# QCD at low energy: The use of many-body methods

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**Abstract.** The exact diagonalization of the kinetic energy-term of the QCD-Hamiltonian, at low energy and restricted to a finite volume, is resumed. The Coulomb interaction term, approximated by a sum of a $1/r$ and a linear potential, is added. Only quarks and anti-quarks are considered, while the gluons are taken into account effectively via the linear potential. The Tamm-Dankoff method is applied to obtain the meson spectrum of a simplified QCD-Hamiltonian at low energy. The current status and first results of our research-program are presented.

## 1. Introduction

In [1] the kinetic energy-term of the QCD Hamiltonian at low energy, derived in [2], was diagonalized exactly. The Coulomb interaction part was approximated by a contact interaction, resulting in a second order Casimir operator of the color group $SU_C(3)$. In this approach, pairs of quark-antiquarks, being a combination of, in general, two different orbitals, play an important role. Recently, we have started to investigate a more realistic interaction, consisting of a combination of a standard Coulomb interaction $(1/r)$, and a linear potential $(r)$, simulating the static contribution from the gluons. The current status of this investigation is the topic of this contribution. We will first resume the results of the exact diagonalization. For details we refer to [1]. The Tamm-Dankoff (TD) method is used, which introduces a linear combination of pairs of quark-antiquarks at low energy. The RPA method and an extended version of it, published in [3], will be treated elsewhere [9].

## 2. The QCD Hamiltonian at low energy and the exact diagonalization of the kinetic energy

In a next step, the contact interaction is extended to a combination of a $1/r$ plus a linear potential

$$V = \frac{\alpha}{\mid \vec{x} - \vec{y} \mid} + \beta \mid \vec{x} - \vec{y} \mid \quad . \tag{1}$$

where $\alpha = -\frac{\pi}{12}$ [4] and $\beta$ is the string tension, which is considered here as a free parameter.

The QCD Hamiltonian at low energy, taking into account only the quark-antiquark part of it, and approximating the Coulomb part by a contact interaction, is given by [1, 2]

$$\boldsymbol{H}_0 \;=\; \int d^3x \boldsymbol{\psi}^\dagger \left(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\boldsymbol{\beta}m_0\right)\boldsymbol{\psi} + V_0 \int d^2x d^3y \boldsymbol{\rho}_a(\vec{x})\boldsymbol{\rho}^a(\vec{y}) \quad , \tag{2}$$

The system is restricted to a finite volume, of the size of a meson, i.e. with a radius of about 0.6 fm. As basis states for the orbital part, the harmonic oscillator functions were used. The last point seems to restrict the validity of the approximations to a non-relativistic theory, but this is only true when individual levels are considered to describe quark states. When the complete basis is used, any relativistic state can be expanded into the non-relativistic basis. This requires to expand the relativistic states into, in general, an infinite sum. When sufficient basis states are included, this sum can be limited in such a way that adding new terms would not modify the results. As shown in [8], already for a maximal number of 20 quanta, the lowest state does not change much under the increase of the number of oscillation quanta. Therefore we shall take here 20 oscillator shells. The advantage of the harmonic oscillator is that the kinetic interaction within this basis only connects states within a fixed column of a given, but otherwise arbitrary, spin $j$, reducing the numerical effort tremendously. Using solutions of the Dirac equation would mix-up all spin states and increase the dimension of the single particle space to be diagonalized.

The quantification of the fields introduces fermion creation- and annihilation-operators, where the creation-operators have the structure

$$\boldsymbol{b}^\dagger_{\alpha(N,l\frac{1}{2})j\lambda,cf} \;=\; \sum_{m\sigma}(lm,\frac{1}{2}\sigma \mid j\lambda)\boldsymbol{b}^\dagger_{\alpha Nlm,\sigma cf} \quad , \tag{3}$$

represented here in their coupled and decoupled form in spin. The first index, $\alpha = \pm\frac{1}{2}$, refers to states in the upper, positive energy (+ sign), and lower, negative energy (− sign), orbital-levels. $N$ refers to the number of oscillation-quanta of the orbital level in the Dirac picture, $l$ is the orbital angular momentum, $j$ is the total spin with projection $\lambda$ and $c$, and $f$ are the components of color and flavor respectively.

