Journal of Physics: Conference Series **641** (2015) 012002 doi:10.1088/1742-6596/641/1/012002

Estimating the trajectory of a space vehicle passing by the Moon using Kalman Filter

A F S Ferreira[1], H K Kuga[1], A F B A Prado[1] and O C Winter[2]

[1] Instituto Nacional de Pesquisas Espaciais, INPE, São José dos Campos, Brazil
[2] Faculdade de Engenharia - Universidade Estadual Paulista, FEG-UNESP, Guaratinguetá, Brazil

E-mail: aleferrazsilva@hotmail.com

Abstract. The trajectory of a spacecraft going to the periapsis of its orbit around the smaller body of a binary system is estimated from the observed data using a Kalman Filter. Simulations of the trajectories will be made to generate the data of the spacecraft position. When observed from the Earth, using a fixed star as a reference, we have the spacecraft position as an angle function ϕ. This is the angle between spacecraft position vector and the star position vector. The result shows that Kalman Filter can be used successfully for trajectory estimation, if adequate accuracy is available for the angle observations.

1. Introduction

When a satellite passes next to a celestial body and uses the gravity of this body to gain or lose energy, there is a maneuver called Swing-by [1,2]. There are different approaches to use this maneuver. It can be propelled or not, can vary the activation point of the propellant and can be applied to circular or elliptical orbits. There are several works available in the literature about this problem [3-7].

In this paper we will estimate the spacecraft trajectory going to the periapsis of its orbit around the smaller body of a binary system. We use observed data and used a Kalman Filter [8,9]. We will estimate only the trajectory part before the passage by the periapsis, because there are multiple options after that point, e. g., the spacecraft can be captured, crash with the body or continue its trajectory.

We consider the orbital plane of the binary system as a reference plane, and the system origin is choose as the center of mass of the two main bodies.

To have the observational data, simulations are made. These simulations were performed by integrating a trajectory around the smaller body of the binary, using the model given by the restricted three body problem. When observed from Earth, using a fixed star as a reference, we have the spacecraft position as a function of an angle, given by the scalar product of the spacecraft position vector ($\mathbf{r}_s$) and the star position vector ($\mathbf{r}_e$).

The data describing the position for each instant of the trajectory, inserted with random errors, will be the observational data to be used by the Kalman Filter, to estimate the correct trajectory.

Therefore, the goal of this work is to obtain an algorithm that estimates the correct trajectory of a spacecraft, around a binary system, from this type of observed data.

2. Methods

The first step was to simulate the observations. We considered the radius of the periapsis of the trajectory of the spacecraft equals to 10 radius of the smaller body of the binary. The canonical system of units was used. Numerical integration were made to obtain the results.

Journal of Physics: Conference Series **641** (2015) 012002 doi:10.1088/1742-6596/641/1/012002

The observations are the angle (ϕ) that describes the spacecraft's position at each instant of time, observing the Earth, having a fixed star as a reference. The ϕ value was calculated by the scalar product of the spacecraft position vector ($\mathbf{r_s}$) (originated on the Earth) and the star position vector ($\mathbf{r_e}$). Being $\mathbf{r_{cm}} = (r_{cm_x}, r_{cm_y}, r_{cm_z})$ the binary system center of mass position vector (origin on the Earth), $\mathbf{r_e} = (r_{e_x}, r_{e_y}, r_{e_z})$ the star position vector, $\mathbf{r_s} = \mathbf{r_{cm}} + \mathbf{r}$ and $\mathbf{r} = (x, y, z)$ the spacecraft position vector with origin at the binary system center of mass. Since, the spacecraft moves in fixed plane $(z = 0)$. Then ϕ is given by:

$$\phi = a\sin\left(\frac{r_{e_x}(r_{cm_x} + x) + r_{e_y}(r_{cm_y} + y) + r_{e_z}(r_{cm_z})}{\left|\vec{r_e}\right|\left|\vec{r_s}\right|}\right) \tag{1}$$

$$\left|\mathbf{r_e}\right| = \sqrt{r_{ex}^2 + r_{ey}^2 + r_{ez}^2} \tag{2}$$

$$\left|\mathbf{r_s}\right| = \sqrt{(r_{cmx} + x)^2 + (r_{cmy} + y)^2 + (r_{cmz})^2} \tag{3}$$

