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Patterns, dynamics and phase transitions in Ising ferromagnet driven by propagating magnetic field wave

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Abstract. The nonequilibrium behaviours of kinetic Ising ferromagnet driven by a propagating magnetic field wave have been studied by Monte Carlo simulation. Two types of propagating magnetic field waves are used here. Namely, the plane wave and the spherical wave. For plane propagating wave passing through the Ising ferromagnet, system undergoes a phase transition from a pinned phase to a propagating phase, as the temperature increases. The transition temperature is found to depend on the amplitude of the propagating magnetic field. A phase boundary is drawn in the plane described by the temperature of the system and amplitude of the propagating field. On the other hand, the nonequilibrium behaviours shown by the Ising ferromagnet driven by spherical magnetic field wave, are different. Here, the system exists in three different dynamical phases. The low temperature pinned phase, the intermediate temperature centrally localised breathing phase and the high temperature extended spreading phase. Here also, the transition temperatures are observed to depend upon the amplitude of the propagating magnetic field wave. The phase boundaries are drawn in the plane represented by temperature of the system and the amplitude of the propagating magnetic field wave. The two boundaries merge at the Onsager value of equilibrium critical temperature in the limit of vanishingly small amplitude of the propagating magnetic field. *This article is mainly a review of earlier works and is based on the invited lecture delivered in the conference STATPHYSKOLKATAVIII, held at SNBNCBS, Kolkata, India in December 1-5, 2014.* **This article is dedicated to Prof. H. Nishimori on the occasion of his 60th birthday.**

1. Introduction

The dynamical phase transition [1, 2] in Ising ferromagnet swept by oscillating magnetic field is an interesting field of research in nonequilibrium statistical mechanics. It was first observed [3] in kinetic Ising ferromagnet driven by oscillating magnetic field from the numerical solution of dynamical meanfield equation. Though this study helps to understand the phenomenon by the motion of the system in a double well free energy in the presence of oscillating field, it has a serious problem. Due to the absence of fluctuation, the dynamic transition in the meanfield study[3] would lead to the existence of the phase boundary even in the static limit. However, truely dynamic transition means such a transition where the transition should disappear in the static limit. This is only possible if the fluctuation is present, where the noise may push the system from one well to another even for inadequate amount of external magnetic field in the static limit, resulting the disappearance of dynamic phase transition. One possible way to study the effect of fluctuation, is the Monte Carlo simulation of the kinetic Ising model driven by oscillating magnetic field. Being motivated by this idea the Monte Carlo study was done and the disappearance of dynamic phase transition was indeed found in the static limit.

In the Monte Carlo simulation of kinetic Ising model driven by oscillating magnetic field the dynamical phase transition was found and a phase boundary was drawn in the plane described by the temperature of the system and the amplitude of oscillating magnetic field. The exponential relaxation of the dynamic order parameter was studied and critical slowing down was observed[4]. This relaxation of dynamic order parameter and the critical slowing down was verified experimentally[5]. The dynamic specific heat was also observed[4] to diverge at the dynamic transition point. The fluctuation of the dynamic order parameter was found [6] to diverge at the critical point. The dynamic transition is closely related to the hysteresis[7]. The dynamic transition by randomly varying field was also studied[8] in details. The existence of tricritical point on the dynamic phase boundary was observed and it is found to be related to the stochastic resonance[9]. The critical growth of dynamic correlation was also found[10]. Very recently, the surface and bulk dynamic critical behaviour was studied in kinetic Ising ferromagnet driven by oscillating magnetic field and the different class of universality was found[11]. The dynamical transition is associated with dynamical symmetry breaking. This is experimentally studied in highly anisotropic Co film on Cu by magneto optic Kerr effect[12].

All the studies mentioned above are of particular type. In all the cases the external magnetic field was oscillating but uniform over the space at any given instant of time. What will happen if the external field has a spatio-temporal variation ? This is precisely the subject of this article. The dynamical behaviours of planar Ising ferromagnet driven by propagating magnetic field wave, were studied recently. Here, two types of propagating magnetic waves are considered, plane[13, 14] and spherical[15].

The article is arranged in the following format: the model and the simulation scheme is described in section 2, the numerical results are described in section 3 and this ends with a summary in section 4.

