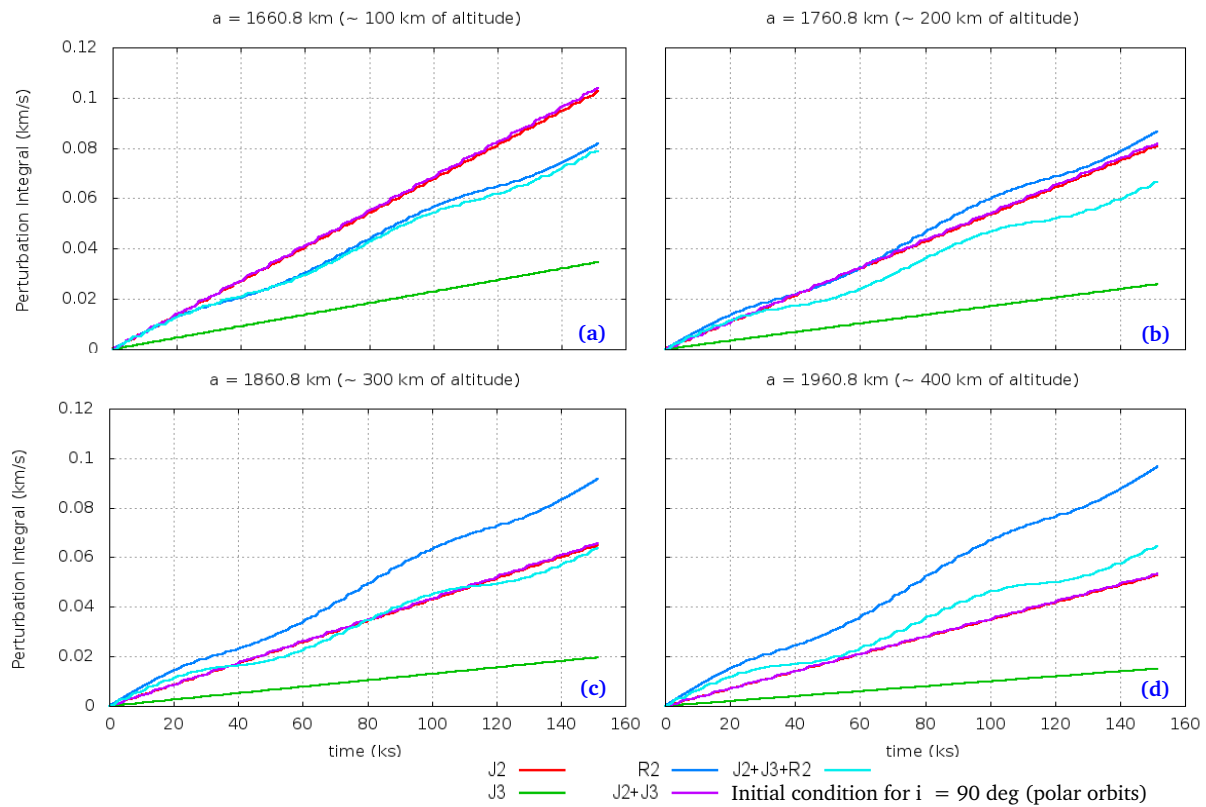result quantifies the qualitative result highlighted in the item (2) (Figure 2);

