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Three-dimensional Two-Body Tether System – Equilibrium Solutions

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Abstract. The tethered spacecraft system was studied. This is a three-body problem formed by the main body with larger mass, m_1 and m_2 are the smaller bodies and they are connected by a cable. The main body is orbited by the center of mass m_1 and m_2. The cable was assumed rigid and massless. The study goal is analyze the existing equilibrium to tether in two situations: using a control law for the cable size and constant cable size. Assuming a control law, the solutions were fixed values independent of eccentricity (e) and true anomaly (ν) of the system. For the constant case there are periodic solutions, continuous and discontinuous for different values of e and ν. The results obtained are angles functions which describe the m_1 and m_2 plane and out plane motion.

1. Introduction

The space tether systems problem has been studied in several published articles. Beletsky and Levin [1] begins by setting the scene for tethers in space. They summarized possible applications and also discussed fact and fiction, analyzing clearly the main parameters and applications for tethers system as the material density, the effective forces, orbital dynamics, mechanics models, attitude and possible disturbances for flexible tethers with end masses, massless and massive variations. Pearson (1979) [2] considered the applications lunar spacecraft formulations for data transportation and communication, using the space elevators concept, the three bodies dynamics, the Lagrangian points $L1$ and $L2$, exploring stability. The space elevator idea as a convenient and low cost way to access space was first formulated in 1960 by the Russian Yuri Artsutanov ([3] and [4]) in "Into the Cosmos without Rockets". This study analyzed the possibility of creating a space elevator connecting the Earth. The minimum requirement for a space elevator is obviously a cable strong enough to support its own weight when hanging from geostationary orbit.

Burov et al. ([5] – [9]) studied the tethers motion considering dumbbell oscillations, bodies in the central field of vibrating forces, showing stability solutions for angles on the chaotic dynamics in elliptical orbit. Analyzing the problem in other aspect of Moon-tethered pendulum there are works considering the two-body tethered system uniform rotations in planar motion and tether length control [10], and Moon-tethered system with variable tether length in restricted three-body problem [11]. In both studies [10] and [11] were suggested methods of controlling the geometric configuration. Other available studies in the literature about tether can be seen in [12] to [16].

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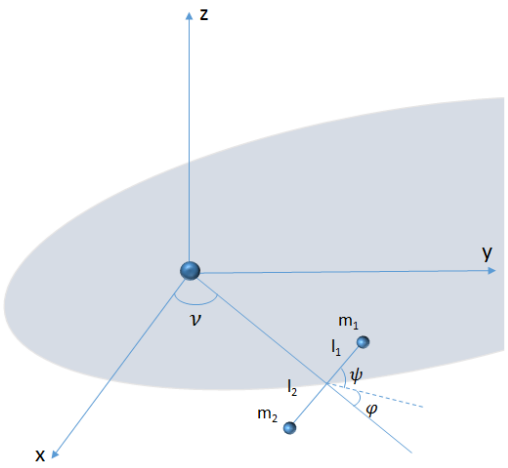


Figure 1. Dumbbell-like system model

This work is a study for a tether system with two bodies connected by a cable, which moves your center of mass in plane and around the main body. The equilibrium analysis is made for m_1 and m_2 plane and out plane movement. Based on the papers mentioned above and in particular the work [10] and [11].

2. Mathematical model

The model is a dumbbell-like system, with the two mass points (m_1 and m_2) connected with a rigid and massless tether. The system's center of mass is described by a Keplerian orbit fixed on an inert axis in primary body's center [13].

The angle ν is the tether true anomaly around primary body. The variable ρ is the distance from primary body to center of mass (CM), l_1 and l_2 are tether lengths from the mass points m_1 and m_2 to CM, respectively. φ is the angle between the plane tether projection and ρ, it describe the plane motion. The ψ is the angle between the tether and primary body plane. It describe the m_1 and m_2 out plane motion. The system geometry is presented in Figure 1.