2. Model and Simulation

The Hamiltonian of a two dimensional Ising ferromagnet (having uniform interaction strength among the nearest neighbours only) in presence of a propagating magnetic field wave (having spatio-temporal variation) can be represented as follows:

$$H(t) = -J\Sigma s(x,y,t)s(x',y',t) - \Sigma h(x,y,t)s(x,y,t) \tag{1}$$

The Ising spin variable, $s(x,y,t)$ assumes value ± 1 at lattice site (x,y) at time t on a square lattice of linear size L. Here, (x',y') represent the coordinates of nearest neighbors of the site (x,y). The parameter $J(>0)$ is the uniform ferromagnetic nearest neighbour interaction strength. The first term represents the Ising spin-spin ferromagnetic interaction. The spin-field interaction is represented by the second term. The $h(x,y,t)$ is the value of the magnetic field (at point (x,y) and at any time t) of the propagating magnetic field wave. The boundary condition, chosen here, is periodic in all directions.

2.1. Plane propagating wave

The plane propagating magnetic field wave is represented as:

$$h(x,y,t) = h_0 \cos(2\pi f t - 2\pi y/\lambda). \tag{2}$$

The phase of the field propagates along the y direction with amplitude h_0, frequency f and wavelength λ. In the case of plane propagating field, the initial ($t=0$) configuration, a high temperature random configuration where 50 percent spins are chosen (randomly) as

$s(x, y, t) = -1$ The spins are updated randomly (a site (x, y) is chosen at random) and spin flip occurs (at temperature T) according to the Metropolis probability of single spin flip (W)

$$W(s \rightarrow -s) = \mathrm{Min}[\exp(-\Delta E/kT), 1], \tag{3}$$

where ΔE is the change in energy due to spin flip and k is Boltzmann constant. Here, L^2 such random updates of spins defines the unit time step here and is called Monte Carlo Step per spin (MCSS). Here, the magnetic field and the temperature are measured in the units of J and J/k respectively. The dynamical steady state is reached by cooling the system slowly (in the presence of the propagating field) in small step ($\delta T = 0.02$ here) of temperature. It may be mentioned here that the same dynamical steady state was observed to be achieved by heating the system from a low temperature ordered configuration. The frequency and wavelength of the propagating magnetic field were kept fixed ($f = 0.01$ and $\lambda = 25.0$) throught the study. The frequency is measured in the unit of MCSS^{-1} and the wavelength is measured in the unit of lattice spacing. The total length of simulation is 2×10^5 MCS and first 10^5 MCS transient data were discarded to achieve the stable dynamical steady state. Since the frequency of the propagating field is $f = 0.01$, the complete cycle of the field requires 100 MCS. So, in 10^5 MCS, 10^3 numbers of cycles of the propagating field are present. The time averaged data over the full cycle (100 MCSS) of the propagating field are further averaged over 1000 cycles.

2.2. Spherically propagating wave

The form of spherically propagating magnetic field wave, is

$$h(x, y, t) = h_0 \frac{\mathrm{e}^{i(2\pi f t - 2\pi r/\lambda)}}{r} \tag{4}$$

Where, $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. The spherical wave originates at (x_0, y_0) and propagates radially outward.

The h_0, $f = \frac{\omega}{2\pi}$ and λ represent the amplitude, frequency and the wavelength respectively of the propagating magnetic field wave. In the present simulation, a $L \times L$ square lattice is considered. The boundary condition, used here, is periodic in both (x and y) directions. In this present study, $L = 100$ is considered.

In the case of spherically propagating magnetic field wave, the initial ($t = 0$) spin configuration is taken here as all the Ising spins are up ($s(x, y, t = 0) = +1, \forall$ x,y). Here, the dynamical steady state is obtained by heating the system slowly (δT=0.05) from fully ordered state. Here also one may achieve the same nonequilibrium steady state by cooling the system slowly from high temperature random state. The random updating of single spin flip with Metropolis rate is used here. The frequency ($f = 0.01$) and the wavelength (λ=15) were kept constant throughout the study.

Here also, the total length of simulation is 2×10^5 MCSS, where initial 10^5 MCSS is discarded and the averaging were calculated over 10^5 MCSS.