Adding the diagonal term $\sum_{Nj}\sqrt{\gamma N+m_0^2}$ to the Hamiltonian, to confine the quarks in a finite volume, gives for the kinetic energy

$$\begin{aligned}\boldsymbol{K} \;=\; & \sum_{Nj}\sqrt{\gamma N+m_0^2}\left[\left(\boldsymbol{b}^\dagger_{\frac{1}{2}(N,j+\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{\frac{1}{2}(N',j+\frac{1}{2},\frac{1}{2})j}\right)-\left(\boldsymbol{b}^\dagger_{-\frac{1}{2}(N,j+\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{-\frac{1}{2}(N',j+\frac{1}{2},\frac{1}{2})j}\right)\right] \\ & +\sqrt{\gamma}\sum_{Nj}k^j_{NN'}\left[\left(\boldsymbol{b}^\dagger_{\frac{1}{2}\Gamma(N,j+\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{-\frac{1}{2}\Gamma(N',j-\frac{1}{2},\frac{1}{2})j}\right)+\left(\boldsymbol{b}^\dagger_{\frac{1}{2}\Gamma(N',j-\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{-\frac{1}{2}\Gamma(N,j+\frac{1}{2},\frac{1}{2})j}\right)\right] \\ & +\sqrt{\gamma}\sum_{Nj}k^j_{NN'}\left[\left(\boldsymbol{b}^\dagger_{-\frac{1}{2}\Gamma(N,j+\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{\frac{1}{2}\Gamma(N',j-\frac{1}{2},\frac{1}{2})j}\right)+\left(\boldsymbol{b}^\dagger_{-\frac{1}{2}\Gamma(N',j-\frac{1}{2},\frac{1}{2})j}\cdot\boldsymbol{b}^{\frac{1}{2}\Gamma(N,j+\frac{1}{2},\frac{1}{2})j}\right)\right]\end{aligned} \tag{4}$$

The indices $\pm\frac{1}{2}=\tau$ refer to the pseudo-spin-components, where the positive sign refers to positive and the negative sign to negative energy states. The $m$, (spin $(j)$ components), $c$ (color components) and $f$ (flavor components), are not shown explicitly. because we have indicated the sum over $m$, $c$ and $f$ by scalar products. The first term in (4) is an ansatz for the kinetic energy of a particle in a restricted volume. The square root expression simulates a relativistic energy, and $m_0$ is the residual mass. In the following expressions $N$ is the number of oscillation-quanta. This implies that a quark in the level with $N$ oscillation-quanta carries, in case of no-interaction, the energy $\sqrt{\gamma N+m_0^2}$.

The factors $k^j_{NN'}$ are defined as

$$k^j_{NN'} = \sqrt{\frac{N-j+\frac{3}{2}}{2}}\delta_{N',N+1} + \sqrt{\frac{N+j+\frac{3}{2}}{2}}\delta_{N',N-1} \quad . \tag{5}$$

Eq. (4) can be written as $(i = 1, 2)$

$$\boldsymbol{K} = \sum_{j\lambda_i N_i l_i} K^j_{\tau_1 N_1 l_1, \tau_2 N_2 l_2} \left( \boldsymbol{b}^\dagger_{\tau_1 (N_1 l_1, \frac{1}{2}) j} \cdot \boldsymbol{b}^{\tau_2 (N_2 l_2 \frac{1}{2}) j} \right) \quad , \tag{6}$$