Figure 1. *PI* as a function of time for $i = 90°$ and different semi-major axes, as denoted in
each panel, for a spacecraft orbiting Europa under influence of its J_2 and J_3 terms of the gravity
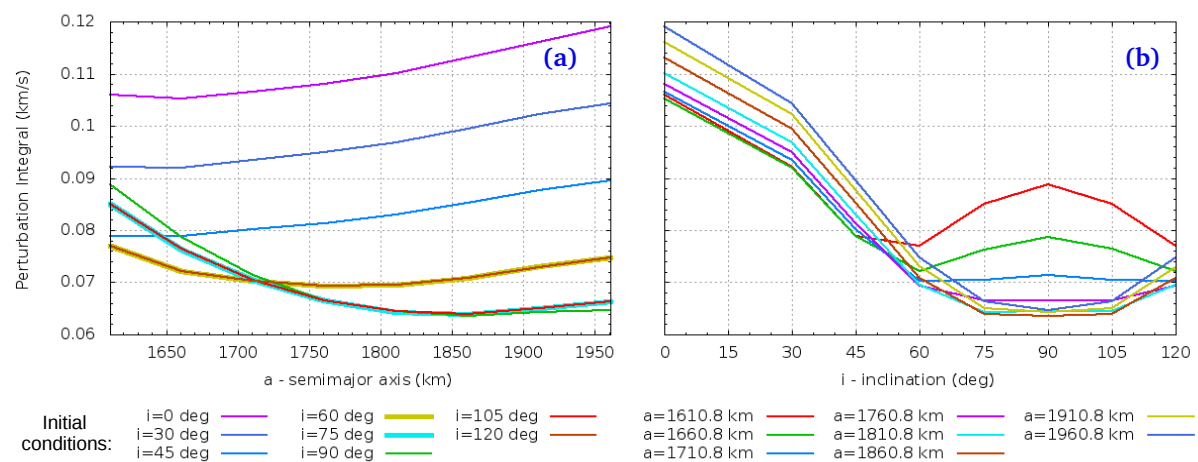field and the Jupiter's disturbing potential.

Figure 2. *PI* as a function of the spacecraft's semi-major axis (a) and orbital inclination (i).
The mean radius of Europa in Table 1 is a value of reference for the altitudes considered.

6) In all the results, the equatorial orbits for the spacecraft have PI with higher values compared to the other ones, due to the fact that the closer the spacecraft is to the bulge, the higher is the perturbations due to the oblateness. The desired orbits for the present study have high inclinations, which allow a better coverage of the Europa's surface (Figure 2);

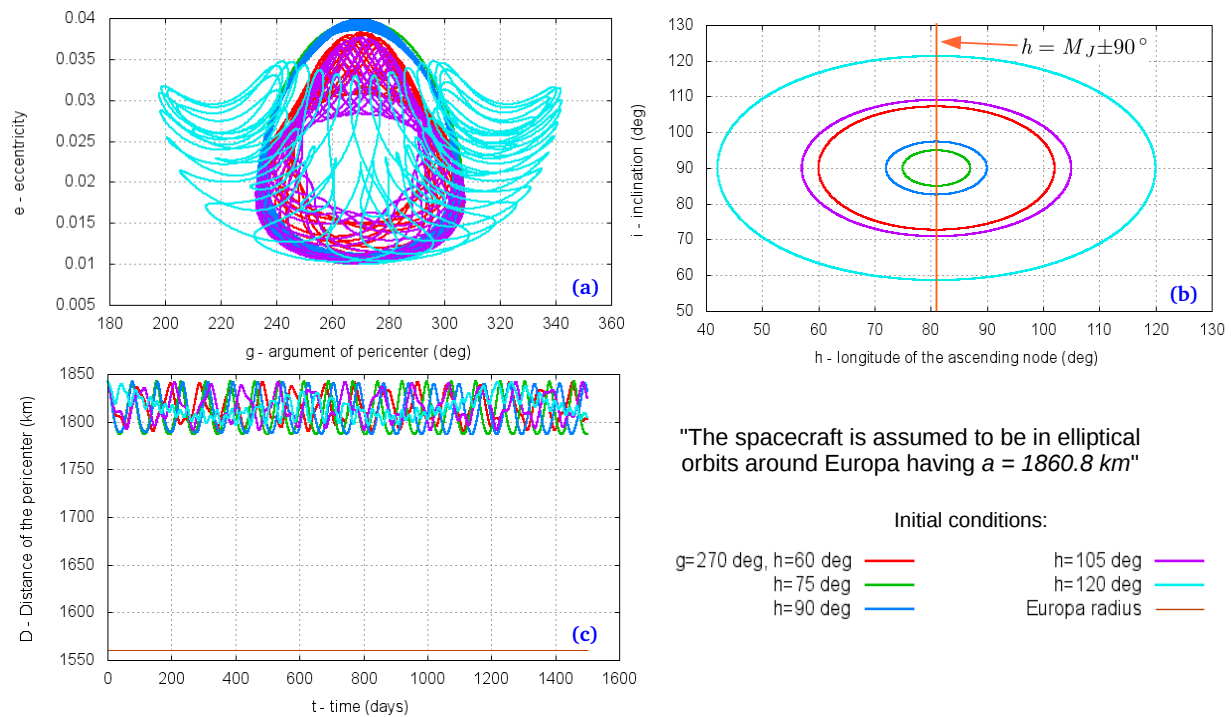7) The existence of symmetry with respect to the orbital inclination (Figure 2.b).