3. Results

3.1. For plane propagating wave

The fundamental physical quantity, which shows the different dynamical behaviours in this case, is the instantaneous magnetisation $m(t)$. In the case of plane propagating wave, the nonequilibrium steady states, depend on the values of the temperatre (T) of the system and the amplitude (h_0) of the propagating wave. For high temperature, the propagation of spin clusters (strip like) is observed. A typical such propagation is shown in a video (http://youtu.be/41TZPM-uxuc). However, for lower temperature no such coherent propagation of strip-like spin clusters was observed. For example, with $T = 1.5$ and $h_0 = 0.6$ the coherent

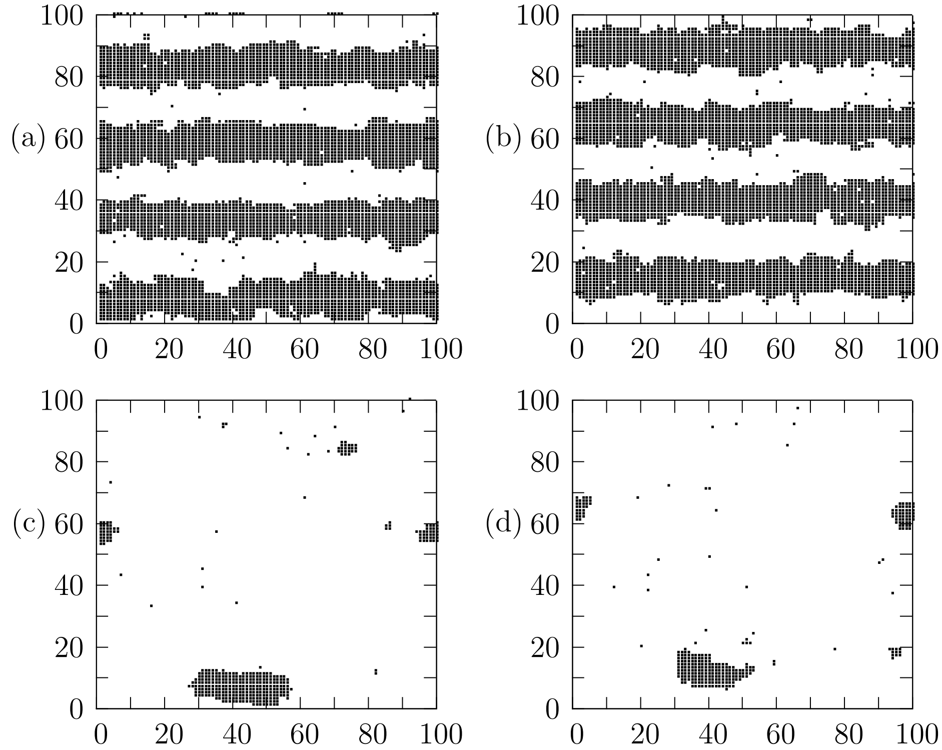

Figure 1. The motion of spin-clusters of down spins (shown by black dots), swept by propagating magnetic field wave, for different values of (a) Time = 100100 MCSS, T=1.5 and $h_0 = 0.6$ (b) Time = 100125 MCSS, $T = 1.5$ and $h_0 = 0.6$ (c) Time = 100100 MCSS, $T = 1.26$ and $h_0 = 0.6$ (d) Time = 100125 MCSS, $T = 1.26$ and $h_0 = 0.6$. See, Acharyya M. 2014 *Acta Physica Polonica B* **45** 1027.

propagation of strip-like spin clusters were found in the direction of the propagating plane magnetic field wave. No such propagation of spin clusters was detected for lower values of temperature ($T = 1.26$). Figure-1 shows some snapshots of such behaviours.

The system transits from a nonequilibrium propagating phase to a nonpropagating phase as the temperature decreases for a fixed value of amplitude of plane propagating magnetic field wave. It may be noted here that the instantaneous magnetisation $m(t) = \frac{1}{L^2}\sum s(x,y,t)$. For a quantitative analysis of this nonequilibrium phase transition, the order parameter has to be defined. Here, the time average magnetisation over the full cycle of the propagating magnetic field, is assumed to serve as the order parameter, $Q = f\oint m(t)dt$. The fluctuation in order parameter is $< (\delta Q)^2 >=< Q^2 > - < Q >^2$. The dynamic energy is $E = f\oint H(t)dt$ and the specific heat is defined as $C = \frac{dE}{dT}$. All these quantities are studied as functions of temperature to observe the nonequilibrium phase transition.

Figure-2 shows such plots for two different values of the amplitudes, i.e., $h_0 = 0.6$ and $h_0 = 0.3$. The order parameter Q is zero in the propagating phase and becomes nonzero at the transition temperature (T_d), as the system is cooled (fig-2(a)). The derivative $\frac{dQ}{dT}$, shows sharp dip (indication of scaling like $Q \sim (T_d - T)^\beta$ where $\beta < 1$) which is believed to be divergences eventually in the $L \to \infty$ limit. This is shown in fig-2(b). The transition is associated with

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the growth of fluctuations near the critical point and is shown in the plot $< (\delta Q)^2 >$ versus temperature in fig-2(c). The $< (\delta Q^2) >$ shows sharp peak (believed to be divergences in $L \to \infty$ limit). The dynamic specific heat C also shows sharp peak near the transition point and plotted in fig-2(d). From all these plots it is clear that the dynamic transition occurs at lower temperaure for higher values of the amplitude of the plane propagating magnetic field wave.

