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Preliminary Analysis of Optimal Round Trip Lunar Missions

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Abstract. A study of optimal bi-impulsive trajectories of round trip lunar missions is presented in this paper. The optimization criterion is the total velocity increment. The dynamical model utilized to describe the motion of the space vehicle is a full lunar patched-conic approximation, which embraces the lunar patched-conic of the outgoing trip and the lunar patched-conic of the return mission. Each one of these parts is considered separately to solve an optimization problem of two degrees of freedom. The Sequential Gradient Restoration Algorithm (SGRA) is employed to achieve the optimal solutions, which show a good agreement with the ones provided by literature, and, proved to be consistent with the image trajectories theorem.

1. Introduction

The problem of transferring a space vehicle from one orbit to another orbit has been growing in importance in last decades. The solution of this problem is of great concern to the space science. Also commercial applications can be found such as: the maintenance of telecommunications satellites, GPS constellations satellites, geostationary satellites and others [7]. However, the science development is the greatest benefit of this field because the trajectories design is one of many keys necessary to perform a space exploration and solve fundamentals issues of science, including the origin of life on Earth.

In the majority of the examples above the actuators of the space vehicle are assumed impulsive, which means that they produce an instantaneous velocity increment to put the vehicle in the desired trajectory. The determination of the velocity increments of the space vehicle depends upon the dynamics constraints utilized in the mathematical modeling of the system. These dynamics constraints can be divided in four class according to the literature [7]: two body model, perturbed two body model, three body model and N-body model. The two body model is the simplest one and it is more studied among all because it provides a good approximation for more complex problems and it holds analytical solutions. The so-called Hohmann transfer appears from this model [8]. The four-body model was studied by several researches [3] [10] [2]. Da Silva Fernandes and Marinho [2] have employed the gradient algorithm coupled with the Newton-Raphson method to obtain low energy Earth-Moon transfer with the Sun's perturbation.

To accomplish the optimization and to obtain the desirable trajectory it is necessary to provide an initial guess for the solution. For lunar or interplanetary mission, the well-known patched-conic approximation can be used to obtain the initial guess. It can be also applied for preliminary mission analysis [1].

The focus of the present work is to calculate and analyze optimal round trip lunar missions based on the patched-conic approximation, which has a more detailed geometry. The solution obtained by this model can be used to feed the optimization algorithm in more complex models.

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2. Objectives

The main purpose of this work is to study a full lunar patched-conic approximation considering a round trip mission and utilize this model to optimize the fuel consumption of the space vehicle by the application of the Sequential Gradient Restoration Algorithm (SGRA).

3. Formulation of patched-conic approximation

The aim of this section is to explain the round trip lunar patched-conic approximation and the optimization problem related to it. To achieve this purpose the outgoing trip is explained first. It consists of transferring a space vehicle from a circular low Earth orbit (LEO) to a circular low Moon orbit (LMO) minimizing the fuel consumption, represented by the total velocity increment. Next, the return flight is formulated similarly to the outgoing trip, but transferring the space vehicle from a LMO to a LEO. The velocity increments are assumed to be instantaneous and tangential to the corresponding orbits.

3.1. Outgoing Trip

The patched-conic approximation is based on the two-body dynamics, in which the motion of the space vehicle is described by a conic, and it uses the concept of sphere of influence. It is a simple strategy of patching two conics to describe the entire trajectory. In the lunar patched-conic approximation an ellipse and a hyperbole are used. The ellipse describes the motion of the space vehicle when the Earth gravitational field is considered (geocentric phase), and, the hyperbole describes the motion when the Moon gravitational field is considered (selenocentric phase). To obtain the complete trajectory of the relative motion of the vehicle, the two conics are connected together at the edge of the sphere of influence of the Moon. Similar to the Hohmann transfer, the patched-conic approximation involves two impulses with the total increment of velocity representing the fuel consumption [4]. A mathematical formulation of the patched-conic approximation for the outgoing trip can be found in [1]. The following hypotheses are assumed:

1) The Earth planet is fixed in space;
2) The Moon orbit around the Earth is circular;
3) The flight of the space vehicle lies in the orbital plane of the Moon;
4) The gravitational field of the Earth and the Moon are central and obey the inverse square law;
5) The transfer trajectory has two distinct phases: the geocentric phase, which it starts immediately after the first velocity increment; and, the selenocentric phase, which begins when the vehicle reaches the sphere of influence of the Moon;
6) The two impulses model is considered. Each velocity increment is applied tangentially to the initial orbit (LEO) and the final (LMO) orbit.

