A semi-analytical model for strong field atomic ionization

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Synopsis A semi-analytical model is proposed to solve the TDSE. The method leads to an integro-differential equation for the initial state amplitude to be numerically solved. Closed expressions for the ionization amplitudes in term of the survival amplitude are obtained. The method is applied to Hydrogen ionization by a laser pulse.

The highly non-linear interaction of atoms with intense laser pulses has demanded increasing effort for either solve the time dependent Schrödinger equation (TDSE), or proposing semi-analytical models keeping simple interpretation while still describing the essential phenomena. Among the first ones an interesting numerical method for solving TDSE in momentum space has been recently proposed [1]. In the second group, most of the best known and successful semi-analytical methods are related to the Strong Field Approximation (SFA) [2].

In [1], the method is able to be improved by adding a number of bound states supported by the model potential. A set of coupled Volterra type integral equations has to be numerically solved. Following this line, a theoretical method accounting for a number of bound states coupled among them and with the continuum is proposed in this study. However, a simplified treatment for the continuum-continuum coupling is made. This simplification is related to that made in the derivation of the SFA [2].

As in [1], the velocity gauge is used. A sine-square envelope laser pulse applied to Hydrogen atom is considered. The Hamiltonian (a.u. are used) is:

$$H = H_{\rm at} + \hat{\mathbf{p}}.\mathbf{A}(t) = \frac{\hat{\mathbf{p}}^2}{2m_e} + V_{\rm T}(\mathbf{r}) + \hat{\mathbf{p}}.\mathbf{A}(t)$$

The TDSE for the state $|\psi(t)\rangle$ with the initial condition $|\psi(0)\rangle = |i\rangle$ has to be solved. $|\psi(t)\rangle$ is expanded in a complete set of $H_{\rm at}$ eigenstates $|\psi(t)\rangle = a_i(t)e^{-i\varepsilon_i t}|i\rangle + \sum_{n\neq i} a_n(t)e^{-i\varepsilon_n t}|n\rangle + \int \mathrm{d}\mathbf{k}\, a_{\mathbf{k}}(t)e^{-i\varepsilon_K t}|\mathbf{k}\rangle$.

Using the above expansion in the TDSE,

$$\dot{a}_i(t) = -\int_0^t dt' K(t,t') a_i(t)$$

is obtained for the initial-state amplitude.
The integro-differential equation kernel is

$$K(t,t') = \sum_{n\neq i}[\hat{\mathbf{p}}_{\rm in}\cdot\mathbf{A}(t)][\mathbf{p}_{\rm ni}\cdot\mathbf{A}(t')]e^{-i\varepsilon_n(t-t')} + \int \mathrm{d}\mathbf{k}[\hat{\mathbf{p}}_{\rm ik}\cdot\mathbf{A}(t)][\hat{\mathbf{p}}_{\rm ki}\cdot\mathbf{A}(t')]e^{-i\varepsilon_k(t-t')}e^{i\mathbf{k}\cdot\boldsymbol{\alpha}(t,t')}$$

Two main approximations are made to obtain this equation. Coupling between the initial-state and all the other states are kept. The other couplings are neglected excepting the continuum-continuum ones which are approximated by $\hat{\mathbf{p}}_{\mathbf{kk}'} \approx \mathbf{k}\,\delta(\mathbf{k}-\mathbf{k}')$. In spite of these approximations, the initial-state decay and the ATI peaks are well described.

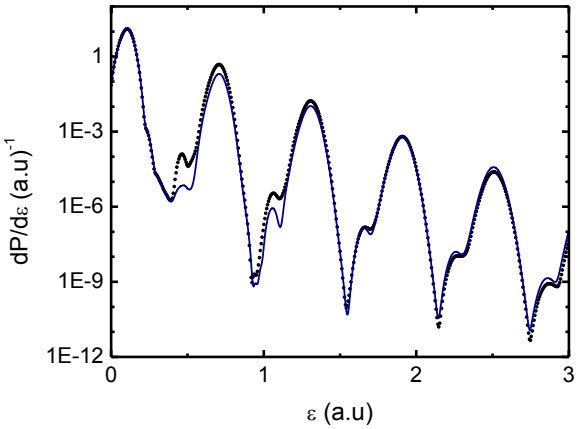

Figure 1. Electron energy spectrum by interaction of the Hydrogen atom with a twenty-cycle sine-square pulse. The electric field peak amplitude is equal to 0.03 a.u., and the photon energy is 0.6 a.u.

The electron spectrum obtained by Hydrogen interacting with a twenty-cycle, 3.1×10^{15} Wcm^{-2} peak-intensity laser pulse is shown in figure 1. The spectrum exhibits several ATI peaks separated by the photon energy. The present results along with those obtained by using the QPROP code for solving the TDSE [3] are displayed.

References

[1] H. M. T. Nganso *et al* 2013 *Phys. Rev. A. Phys.* **87** 013420
[2] F. Kraus and M. Ivanov 2009 *Rev. Mod. Phys.* **81** 163
[3] D. Bauer and P. Koval, 2006 *Comput. Phys. Commun.* **174** 296

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