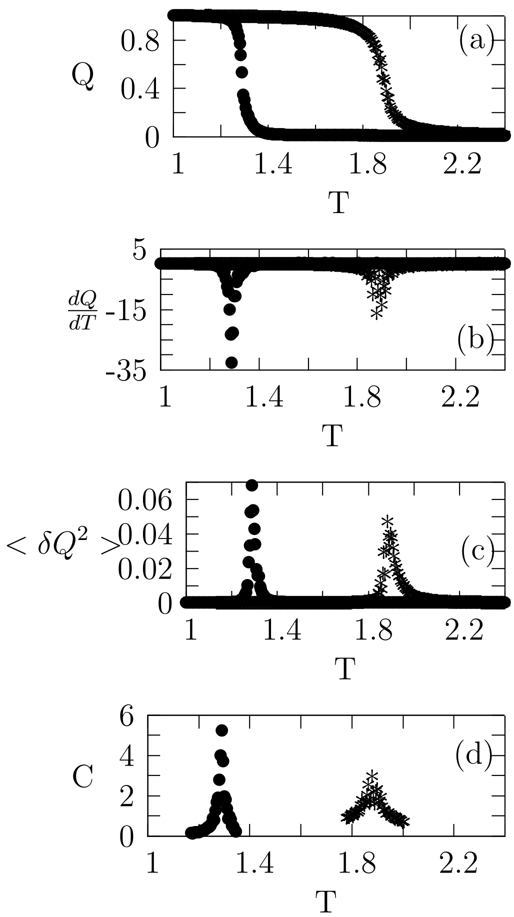

Figure 2. The temperature (T) dependences of the (a) Q, (b) $\frac{dQ}{dT}$, (c) $< (\delta Q)^2 >$ and (d) C, for two different values of h_0 for *propagating* magnetic field wave having $f = 0.01$ and $\lambda = 25$. In each figure, $h_0 = 0.3(*)$ and $h_0 = 0.6(\bullet)$. See, Acharyya M. 2014 *Acta Physica Polonica B* **45** 1027.

The transition temperatures are found to be function of the amplitude of the propagating magnetic field (for given f and λ). A comprehensive phase boundary is obtained and plotted in figure-3.

3.2. For spherically propagating wave

The dynamical responses of a two dimensional Ising ferromagnet driven by spherical magnetic field wave is also studied. The system shows three main dynamical states. For small value of field amplitude of propagating spherical wave it remains pinned. For some intermediate value of the amplitude it shows dynamically breathing phase. Figure-4 shows a typical snapshots of

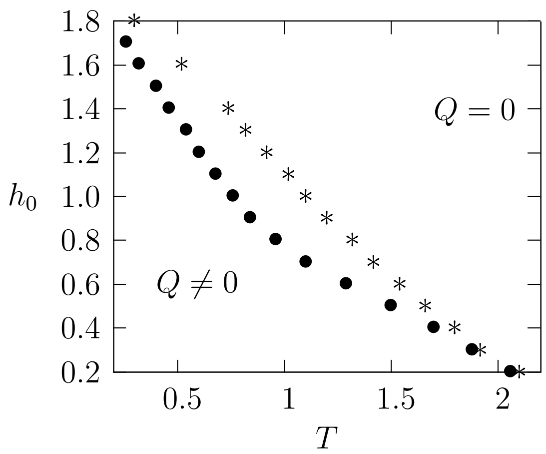

Figure 3. The phase diagram for dynamic phase transition by propagating magnetic field wave for two different values of wavelengths, $\lambda = 25(\bullet)$ and $\lambda = 50(*)$. Here, $f = 0.01$. See, Acharyya M. 2014 *Acta Physica Polonica B* **45** 1027.

these types (pinned and breathing) of dynamical phases. A video of spreading phase may be found in http://youtu.be/S_rGUQNjcig.

A centrally localised dynamical mode is found. For high value of field amplitude a spreading dynamical mode was observed. In this spreading phase the ring shaped spin clusters were found to spread over the whole lattice. Figure-5 shows the dynamically spreading phase.