The patched-conic approximation for the outgoing trip is completely determined by four variables: the radial distance r_0, the velocity v_0, the flight path angle φ_0 of the space vehicle at the point of insertion in the geocentric trajectory; and, the phase angle λ_1 of the vehicle with the Earth relatively to the Moon at the moment when the space vehicle reaches the sphere of influence of the Moon. From the hypothesis 6), $\varphi_0 = 0$.

3.2. Return Trip

The mathematical formulation of the return trip is basically the same as the outgoing mission, but with the selenocentric trajectory as the starting path. The hypotheses remain the same; however, the fifth hypothesis can be better stated as it follows:

5) The transfer trajectory has two distinct phases: the selenocentric phase, which starts immediately after the first velocity increment; and, the geocentric phase, which begins when the vehicle reaches the edge of the sphere of influence of the Moon.

The return trip based on the patched-conic approximation is completely solved when the set of variables $(r_0, v_0, \varphi_0, \lambda_1)$ is known. Unlike the outgoing trip, these variables are related with the beginning of the return mission so that the radial distance r_0, the velocity v_0 and the flight path angle φ_0 are related at the point of insertion in the selenocentric trajectory. λ_1 is the same phase angle but with a distinct configuration. Again, according to the hypothesis 6), the flight path angle φ_0 is 0.

3.3. Optimization Problem based on Patched-Conic Approximation

The patched-conic approximations for the outgoing and return trips, as mentioned before, are determined through a boundary value problem, in which the iteration variable is v_0. The variables r_0, φ_0 and λ_1 must be previously specified. In order to solve this problem one can use the Hohmann transfer as initial guess of v_0. If the set (r_0, φ_0) is prescribed, then there is just one solution for each value of λ_1 if the solution exists. In this way, an optimization problem can be enunciated in order to determine some value of λ_1 which minimizes the total fuel consumption, represented by the total velocity increment. The complete optimization problem is stated as below:

"Determine the set of variables (λ_1, v_0) which minimizes the function

$$F(\lambda_1, v_0): \Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$$

subjected to the constraint

$$g(\lambda_1, v_0): r_f - r_p = 0$$

where Δv_{LEO} is the velocity increment at the circular low Earth orbit (LEO) and Δv_{LMO} is the velocity increment at the circular low Moon orbit (LMO). Note that $r_p = r_{pM}$ for the outgoing trip, and, $r_p = r_{pE}$ for the return trip, where r_{pM} and r_{pE} are, respectively, the distance at the periapsis of the selenocentric trajectory in the outgoing trip, and, the distance at the periapsis of the geocentric trajectory in the return trip. In both cases r_f is the prescribed radius"

The Sequential Gradient Restoration Algorithm (SGRA) is employed to solve this two degrees of freedom optimization problem. This algorithm has two distinct phases [6]: in the gradient phase, a displacement is applied in order to reduce the value of the function, but only satisfying the constraints at first order. In the restoration phase, a displacement is applied in order to restore the constraints, which is originally not completely satisfied due to the displacement of the gradient phase. To initialize this algorithm, an initial guess that satisfies the constraint must be provided [5]. Therefore, the boundary value problem must be solved in order to obtain this initial guess and to ensure the algorithm's convergence. The boundary value problem consists of determining v_0 to a specified set (r_0, φ_0) such that it satisfies the final condition $r_p = r_f$ to a certain λ_1.

4. Results

This section shows the main results obtained to the problem of minimizing the fuel consumption of a space vehicle in a round trip mission from Earth to Moon. The outgoing and the return trips are considered separately by patched-conic approximations. The path of the space vehicle in the LMO can be clockwise or counterclockwise. All the trajectories calculated are direct ascent. The LEO altitude is specified at 463 km, which corresponds to the altitude of the International Space Station, and the LMO altitude is set at 100 km. The table 1 illustrates a list with the values of others parameters necessary to specify the entire problem.