2.1 Kalman Filter

Kalman filter is a mathematical method developed by Rudolf Kalman, in 1960, to estimate states that tend to be closer to actual values, from measurements along the time. There are related different versions of the filter, and in this work we used the extended Kalman filter, that treats nonlinear models. This version generates reference trajectories that are updated with every processing measurements. Given the dynamical system:

$$\dot{X} = f(X) + \omega = \begin{bmatrix} x_3 \\ x_4 \\ \frac{-(1-\mu)(x_1+\mu)}{((x_1+\mu)^2+x_2{}^2)^{3/2}} - \frac{-\mu(x_1-1+\mu)}{((x_1-1+\mu)^2+x_2{}^2)^{3/2}} \\ \frac{-(1-\mu)x_2}{((x_1+\mu)^2+x_2{}^2)^{3/2}} - \frac{\mu x_2}{((x_1-1+\mu)^2+x_2{}^2)^{3/2}} \end{bmatrix} + \omega, \tag{4}$$

the spacecraft state is $X = [x\ y\ \dot{x}\ \dot{y}] = [x_1\ x_2\ x_3\ x_4]$, the equations of motion are given by the restricted three body problem and ω is noise of the process that has normal distribution with zero mean and covariance Q ($\omega = N(0, Q)$).

The measurement model is given by:

$$g = h(X) + \nu \tag{5}$$

being y the measurements vector, $h(X) = \phi(X)$, where $h(X)$ relates the actual states with the observed states and ν is the measurements noise, assumed to be a white noise with zero mean and covariance R ($\nu = N(0, R)$).

The Kalman filter is divided in time-update and measurement-update cycles. The time-update process propagates the state and the covariance from the time t_{k-1} to t_k, using the equations:

Journal of Physics: Conference Series **641** (2015) 012002 doi:10.1088/1742-6596/641/1/012002

$$\dot{\overline{X}} = f(\overline{X}) \tag{6}$$

$$\overline{P}_k = \varphi_{k,k-1}\hat{P}_{k-1}\varphi_{k,k-1}^t + \Gamma_k Q_k \Gamma_k^t \tag{7}$$

where $\overline{X}$ is the propagated state vector, $\overline{P}$ is the propagated covariance matrix, $\hat{P}$ is the former updated covariance matrix and φ is the transition matrix, given by $e^{F\Delta t}$, being F the jacobian matrix of f with respect to X ($F = \frac{\partial f}{\partial X}$) and $\Delta t = t_k - t_{k-1}$ is the time interval. The measurement-update process, at this stage, corrects the state and the covariance for the instant t_k in accordance with the measurement g_{t_k}, according to the equations:

$$K = \overline{P}H^t(H\overline{P}H^t + R)^{-1} \tag{8}$$

$$\hat{P} = (I - KH)\overline{P} \tag{9}$$

$$\hat{X} = \overline{X} + K[g - h(\overline{X})] \tag{10}$$

where H is the jacobian matrix of $h(X)$ with respect to X ($H = \frac{\partial h}{\partial X}$), K is the Kalman gain, I is the identity matrix and $\hat{X}$ is the estimated state vector.

3. Results

For the results that follow we adopt the initial covariance matrix:

$$\overline{P}_0 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix},$$

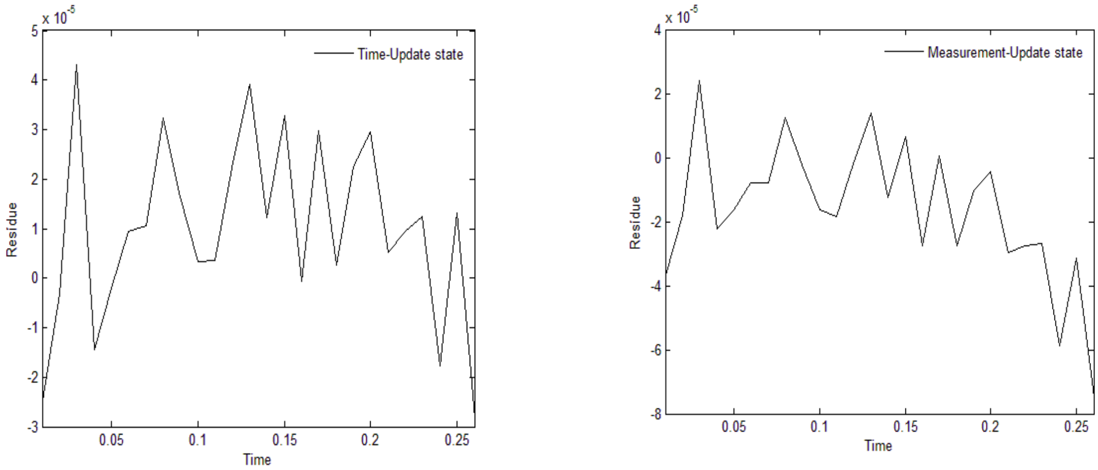
the matrix Q of the process noise is given by