The dynamical responses of a two-dimensional Ising ferromagnet driven by spherical magnetic field wave are characterised as follows: The density of instantaneous local magnetisation in the circle of radius $\lambda/2$ is $m_b(t) = \sum s(x, y, t)/(N_b)$, where the sum is carried over the number of sites (N_b) lying within the circle of radius $\lambda/2$ centered at the centre (x_0, y_0) of lattice. In the present study, the lattice size L is taken equal to 100. So, the coordinates of the centre is (x_0=50.5,y_0=50.5). and N_b is the total number of lattice sites within this circle. The order parameter of the dynamic breathing transition is defined as $Q_b = \frac{\omega}{2\pi} \oint m_b(t)dt$. The statistical fluctuations of the dynamic order (for breathing) parameter is $< \delta {Q_b}^2 >=< {Q_b}^2 > - < Q_b >^2$. The average value of Q_b and the fluctuation are calculated over $n_c = 1000$ number of random samples. It is also checked that this number of samples is sufficient to have the nonequilibrium steady state as well as to have the data within the specified accuracy.

To study the dynamical spreading transition the releavant quantities are: The instantaneous magnetisation density within a ring of width $\lambda/2$, which thouches the boundary of the lattice, is $m_s(t) = \sum s(x, y, t)/N_s$, where the summation is carried over the sites residing within the ring of width $\lambda/2$ touching the boundary, and N_s is the total number of lattice sites within that ring. It is a region bounded between two concentric ($x_0 = 50.5, y_0 = 50.5$) circles having outer radius $r_{out} = L/2$ and the inner radius $r_{in} = L/2 - \lambda/2$. The dynamic order parameter of spreading transition $Q_s = \frac{\omega}{2\pi} \oint m_s(t)dt$. The fluctuation in dynamic order parameter (for spreading) is $< \delta {Q_s}^2 >=< {Q_s}^2 > - < Q_s >^2$.

The dynamic breathing and spreading order parameters are plotted against temperature and shown in figure-6. It is observed that, for a fixed set of values of h_0, f and λ of the field wave, the breathing transition occurs at lower temperature than that for spreading transintion. As the amplitude of the magnetic field wave increases both the transitions occur at lower temperatures.

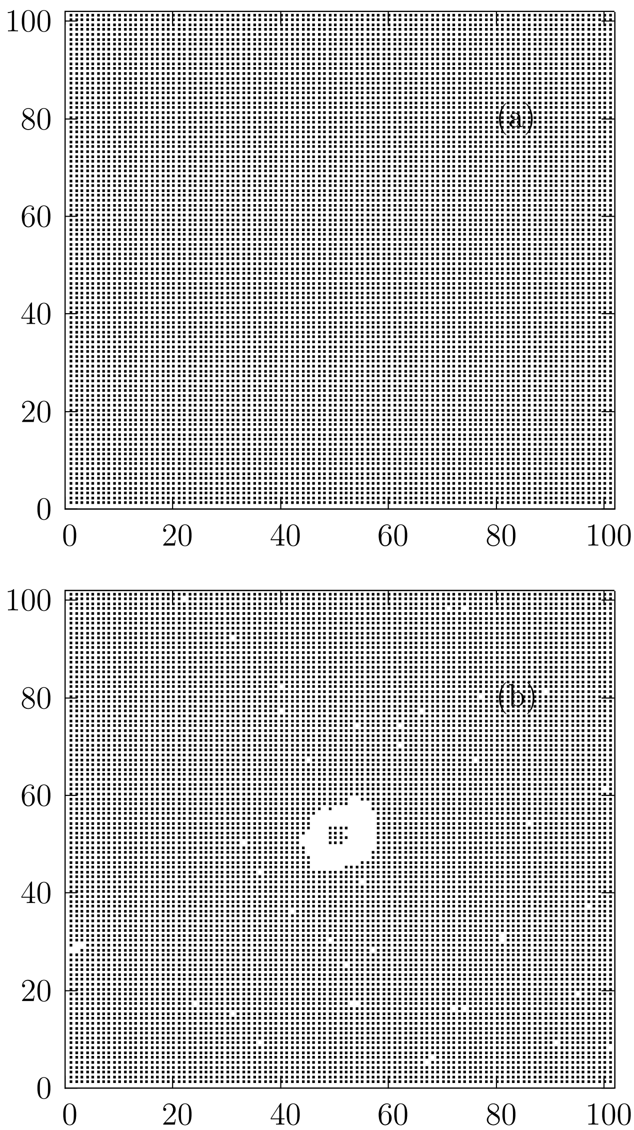

Figure 4. The pinned (a) and breathing (b) modes on the lattice. Dots represents up spins only. (a) $h_0 = 2.5$, $T = 0.30$ and (b) $h_0 = 2.5$ and $T = 1.45$. Here, $\lambda = 15.0$, $f_0 = \frac{\omega_0}{2\pi} = 0.01$ and $t = 4000$ MCSS. See, Acharyya M. 2014 *J. Magn. Magn. Mater.* **354** 349.