2.1. System Positions

Since the model chosen is based on masses points and a massless tether. This allows the model to be described by Keplerian motion:

$$l_1 = \frac{m_2 l}{m_1 + m_2} \tag{1}$$

$$l_2 = \frac{m_1 l}{m_1 + m_2} \tag{2}$$

$$\rho = \frac{p}{(1 + e\cos(\nu))} \tag{3}$$

being p the focal parameter, e the eccentricity and ν the true anomaly. For the coordinates system the components of center of mass position vector (x_0, y_0, z_0) and components of point

mass i position vector $(x_i,\, y_i,\, z_i)$ are:

$$\begin{cases} x_0 = \rho\cos(\nu) \\ y_0 = \rho\sin(\nu) \\ z_0 = 0 \\ x_1 = x_0 + l_1\cos(\nu+\varphi)\cos(\psi) \\ y_1 = y_0 + l_1\sin(\nu+\varphi)\cos(\psi) \\ z_1 = l_1\sin(\psi) \\ x_2 = x_0 - l_2\cos(\nu+\varphi)\cos(\psi) \\ y_2 = y_0 - l_2\sin(\nu+\varphi)\cos(\psi) \\ z_2 = -l_2\sin(\psi) \end{cases} \tag{4}$$

2.2. Lagrange Motion Equations

The Lagrange Motion Equations, second-order ordinary differential equations, which describe the motions mechanical systems under the action of forces, can be obtained from the Lagrangian of the system given by $L = T - V$. Being T the kinetic energy and V potential energy.

For the following analysis, the generalized coordinates are φ and ψ. The system is only under the gravity-gradient. It is forces a conservative system. Two equations can be obtained based on those coordinates.

Defining φ or l allows to control the system, as can be seen in [2]. It has been chosen to define φ and consecutively obtain the l behavior.

$$\cos(\psi)\big(4p^3\dot{l}\cos(\psi)\,(\dot{\nu}+\dot{\varphi}) + l\big(3\mu_0\sin(2\varphi)\cos(\psi)(1+e\cos(\nu))^3 + \\ + 2p^3\big(-2\dot{\psi}\sin(\psi)\,\big(\dot{\nu}+\dot{\varphi}\big) + \cos(\psi)\,(\ddot{\nu}+\ddot{\varphi})\big)\big)\big) = 0 \tag{5}$$

$$\sin(2\psi)\left(3\mu_0\cos^2(\varphi)(1+e\cos(\nu))^3 + p^3\,(\dot{\nu}+\dot{\varphi})^2 + 2p^3\ddot{\psi}\right) + 4p^3\frac{\dot{l}}{l}\dot{\psi} = 0 \tag{6}$$

The motion in the spherical coordinates, ψ, φ, ν and l, where l represents the cable length, ν and φ in-plane and ψ out-plane rotation, see Figure 1. The equations 5 and 6 have been written in the time domain. Taking the true anomaly as an independent variable, assuming l behaves according to $l(\nu) = \frac{\eta(\nu)l_0}{1+e\cos(\nu)}$, being l_0 the tether initial length and $\eta(\nu)$ a new variable introduced to describe the cable behavior. The equations system is rewritten in ν domain as:

$$\frac{1}{2}l_0\cos(\psi)(1+e\cos(\nu)) \\ \left(\left(\cos(\psi)\left(\frac{3\sin(2\varphi)}{1+e\cos(\nu)} + 2\varphi''\right) - 4\left(\varphi'+1\right)\psi'\sin(\psi)\right) + \frac{4\eta'\left(\varphi'+1\right)\cos(\psi)}{\eta}\right) = 0 \tag{7}$$

$$2l_0(1+e\cos(\nu)) \\ \left(\frac{2\eta'\psi'}{\eta} + \psi''\right) + \frac{1}{2}l_0\sin(2\psi)\left(2\varphi'\left(\varphi'+2\right)(1+e\cos(\nu)) + 2e\cos(\nu) + 3\cos(2\varphi) + 5\right) = 0 \tag{8}$$

Assuming that the length l is also parameterized with the true anomaly ν, the equations 7 and 8 are the tethers oscillations equations.