with

$$K^j_{\tau_1 N_1 l_1, \tau_2 N_2 l_2} = \begin{cases} \sqrt{\gamma} k^j_{N_1 N_2} & \text{if } l_1 = j + \frac{1}{2} \ , \ l_2 = j - \frac{1}{2} \ , \ \tau_1 \neq \tau_2 \\ \sqrt{\gamma} k^j_{N_2 N_1} & \text{if } l_1 = j - \frac{1}{2} \ , \ l_2 = j + \frac{1}{2} \ , \ \tau_1 \neq \tau_2 \\ \sqrt{\gamma N_1 + m_0^2} & \text{if } (N_1 l_1) = (N_2 l_2) \ , \ \tau_1 = \tau_2 = +\frac{1}{2} \\ -\sqrt{\gamma N_1 + m_0^2} & \text{if } (N_1 l_1) = (N_2 l_2) \ , \ \tau_1 = \tau_2 = -\frac{1}{2} \\ 0 & \text{in all other cases} \end{cases} \tag{7}$$

The diagonalization of the kinetic energy is achieved by applying a unitary transformation of the form

$$\boldsymbol{b}^\dagger_{\tau (N, l, \frac{1}{2}) j m f c} = \sum_{\lambda k} \left( \alpha^j_{\tau N l, \lambda k} \right)^* \hat{\boldsymbol{b}}^\dagger_{\lambda k j m f c} \quad . \tag{8}$$

The kinetic energy is diagonalized by diagonalizing the matrix $K^j_{\tau_1 N_1 l_1, \tau_2 N_2 l_2}$, given in (7). The eigenvalues are positive and negative, due to the structure of the matrix. The vacuum state is defined by filling-in all negative energy levels. The matrix elements $\alpha$ have the structure

$$\begin{aligned} \alpha^j_{\frac{1}{2} N l, \frac{1}{2} k} &= \gamma^j_{N l, k} \ , \ \alpha^j_{-\frac{1}{2} N l, \frac{1}{2} k} = \beta^j_{N l, k} \\ \alpha^j_{\frac{1}{2} N l, -\frac{1}{2} k} &= -\beta^j_{N l, k} \ , \ \alpha^j_{-\frac{1}{2} N l, -\frac{1}{2} k} = \gamma^j_{N l, k} \quad . \end{aligned} \tag{9}$$

The index $(\frac{1}{2}k)$ refers to the positive energy solutions after diagonalization and the index $(-\frac{1}{2}k)$ to the negative energy solutions. The index $\frac{1}{2}Nl$ refers to the *component* of the *former positive energy solutions* (the basis) and $-\frac{1}{2}Nl$ to the basis states of the *former negative energy solutions*. In practice, we take only the $\alpha$ components for $\lambda = +\frac{1}{2}$ and fix via (9) the $\lambda = -\frac{1}{2}$ components. All transformation coefficients are real. Identifying the creation (annihilation) operators in a positive energy state $(\lambda = +\frac{1}{2})$ as *quark* operators, and the creation (annihilation) operators in a negative energy state $(\lambda = -\frac{1}{2})$ as *antiquark* operators, the old creation and annihilation operators can be written in terms of the quark (antiquark) operators as:

$$\begin{aligned} \boldsymbol{b}^\dagger_{\frac{1}{2}(N l, \frac{1}{2}) j m f c} &= \sum_k \left( \gamma^j_{N l, k} \boldsymbol{b}^\dagger_{k j m f c} - \beta^j_{N l, k} \boldsymbol{d}_{k j m f c} \right) \\ \boldsymbol{b}^\dagger_{-\frac{1}{2}(N l, \frac{1}{2}) j m f c} &= \sum_k \left( \beta^j_{N l, k} \boldsymbol{b}^\dagger_{k j m f c} + \gamma^j_{N l, k} \boldsymbol{d}_{k j m f c} \right) \\ \boldsymbol{b}^{\frac{1}{2}(N l, \frac{1}{2}) j m f c} &= \sum_k \left( \gamma^j_{N l, k} \boldsymbol{b}^{k j m f c} - \beta^j_{N l, k} \boldsymbol{d}^{\dagger \ k j m f c} \right) \\ \boldsymbol{b}^{-\frac{1}{2}(N l, \frac{1}{2}) j m f c} &= \sum_k \left( \beta^j_{N l, k} \boldsymbol{b}^{k j m f c} + \gamma^j_{N l, k} \boldsymbol{d}^{\dagger \ k j m f c} \right) \quad . \end{aligned} \tag{10}$$