$$Q = \begin{bmatrix} 10^{-5} & 0 & 0 & 0 \\ 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 10^{-4} \end{bmatrix},$$

and $R = 10^{-10}$.

The following data will show the estimated trajectory. Figure 1 shows the measurement residues of the time-update and measurement-update state as a time function. The residue is given by the difference between the measurements and the function that relates the actual states with the observed states, i.e., $g - h(\overline{X})$ for the residue of the propagated state vector (time-update process) and $g - h(\hat{X})$ for residue of the estimated state vector (measurement-update process). The data are in canonical units.

Journal of Physics: Conference Series **641** (2015) 012002 doi:10.1088/1742-6596/641/1/012002



(a) (b)

Figure 1. Measurement residues of the (a) time-update and (b) measurement-update state as a time function.

Figure 2 and 3 show the standard deviation and error for position and velocity, respectively. The error is the difference between the actual state and the estimated state.

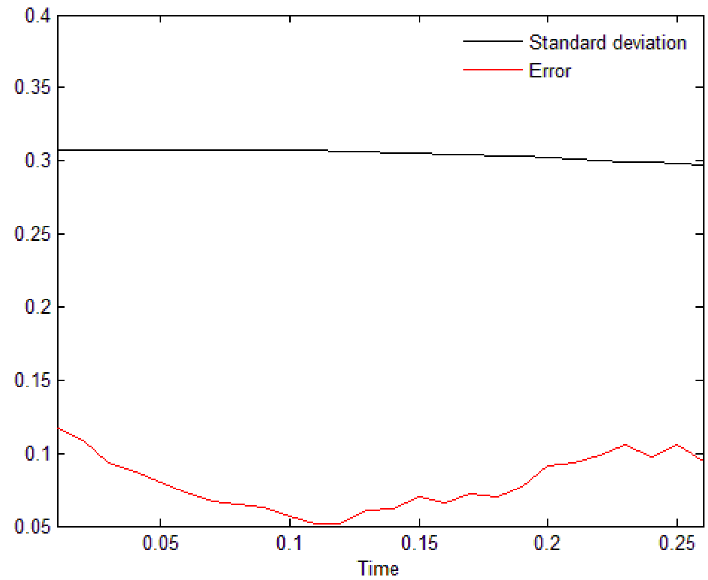


Figure 2. Standard deviation and error of the measurement-update of position.

Journal of Physics: Conference Series **641** (2015) 012002 doi:10.1088/1742-6596/641/1/012002

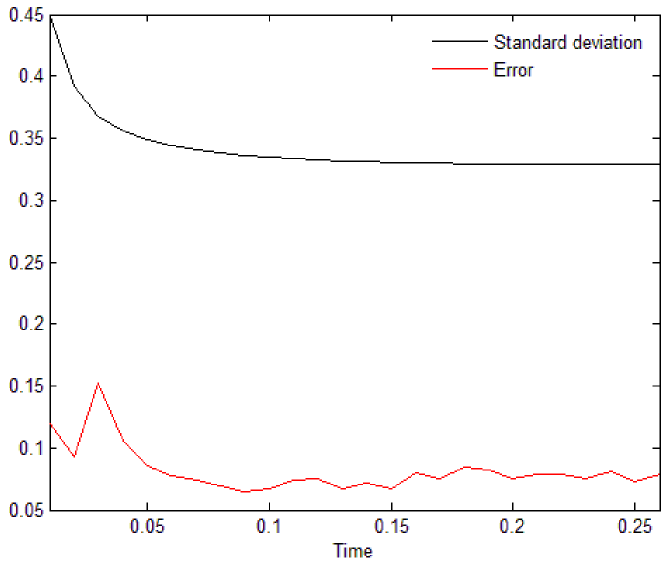


Figure 3. Standard deviation and error of the measurement-update of velocity.