3. Equilibrium points

To find equilibrium points analyzes the system with the conditions following: $\varphi'[\nu] = 0$, $\eta'[\nu] = 0$, $\psi'[\nu] = 0$ replaced in Equation 7 and 8.

$$\varphi'' = -\frac{3\sin(2\varphi)}{2(1 + e\cos(\nu))} \tag{9}$$

$$\psi'' = -\frac{\sin(2\psi)(5 + 2e\cos(\nu) + 3\cos(2\varphi))}{4(1 + e\cos(\nu))} \tag{10}$$

The equilibrium solutions to the tether in this configuration, imposes $\varphi'' = 0$ and $\psi'' = 0$ (Eq.9 and 10), the solutions obtained are: i) in plane $\psi = k\pi$ and $\varphi = \left\{k\pi, \frac{\pi}{2} + k\pi\right\}$; ii) out plane $\psi = \frac{\pi}{2} + k\pi$ and $\varphi = \left\{k\pi, \frac{\pi}{2} + k\pi\right\}$, for $k \in Z$. See Figures 2 and 3.

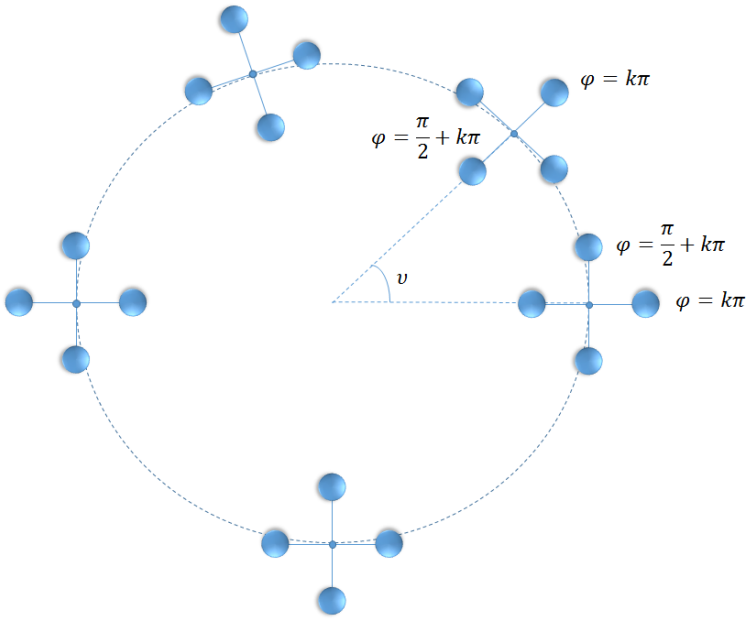


Figure 2. The geometry solutions in plane $\psi = k\pi$ and $\varphi = \left\{k\pi, \frac{\pi}{2} + k\pi\right\}$

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The Eq. 11 and 12 for the system equilibrium points were obtained by substituting variables in Eq. 5 and 6, to change the time domain to the ν domain and assuming a constant l. The solutions in plan (φ angle) for eccentricities $e \leq 0.75$ are continuous and periodic (in true anomaly ν), see Figures 4 and 5, for eccentricity $e > 0.75$ there is no physically possible motion because the function $\varphi(\nu)$ is discontinuous, however periodic.

$$\varphi'' = \frac{4e\sin(\nu) - 3\sin(2\varphi)}{2(1 + e\cos(\nu))} \tag{11}$$

$$\psi'' = -\frac{\sin(2\psi)(5 + 2e\cos(\nu) + 3\cos(2\varphi))}{4(1 + e\cos(\nu))} \tag{12}$$

Figures 4 and 5 presented the solution for $\sin(2\psi) = 0$ in the Eq. 11 and 12, which implies $\psi = \left\{k\pi, \frac{\pi}{2} + k\pi\right\}$ (Figure 2 and 3).