Plugging this into the kinetic energy and denoting the diagonal values of the matrix $K^{j}_{k_1,k_2}$ as $\tilde{\varepsilon}_{kj}$, we obtain the diagonalized energy as

$$\boldsymbol{K} \;=\; \sum_{kj} \tilde{\varepsilon}_{kj} \left[ \left( \boldsymbol{b}^{\dagger}_{kj} \cdot \boldsymbol{b}^{kj} \right) - \left( \boldsymbol{d}_{kj} \cdot \boldsymbol{d}^{\dagger\ kj} \right) \right] \quad . \tag{11}$$

## 3. Introducing a more general interaction to the quark-antiquark system

The interaction is written in second quantized form. For that, the fermion fields $\boldsymbol{\psi}$ and $\boldsymbol{\psi}^{\dagger}$ are expanded in the harmonic oscillator basis, with the coefficients representing the creation and annihilation operators of quark and antiquarks. The details will be published elsewhere [9]. The result for the interaction potential is of the form

$$\begin{aligned}
&\textstyle\sum_{\{N_i l_i j_i\}} V^{\Gamma}_{\{N_i l_i j_i\}} \left[ \left[ \boldsymbol{b}^{\dagger}_1 \boldsymbol{b}_2 \right]^{\Gamma} \left[ \boldsymbol{b}^{\dagger}_3 \boldsymbol{b}_4 \right]^{\Gamma} \right]^{0}_{0} \\
&= \textstyle\sum_{\{k_i j_i\}} \left\{ E^{\Gamma}_{\{k_i j_i\}} \left[ \left( \left[ \mathbf{b}^{\dagger}_1 \boldsymbol{b}_2 \right]^{\Gamma} - \left[ \boldsymbol{d}_1 \boldsymbol{d}^{\dagger}_2 \right]^{\Gamma} \right) \left( \left[ \mathbf{b}^{\dagger}_3 \boldsymbol{b}_4 \right]^{\Gamma} - \left[ \boldsymbol{d}_3 \boldsymbol{d}^{\dagger}_4 \right]^{\Gamma} \right) \right] \right. \\
&\quad + F^{\Gamma}_{\{k_i j_i\}} \left[ \left( \left[ \mathbf{b}^{\dagger}_1 \boldsymbol{b}_2 \right]^{\Gamma} - \left[ \boldsymbol{d}_1 \boldsymbol{d}^{\dagger}_2 \right]^{\Gamma} \right) \left( \left[ \mathbf{b}^{\dagger}_3 \boldsymbol{d}^{\dagger}_4 \right]^{\Gamma} + \left[ \boldsymbol{d}_3 \boldsymbol{b}_4 \right]^{\Gamma} \right) \right] \\
&\quad + F'^{\Gamma}_{\{k_i j_i\}} \left[ \left( \left[ \mathbf{b}^{\dagger}_1 \boldsymbol{d}^{\dagger}_2 \right]^{\Gamma} + \left[ \boldsymbol{d}_1 \boldsymbol{b}_2 \right]^{\Gamma} \right) \left( \left[ \mathbf{b}^{\dagger}_3 \boldsymbol{b}_4 \right]^{\Gamma} - \left[ \boldsymbol{d}_3 \boldsymbol{d}^{\dagger}_4 \right]^{\Gamma} \right) \right] \\
&\quad \left. + G^{\Gamma}_{\{k_i j_i\}} \left[ \left( \left[ \mathbf{b}^{\dagger}_1 \boldsymbol{d}^{\dagger}_2 \right]^{\Gamma} + \left[ \boldsymbol{d}_1 \boldsymbol{b}_2 \right]^{\Gamma} \right) \left( \left[ \mathbf{b}^{\dagger}_3 \boldsymbol{d}^{\dagger}_4 \right]^{\Gamma} + \left[ \boldsymbol{d}_3 \boldsymbol{b}_4 \right]^{\Gamma} \right) \right] \right\} \quad ,
\end{aligned} \tag{12}$$