A decrease in the value of the standard deviation means that the estimated value approach the actual value. Figure 4 shows the angle ϕ (in rad), for the actual, estimated and measured trajectory, as a function of time.

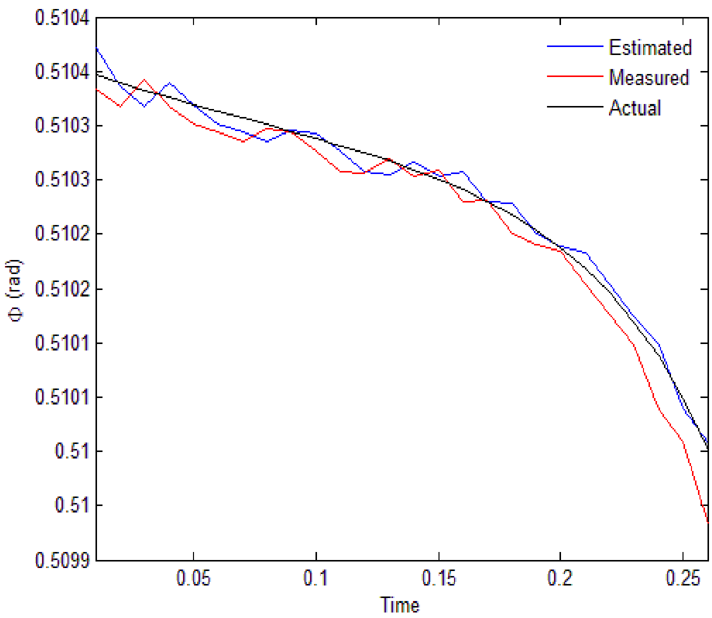


Figure 4. Position of the spacecraft given by the angle ϕ (in rad).

4. Conclusions

The spacecraft trajectory that makes a Swing-By maneuver around a binary system, estimated by the Kalman filter, was presented. The maneuver was estimated until the orbit periapsis, because there are multiple options for the sequence from there, like the capture of the spacecraft, collision or escape.

Measurements were obtained from observations made from Earth, given by the angle between the spacecraft's position and a fixed star.

It is shown that such a technique can be used successfully for trajectory estimation, if adequate accuracy is available for the angle observations. Besides, the Kalman filter was important to reduce the error in estimating the correct trajectory.

Acknowledgements

The authors wish to express their appreciation for the support provided by grants #473387/2012-3, 473164/2013-2 and 304700/2009-6 from the National Council for Scientific and Technological Development (CNPq); grants # 2011/08171-3, 2011/13101-4, 2014/06688-7 and 2012/21023-6 from São Paulo Research Foundation (FAPESP) and the financial support from the National Council for the Improvement of Higher Education (CAPES).

References

[1] Prado A F B A and Broucke R A 1995 A Classification of Swing-By Trajectories Using the Moon *Appl. Mech. Rev.* **48** 4

[2] Prado A F B A 1996 Powered Swing-By *J. of Guid., Control and Dyn.* **19** 5 pp. 1142-47

[3] Broucke R A 1988 The Celestial mechanics of gravity assist In:AIAA/AAS Astrodynamics Conference, Minneapolis, MN, (AIAA paper 88-4220)

[4] Mcconaghy T T, Debban T J, Petropoulos A E and Longuski J M 2003 Design and optimization of low-thrust gravity trajectories with gravity assist *J. Spac. and Rock.* **40** 380-387

[5] Silva A F, Prado A F B A and Winter, O C 2013a Optimal impulsive control in a powered Swing-By In: AIAA Guidance, Navigation, and Control (GNC) Conference

[6] Silva A F, Prado A F B A and Winter O C 2013b Powered Swing-By Maneuvers around the Moon *J. Phy.: Conf. Ser.* **465** 012001

[7] Strange N J and Longuski J M 2002 Graphical method for gravity-assist trajectory design *J. Spac. and Rock.* **39** 9-16

[8] Kuga H K 2005 Practical notions of estimation techniques (in Portuguese) Class Notes on Optimization of Dynamical Systems - Part II

[9] Maybeck P S 1979 Stochastic models, estimation and control. New York: Academic Press