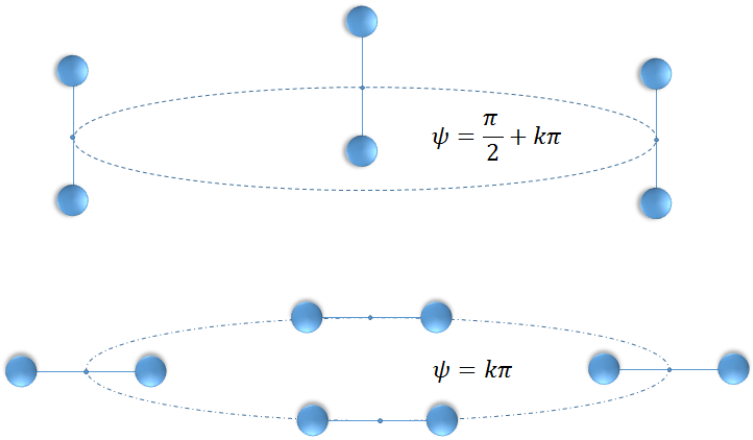


Figure 3. The geometry solutions out plane $\psi = \{k\pi, \frac{\pi}{2} + k\pi\}$ and $\varphi = \{k\pi, \frac{\pi}{2} + k\pi\}$, for $k \in Z$.

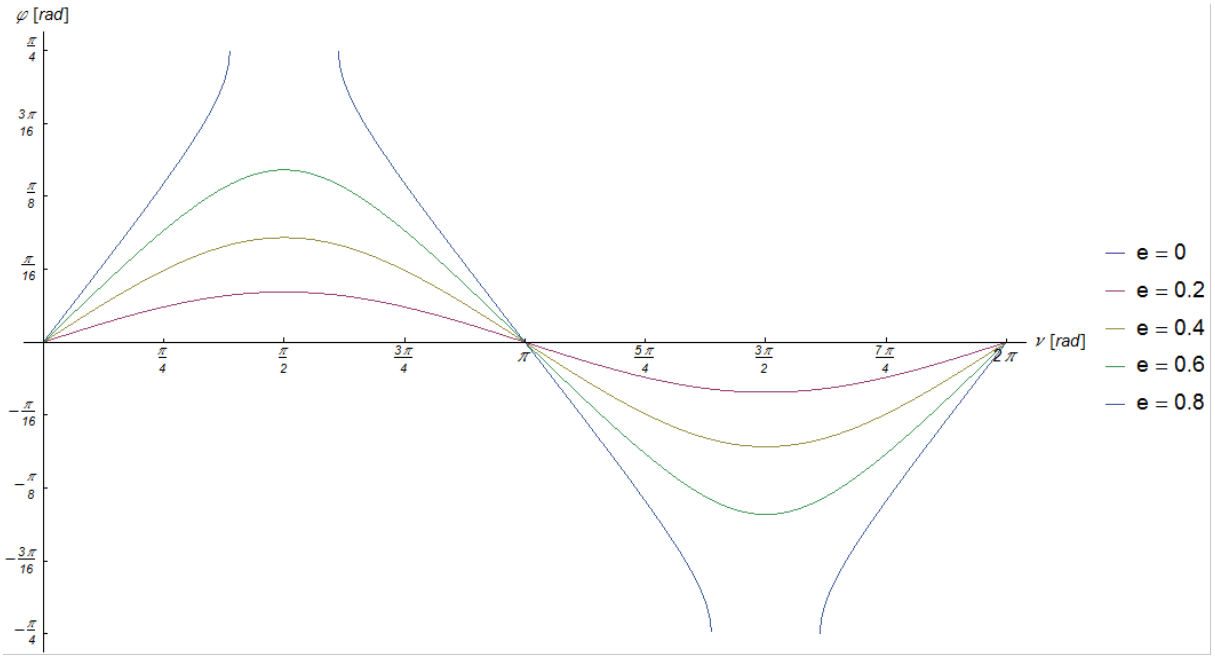


Figure 4. The periodical solutions in plane for eccentricities of the reference orbit.