The fluctuations in dynamic breathing and spreading order parameters are shown in figure-7. The peaks of the fluctuations are indicating the breathing and spreading transition temperatures. The transition temperatures are observed to decreases as the amplitude of the magnetic wave increases. Hence, one can obtain the transition temperatures as functions of the amplitude of the magnetic wave and a comprehensive phase boundary can be drawn in the plane described by the temperature of the system and the amplitude of the magnetic field wave.

The comprehensive phase boundary (for fixed f and $\lambda = 15$) are shown in figure-8. Here the regions of three different nonequilibrium phases are shown in the plane described by the

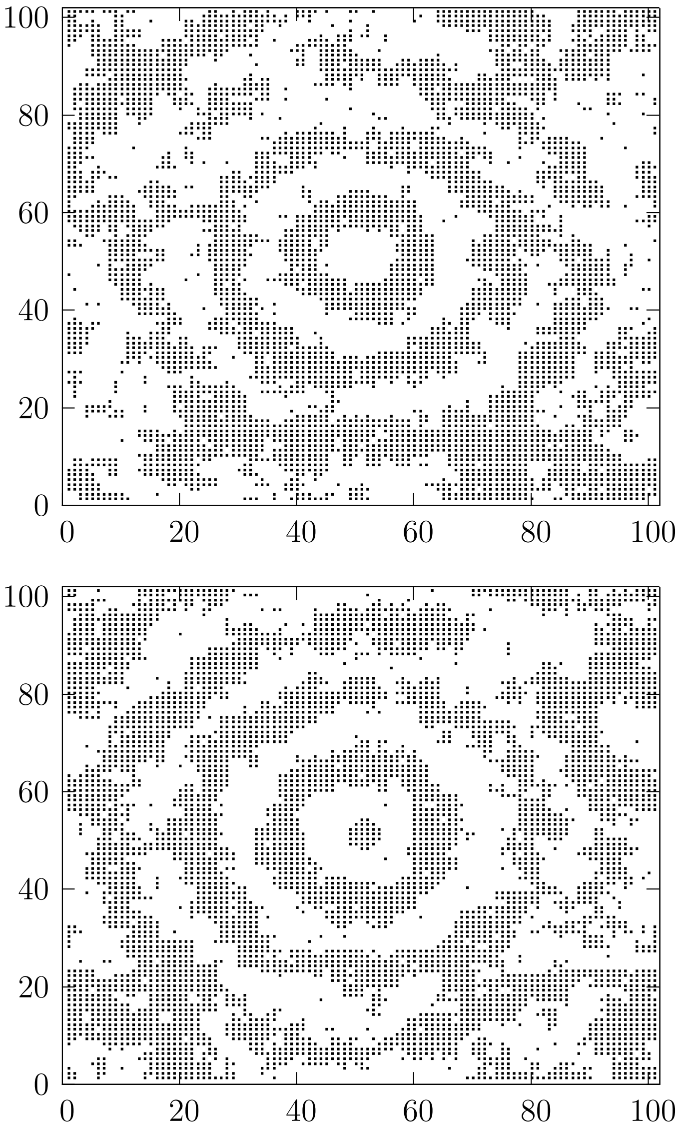

Figure 5. A high field ($h_0 = 10.0$)-high temperature ($T = 2.30$) spin wave propagation mode. The up spins ($S_i = +1$) are shown by black dot. The bottom one shows a spin configuration at 3970 MCSS and the top one shows that at 4000 MCSS. Here, $f_0 = 0.01$ and $\lambda = 15.0$. See, Acharyya M. 2014 *J. Magn. Magn. Mater.* **354** 349.

temperature of the system and the amplitude of spherically propagating magnetic field wave. It was observed that the two phase boundaries merge to a value (the equilibrium ferro-para transition temperature $T_{eq} = 2.269...$, calculated exactly by Onsager) in the limit of vanishingly small amplitude of magnetic wave.