In the lunar patched-conic approximation, the derivatives of the expressions involved in the boundary value problems and in the optimization problem can be obtained analytically and numerically (centered differences). These two approaches are implemented in this work and good agreement of the results has been noticed. Before the optimization problem, the boundary value problem has been solved to verify the behavior of the curve of the total velocity increment with respect to λ_1 as it is illustrated in figure 1. The results of the outgoing trip are analyzed first.

Table 1. Parameters specifications

ω_M(Moon's angular velocity)	$2.649 \times 10^{-6}\ rad/s$
v_M(Moon's velocity)	1.018 km$/s$
R_E (Earth radius)	6378.2 km
R_M(Moon radius)	1738 km
μ_E (Earth's gravitational parameter)	398600 km$^3/s^2$
μ_M(Moon's gravitational parameter)	4902.83 km$^3/s^2$
D (Earth-Moon mean distance)	384400 km
R_s (Moon's sphere of influence radius)	66300 km

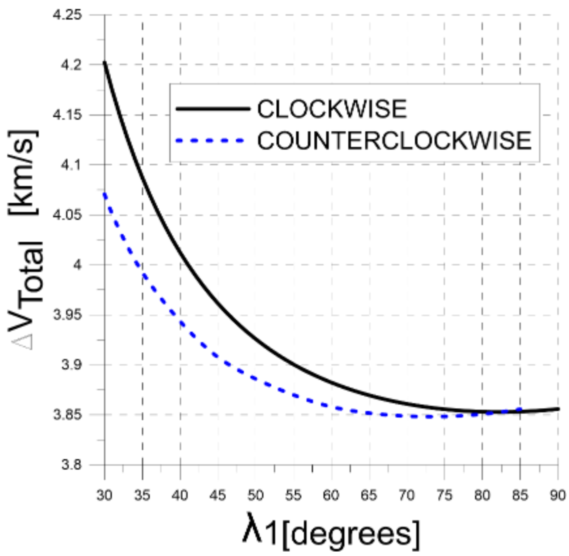

Figure 1. Outgoing trip. LEO = 463 km, LMO = 100 km.

The aspect of the curve shown in figure 1 remains the same for the others values of LMO and LEO altitudes. Note from this figure that it is possible to find a value to λ_1 which minimizes Δv_{Total} in the both cases: clockwise and counterclockwise arrival. So, the SGRA is employed to perform this task. The tolerance in the Newton-Raphson algorithm is 10^{-8}. In the optimization problem it is used a tolerance of 10^{-12} with numerical derivatives and a tolerance of 10^{-17} with analytical derivatives. The tolerance of the restoration phase in the SGRA is set in 10^{-15}.

The table 2 compares the optimized results of the patched-conic approximation obtained in this work with some results provided by the literature. The symbol '[4]' in the tables refers to the results given by [1]; and the expression 'Miele' corresponds to the results given by [6]. According to this table, a good agreement is noticed between the results obtained in the present work with the results provided by literature, mostly with the results given by [1]. When the same results is compared with the ones found by [6], a slight discrepancy occurs among the initial phase angles γ_0. Such variable is very sensitive to the algorithm precision, so its value can change with the algorithm tolerance.

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The return trip optimization problem based on the patched-conic approximation is basically the same as the outgoing mission. Furthermore, the image trajectory theorem is easily verified with the round trip trajectory in a rotating reference frame with origin coincident with the center of Earth, as shown in figures 2-5.

All the trajectories obtained in this paper are direct ascent maneuvers and feasible, i.e., there is no collision with the Moon. Figure 1 shows there is only one minimum in the total velocity increment curve. However, others sets of minimum can be found if the time of flight is considerably increased and more revolutions are performed by the vehicle around the Earth. Topputo [9] develops a systematic analysis of these minimum in a four-body model.

Table 2. Altitude LEO: 463 km. Altitude LMO: 100 km. Couterclockwise arrival. Outgoing trip.