4. Conclusion

The equilibrium of two mass points connected by a cable, orbiting a common center of mass, the center of mass fixed in line with the primary body was searched. The generalized coordinates of motion Lagrange Equation describing the motion in plane (φ) and out plane (ψ). This is a conservative system. Time domain change to the ν domain, due to the center of mass has a Keplerian orbit.

Mathematically, a point is in equilibrium when its speed and acceleration are equal to zero, then assume $\varphi' = \psi' = l' = \eta' = 0$ and $\varphi'' = \psi'' = l'' = \eta'' = 0$. The first case analysed was l assuming a control law dependent of e, ν, cable initial value (l_0) and a parameter η. The solutions are fixed values and independent e and ν. For constant l the equilibrium is dependent e and ν and they are continuous and periodic for $e \leq 0.75$, when e larger than this value the

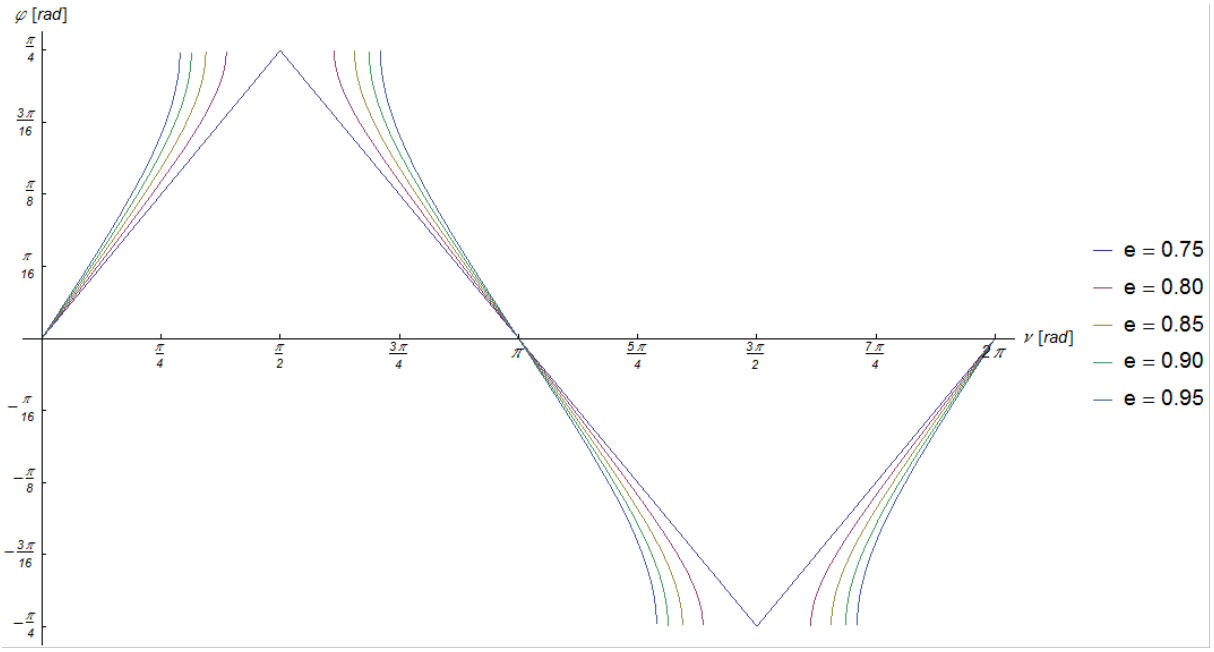


Figure 5. The periodical solutions in plane for eccentricities $e \geq 0.75$.

solutions become discontinuous in true anomaly, not allowing motion tether.

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