where Eq. (10) has been used and the coefficients in (12 result to be

$$\begin{aligned}
E^{\Gamma}_{\{k_i j_i\}} &= \tfrac{1}{2} \textstyle\sum_{\{N_i l_i\}} V^{\Gamma}_{\{N_i l_i j_i\}} \quad \left( \gamma^{j_1}_{N_1 l_1, k_1} \gamma^{j_2}_{N_2 l_2, k_2} + \beta^{j_1}_{N_1 l_1, k_1} \beta^{j_2}_{N_2 l_2, k_2} \right) \\
&\qquad \left( \gamma^{j_3}_{N_3 l_3, k_3} \gamma^{j_4}_{N_4 l_4, k_4} + \beta^{j_3}_{N_3 l_3, k_3} \beta^{j_4}_{N_4 l_4, k_4} \right) \\
F^{\Gamma}_{\{k_i j_i\}} &= \tfrac{1}{2} \textstyle\sum_{\{N_i l_i\}} V^{\Gamma}_{\{N_i l_i j_i\}} \quad \left( \gamma^{j_1}_{N_1 l_1, k_1} \gamma^{j_2}_{N_2 l_2, k_2} + \beta^{j_1}_{N_1 l_1, k_1} \beta^{j_2}_{N_2 l_2, k_2} \right) \\
&\qquad \left( \gamma^{j_3}_{N_3 l_3, k_3} \beta^{j_4}_{N_4 l_4, k_4} - \beta^{j_3}_{N_3 l_3, k_3} \gamma^{j_4}_{N_4 l_4, k_4} \right) \\
F'^{\Gamma}_{\{k_i j_i\}} &= \tfrac{1}{2} \textstyle\sum_{\{N_i l_i\}} V^{\Gamma}_{\{N_i l_i j_i\}} \quad \left( \gamma^{j_1}_{N_1 l_1, k_1} \beta^{j_2}_{N_2 l_2, k_2} - \beta^{j_1}_{N_1 l_1, k_1} \gamma^{j_2}_{N_2 l_2, k_2} \right) \\
&\qquad \left( \gamma^{j_3}_{N_3 l_3, k_3} \gamma^{j_4}_{N_4 l_4, k_4} + \beta^{j_3}_{N_3 l_3, k_3} \beta^{j_4}_{N_4 l_4, k_4} \right) \\
G^{\Gamma}_{\{k_i j_i\}} &= \tfrac{1}{2} \textstyle\sum_{\{N_i l_i\}} V^{\Gamma}_{\{N_i l_i j_i\}} \quad \left( \gamma^{j_1}_{N_1 l_1, k_1} \beta^{j_2}_{N_2 l_2, k_2} - \beta^{j_1}_{N_1 l_1, k_1} \gamma^{j_2}_{N_2 l_2, k_2} \right) \\
&\qquad \left( \gamma^{j_3}_{N_3 l_3, k_3} \beta^{j_4}_{N_4 l_4, k_4} - \beta^{j_3}_{N_3 l_3, k_3} \gamma^{j_4}_{N_4 l_4, k_4} \right) \quad .
\end{aligned} \tag{13}$$