Altitude LMO [km]	Model	Δv_{Total} [km/s]	Δv_{LEO} [km/s]	Δv_{LMO} [km/s]	*Time of flight* [days]	γ_0 [degrees]	Feasibility
100	*Patched-conic* [4]	3.8482	3.0655	0.7827	4.794	-115.723	Yes
	Patched-conic	3.8482	3.0655	0.7827	4.797	-115.691	Yes
	PCR3BP simplified [4]	3.8758	3.0649	0.8109	4.564	-116.800	Yes
	PCR3BP classic [4]	3.8777	3.0658	0.8119	4.573	-116.410	Yes
	Miele	3.8760	3.0650	0.8110	4.370	-118.980	—

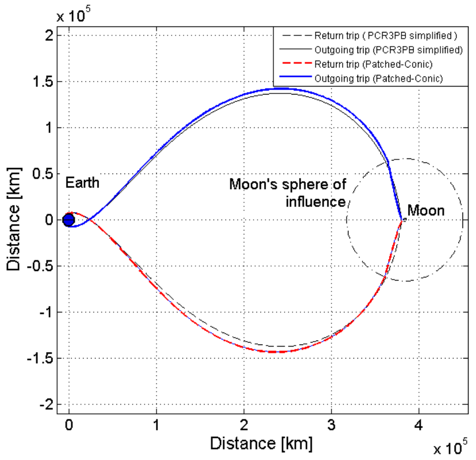

Figure 2. Round trip in the rotating reference frame. LEO = 463 km, LMO = 100 km. Counterclockwise.

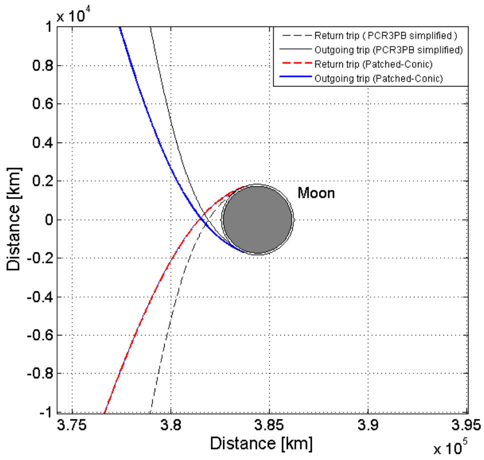

Figure 3. Zoom at LMO. Counterclockwise case.

Note that the flight of time obtained for the optimal trajectories is larger than that of Apollo missions (about 3 days), however, these last trajectories are not an optimal solution with minimum fuel consumption [6].

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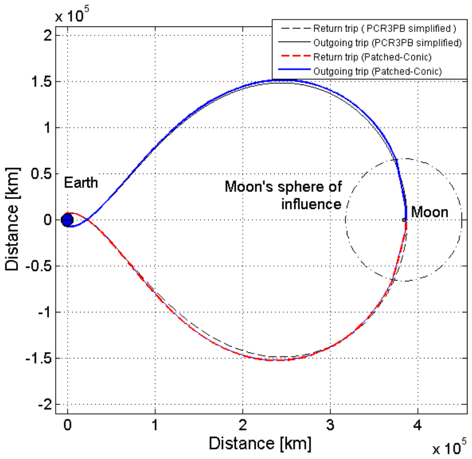

Figure 4. Round trip in the rotating reference frame. LEO = 463 km, LMO = 100 km. Clockwise.

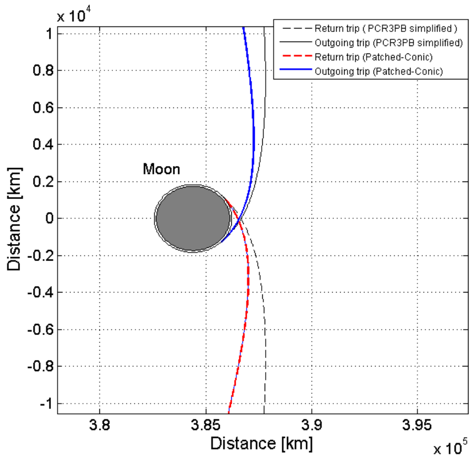

Figure 5. Zoom at LMO. Clockwise case.

5. Conclusion

In this paper, a full lunar patched-conic approximation is presented. The round trip journey of the space vehicle is obtained through optimal impulsive trajectories based on these patched-conic approximations. The Sequential Gradient Restoration Algorithm is utilized in the optimization problem, which is conducted separately for the outgoing and the return trip. The results demonstrate to be consistent with those provided by the three body problem and found in the literature. The image trajectories theorem is proven due to the symmetry between the outgoing trajectory path with the return trajectory path in the rotating reference frame. Therefore, the patched-conic approximations can be applied for preliminary mission analysis as well as used as initial guess for more complex models.

6. Acknowledgment

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