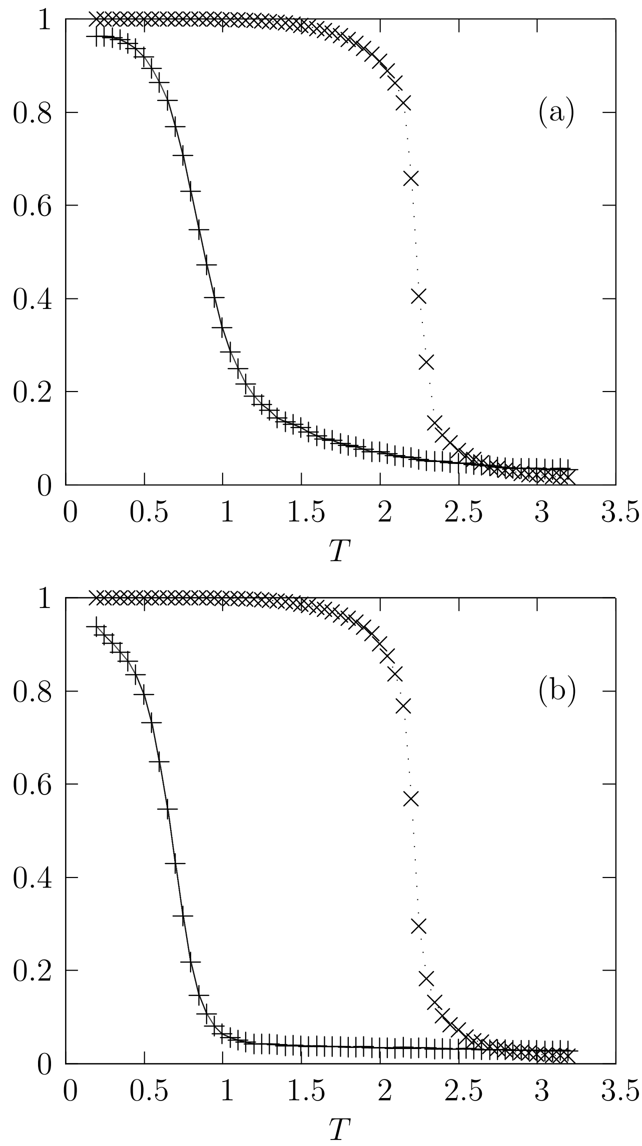

Figure 6. The plots of $Q_b(+)$ and $Q_s(\times)$ as a function of temperature T. (a) $h_0 = 2.5$, $f = 0.01$ and $\lambda = 15.0$ and (b) $h_0 = 3.5$, $f = 0.01$ and $\lambda = 15.0$. See, Acharyya M. 2014 *J. Magn. Magn. Mater.* **354** 349.

4. Summary

In this article, the nonequilibrium responses of planar Ising ferromagnet driven by propagating (plane wave and spherical wave) magnetic field wave are discussed. For plane propagating magnetic field wave, Ising ferromagnet shows two nonequilibrium phases, namely pinned and propagating phase, depending on the values of temperature of the system and the amplitude of the propagating magnetic field wave. The dynamic transitions are marked by studying the dynamic order parameter, its derivative, its fluctuations and the dynamic specific heat as functions of temperatures. A comprehensive phase boundary is plotted in the plane described