We have calculated the values of the interaction-coefficients and it turned out that the main contributions come from terms with $(k_1 j_1) = (k_2 j_2)$, $(k_3 j_3) = (k_4 j_4)$ and $L = 0$. The other interaction factors are at least 10% of these values and for $L > 1$ much smaller. If we restrict to the dominant terms, the interaction has a similar structure as the second order Casimir operator of the color group

$$\boldsymbol{C}_C(SU_C(3)) \;=\; \sum_{13} 9\sqrt{8(2j_1+1)(2j_3+1)}\left[\left[\boldsymbol{b}_1^\dagger\otimes\boldsymbol{b}_1\right]^{0L(0,0)(1,1)_1}\otimes\left[\boldsymbol{b}_3^\dagger\otimes\boldsymbol{b}_3\right]^{0L(0,0)(1,1)_1}\right]_{0000}^{00(0,0)(00)_1} \tag{14}$$

The structure of the Coulomb interaction in (12) suggests that we can write it as the sum of the second order Casimir operator, multiplied by a factor, and the difference of the total interaction to this Casimir operator times this factor, i.e.,

$$[V-\mathcal{V}_0\boldsymbol{C}_2(SU_C(3))]+\mathcal{V}_0\boldsymbol{C}_2(SU_C(3)) \quad . \tag{15}$$

The factor $\mathcal{V}_0$ is chosen such that the interaction $V$ is reproduced *in average*. Because the second order Casimir operator of the color group does not act on the physical states with color zero (it just gives zero), we are left with a reduced interaction. Note, that the factor $\mathcal{V}_0$ is not a parameter of the model, but only serves here to measure the difference, in average, between the interaction $\boldsymbol{V}$ and the second order Casimir operator (14). However, at this point we have to mention that with the approximations made, the interaction will not depend on the total flavor and spin of the meson pair, to which the Hamiltonian acts. This will result in an unnatural degeneration, which can be lifted by including at least interactions with $L=1$. They depend explicitly on the flavor and spin of the configurations upon which it acts. Because this would imply the use of a larger space ,and lengthy calculations, we opted at this stage to restrict to $L=0$. Work on including $L=1$ is in progress [9].

## 4. Tamm-Dankoff

In this section we apply the Tamm-Dankoff method, which consist of defining new pairs of quark anti-quarks, as a linear combination of the pairs $\gamma^\dagger_{12,\Gamma\mu}=\left[\boldsymbol{b}_1^\dagger\otimes\boldsymbol{d}_2^\dagger\right]^\Gamma_\mu$, where $\Gamma$ is a short hand notation for the coupling to spin $L$, flavor $(\lambda_f,\lambda_f)$ ($\lambda_f$=0,1) and color zero $(0,0)$. Again, the indices 1 and 2 refer to $(k_1,j_1)$ and $(k_2,j_2)$, respectively. The new pair is given by

$$\tilde{\gamma}^\dagger_{\kappa\Gamma\mu} \;=\; \sum_{12} C^{\kappa\Gamma}_{12}\gamma^\dagger_{12,\Gamma\mu} \quad , \tag{16}$$

which represents a mixing between all pairs formed by all possible combinations of two orbital. The index $\kappa$ labels the new pairs. It runs from zero to the number of possibilities to form pairs. The old and new vacuum coincide, because $\tilde{\gamma}^{\kappa\Gamma\mu}$ is a linear combination of the old annihilation operators. The resulting matrix equation is of the form

$$\begin{aligned}\sum_{1'2'} M^\Gamma_{12,1'2'}C^{\kappa\Gamma}_{1'2'} &= \hbar\Omega^{\kappa\Gamma}C^{\kappa\Gamma_{12}}\\ \text{with}&\\ M^\Gamma_{21,1'2'} &= \langle\tilde{0}\mid\left[\gamma^{21,\Gamma\mu},\left[\boldsymbol{H},\gamma^\dagger_{1'2',\Gamma\mu}\right]\right]\mid\tilde{0}\rangle \quad ,\end{aligned} \tag{17}$$

where the matrix $M$ does not depend on the magnetic numbers $\mu$. $M$ is well defined, using (12) and (13).