Figure 3. Time evolution of a spacecraft orbiting Europa. Initial conditions: $i = 90°$, $e = 0.01$, $g = 270°$, $a = 1860.8$ km. Integrations are performed for a period of 1500 days.

The PI can also be used as a good tool to search initial conditions for the Keplerian elements that will produce less perturbed orbits. Figures 1 and 2 show that lower values of PI are obtained with $a = 1860.8$ km and $i = 90°$ (polar orbits). Thus, orbits with different eccentricities are obtained using these values in a dynamical model presented in [16, 17]. This model presents the spacecraft's evolution around Europa subject to perturbation from Jupiter, assumed to be in a circular orbit. It is good approximation once $e_J = 0.0094$. The results are shown in Figure 3 and present near-circular ($e \approx 0$), near-polar (i ranging close to $90°$) AF orbits with altitudes near 300 km ($a \approx 1860.8$ km). These results were obtained by numerical integration of the Lagrange planetary equations where the disturbing potential $R = R_2 + R_{J2} + R_{J3}$ was taken into account. The analysis performed with the force function was used to obtain initial conditions that generated the AF orbits. Figure 3.a shows the eccentricity and the argument of pericenter librating around the equilibrium point for the initial value of $g = 270°$. Note that the orbits have a variation of the eccentricity around $\Delta e = 0.03$, but these orbits do not collide with the Europa's surface, where the distance of the pericenter versus time is visible in Figure 3.c. Frozen orbits are orbits having no secular perturbations in the eccentricity, inclination and argument of pericenter [18]. These orbits are periodic, except for the orbital plane of precession, and are therefore called frozen orbits [19]. Once the present orbits have a significant range in the argument of pericenter g, they cannot be considered frozen orbits, but we consider them as AF orbits ($\Delta e \approx 10^{-2}$ and $\Delta i \leq 10°$ can be smaller for values of $h \approx M_J \pm 90°$).

Average methods were not considered in the present paper ([16] did not used it for the Jupiter's motion), so the results are dependent of the initial conditions of the bodies involved. Simulations not showed here due to the limitation of space confirm that the less perturbed orbits depend on the initial position of Jupiter, so the results shown in here are valid for $M_J = 171.016\,°$ (see Figure 3.b). New calculations needs to be done if different values are used. New numerical simulations confirmed that the less perturbed orbits follows the rule $h = M_J \pm 90\,°$. It comes from the geometry of the system. This rule gives the orbit that has a maximum averaged distance Jupiter-spacecraft, so the gravitational perturbation from Jupiter is minimized.

4. Conclusions

In the present paper, the method of *Perturbation Integral* is used to evaluate the effects of some perturbations over the spacecraft's orbit and to obtain orbits that are less perturbed. A study is performed to know the effects of the main terms of the potential of Europa (J_2 and J_3) and the third-body perturbation due to Jupiter. The disturbing body is considered to be in an elliptical-planar orbit. Low-altitude, near-polar, AF orbits which are very desirable for scientific missions are found using those minimum values as guidance. This study can be applied to study orbits for spacecrafts around other celestial bodies where the existence of higher eccentricities for the disturbing body will require a more accurate model for the spacecraft's orbital evolution [20]. Theoretical interests of this type of research are: studying the effect of the non-sphericity of Europa and other celestial bodies, third-body perturbations, force functions, frozen orbits, lifetime. Practical interests are: applications to science missions to planetary moons and other small bodies of the Solar System, like dwarf planets and asteroids. The present work is an extension of the work developed in [3], which is based in the theory presented by [7].

5. Acknowledgments

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