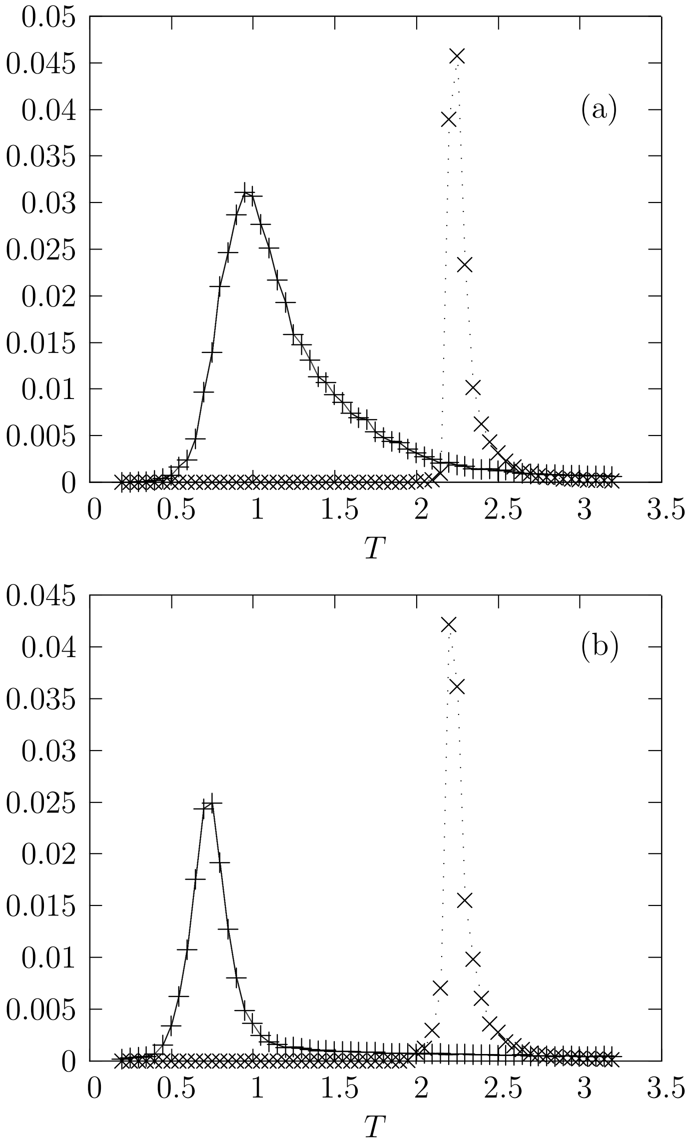

Figure 7. Fluctuations of $Q_b(+)$ and $Q_s(\times)$ plotted against temperature (T) for two different values of field amplitude (h_0) of propagating field. (a) $h_0 = 2.5$ and (b) $h_0 = 3.5$. Here, $\lambda = 15.0$ and $f = 0.01$. See, Acharyya M. 2014 *J. Magn. Magn. Mater.* **354** 349.

by the temperature and the amplitude of the plane propagating magnetic field wave.

Now, if the system is irradiated by a spherically propagating magnetic field wave whose origin is at the centre of the lattice, the nonequilibrium responses show variety of interesting phenomena. Here three different nonequilibrium phases are observed. For very small values of temperature and the amplitude of the magnetic field wave, the system remains in a pinned phase. For intermediate values of these two parameters, the system shows an interesting centrally localised dynamical phase (breathing phase) and for higher values of the temperature and field amplitude, it shows a dynamically extended spreading phase. The transition temperatures

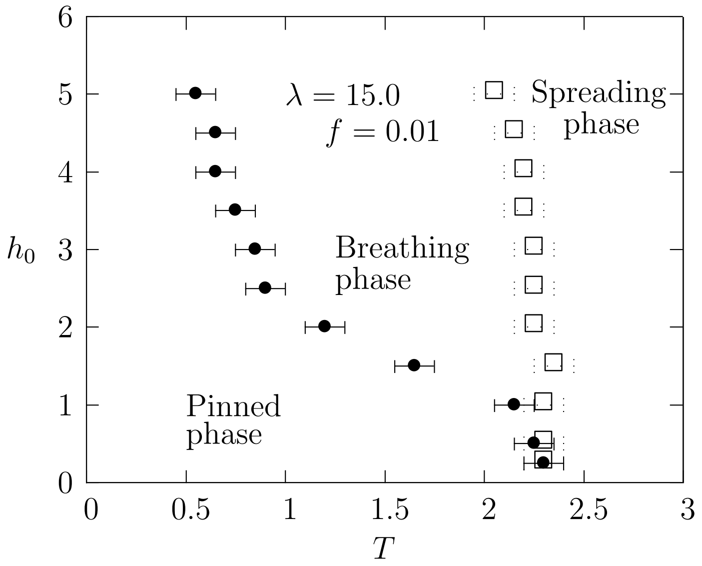

Figure 8. The phase diagram for symmetry breaking dynamic breathing and spreading transitions. Here, $f_0 = 0.01$ and $\lambda = 15.0$. See, Acharyya M. 2014 *J. Magn. Magn. Mater.* **354** 349.

for a given value of field amplitude are estimated from the peak positions of the fluctuations of dynamic breathing and spreading order parameters. The comprehensive phase boundaries are plotted in the plane described by the temperature of the system and the amplitude of the magnetic field wave. The two boundaries are observed to merge at equilibrium ferro-para transition temperature in the limit of vanishing amplitude of the magnetic field wave.

It may be mentioned here that recent experiments on permalloy (50 nm thick) excited by ultrashort (150 fs) laser pulse gives rise to the propagation of circular spin waves[16].

5. Acknowledgements
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