In order to obtain the spectrum, we use parameters given in [1, 8]. In the example presented here, we take for $\alpha$=-0.26 GeV ($\approx -\frac{\pi}{12}$ [4]) and $\beta$=0.094 GeV. One feature we noted is that the $1/r$ interaction does not have a big influence on the results, which is due to the low energy,

when long-range effects are dominant. In the sample calculation presented, we restricted to the $j = \frac{1}{2}$ column, and take the maximal number of oscillation quanta to be $N = 20$. The lowest pre-diagonalized single-particle states are at 0.270 GeV and the next at 0.599 GeV. In this scheme and without interaction, the lowest meson state is at 0.540 GeV. In the absence of interactions ($\alpha = \beta = 0$ GeV) there is, for the first states, a four- fold degeneracy for each combination of flavor and spin. This is explained as follows: the pre-diagonalized energies are two-fold degenerate. Then, there are four possibilities to distribute a quark and an antiquark in two states for each coupling to flavor (0,0), (1,1) and spin 0,1. With interaction, this degeneracy is barely lifted within each flavor and spin.

Once the kinetic energy is diagonalized, it provides us with a new set of basis states, called pre-diagonalized basis. The Hamiltonian *with* the interaction is then diagonalized within a subspace, which is increased in order to check convergence. Our finding is that using only up to 14 states within $j = \frac{1}{2}$ is enough to achieve convergence.

The interaction lowers significantly the states in energy, the lowest one to approximately 0.140 GeV. In the sample calculation, the degeneracy for the same flavor and spin sector is broken but, as already mentioned, not sufficiently. Also, the density of states does not correspond to the observed meson spectrum, which indicates the need of further interactions.

## 5. Conclusions

We have resumed the exact diagonalization of the kinetic energy of the QCD Hamiltonian at low energy. In the sample calculation, we restricted to a finite volume of the size of a meson (0.6 fm) and the effective mass of the quark was set to be zero. The maximal number of oscillation quanta is 20 and only states within the $j = \frac{1}{2}$ column were taken into account. The QCD-Coulomb interaction was approximated by the combination of a $1/r$ and a linear potential. We found that the dominant contributions come from terms which also appear in the second order Casimir operator. Subtracting the average to these terms did lead to an effective interaction, easy to handle numerically. The resulting interaction, however, does not differentiate significantly between different flavor and spin of the meson state. This is reflected in the results: Applying the Tamm-Dankoff method, for each flavor-spin combination there is a four-fold degeneracy of the states at low energy, for the case of no interaction. Including the QCD-Coulomb interaction breaks this degeneracy but not sufficiently. This gives a hint that further interactions have to be included, as the part of the Coulomb interaction with the intermediate coupling to L=1. Work on this line is in progress.

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[1] T. Yépez-Martínez, P. O. Hess, A. P. Szczepaniak and O. Civitarese, Phys. Rev. C *81* (2010), 045204.
[2] A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **65** (2001), 025012.
[3] O. Civitarese, P. O. Hess and D. A. Amor Quiroz, Int. J. Mod. Phys. E **22** (2013), 1350071.
[4] M. Lüscher, Nucl. Phys. B **180** (1981), 317.
[5] T. D. Lee, *Particle Physics and Introduction to Field Theory*, (World Scientific, Singapue, 1981).
[6] J. Escher and J. P. Draayer, J. Math. Phys. **39** (1998), 5123.
[7] J. P. Draayer and Y. Akiyama, J. Math. Phys. **14** (1973), 1904.
[8] T. Yépez-Martínez, *Un Modelo motivado de la Chromodinámica Cuántica a bajas energías para los niveles orbitales s y p*, PhD theis, Posgrado de Ciencias Físicas, UNAM (2011).
[9] (to be published)