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Effects of magnetic field and Hall current to the blood velocity and LDL transfer

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Abstract. The magnetic field and Hall current effects have been considered on blood velocity and concentration of low-density lipoprotein (LDL). It is important to observe those effects to the flowing blood in a stenosed artery. The analysis from the obtained results may be useful to some clinical procedures, such as MRI, where the radiologists may have more information in the investigations before cardiac operations could be done. In this study, the uniform magnetic field and Hall current are applied to the Newtonian blood flow through an artery having a cosine-shaped stenosis. The governing equations are coupled with mass transfer and solved employing a finite difference Marker and Cell (MAC) method with an appropriate initial and boundary conditions. The graphical results of velocity profiles and LDL concentration are presented in this paper and the results show that the velocity increases and concentration decreases as Hall parameter increased.

1. Introduction

Cardiovascular disease which is also known as heart disease is proved as a leading cause of death globally from the study conducted by World Health Organization [1]. The heart failure is a condition in which the heart unable to pump enough blood to meet the body's needs [2]. This situation can be caused by aortic stenosis, a narrowing of the aortic orifice of the heart or the aorta near the valve. The stenosis is caused by atherosclerosis or the accumulation of macromolecules such as low-density lipoprotein (LDL) in the arterial wall which is known as plaque. The development of atherosclerosis is strongly related to the characteristics of the blood flow in the arteries [3].

Chakravarty et al. [4-6] investigated the flow behavior of blood in a different shape of stenotic artery. The blood is considered as a Newtonian fluid. Further studies of unsteady blood flow coupled with mass transfer through arterial stenosis have been reported by [7-9]. Mass transport is a movement of atherogenic molecules such as LDL and oxygen within the blood flow and arterial wall. This movement has been proved to contribute in the formation of stenosis.

Dynamics of biological fluid in the presence of magnetic field has developed great interest among researchers due to its importance in various implications in the bioengineering and medical technology. The development of magnetic devices for cell separation, targeted transport of magnetic particles as cancer tumor treatment, magnetic wound or drug carriers causing magnetic hyperthermia, reduction of

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bleeding during surgeries and the development of magnetic tracers, are well-known applications in this domain of research [10-11]. The most characteristic biological fluid is blood, due to the complex interaction in which behaves as a magnetic fluid. If a magnetic field is applied to a moving and electrically conducting fluid, it will induce electric and magnetic field and the interaction of these fields produces Lorentz force. By Lenz's law, the Lorentz force opposes the motion of conducting fluid and since blood is an electrically conducting fluid, the magnetohydrodynamic (MHD) principles may be used to decelerate the flow of blood in human arterial system and thereby it is useful in the treatment of certain cardiovascular disorders.

The current trend for MHD flow is towards a strong magnetic field, so that the influence by electromagnetic force is noticeable. When the applied magnetic field strength is large and the conducting fluid is an ionized gas where the density is low, the conductivity normal to the magnetic field is reduced due to the spiraling of electrons and ions about the magnetic lines of force before collisions take place and a current is induced in a direction normal to both the electric and magnetic fields. This phenomenon is known as the Hall effect [12-14]. Fluid flow characteristics of blood flow in stenosed arteries with the presence of magnetic field are investigated by [15-16]. Recent study by [17] investigated the influence of magnetic field with Hall currents on blood flow through a stenotic artery. The results obtained show that the axial velocity increases as the Hall parameter increases.

Therefore, the aim of this present paper is to investigate the distribution of the velocity and mass concentration with the presence of magnetic field and Hall current. It is assumed that the arterial segment to be a cylindrical tube with a cosine-shaped stenosis and the blood flow through it to be a Newtonian fluid. The corresponding governing equations consists of continuity, axial and radial momentum and also mass transfer equations are solved using finite-difference scheme known as MAC method. This study will explore the effects of Hall parameter, Hartmann, Reynolds and Schmidt numbers on axial and radial velocity as well as mass transfer concentration.

2. Mathematical formulation

2.1. The geometry of stenosis

The geometry of stenosis, $R(z,t)$ for this study is described mathematically as

$$R(z,t)=\begin{cases} a(t)\left\{1-\dfrac{\delta}{2R_0}\left[1+\cos\left\{\dfrac{\pi(z-z_1)}{z_0}\right\}\right]\right\}, & z_1-z_0\le z\le z_1+z_0 \\ a(t), & otherwise \end{cases} \tag{1}$$

where R_0 is a constant radius of normal artery in the non-stenotic region, z is axial direction of stenosis, z_0 is half-length of stenosis, z_1 is centre of stenosis and δ is critical height of the stenosis. Time-variant parameter in (1), $a(t)$ is given by

$$a(t) = 1 + k_R \cos(\omega t - \varphi)$$

where k_R is a constant, t is time, $\omega = 2\pi f$ is the angular frequency with f is the pulse frequency and φ is the radius phase angle. The diagram of the geometry of stenosis is shown in figure 1.

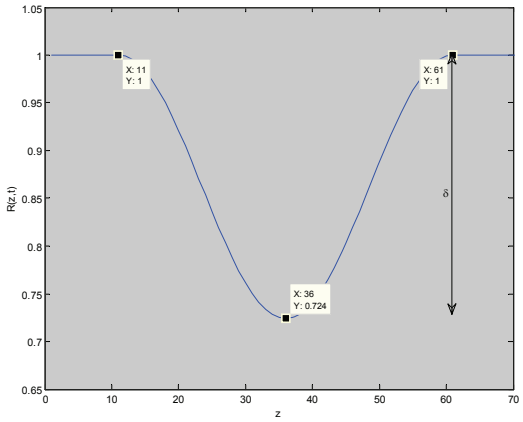

Figure 1. The schematic diagram of cosine-shaped stenosis.

2.2. Governing equations

Let us consider (r, z, θ) to be the coordinates of the material point in the cylindrical coordinates system where the z-axis is taken along the axis of the artery while r and θ are taken along the radial and circumferential directions, respectively. In this study, blood flow in the constricted artery segment is considered to be two-dimensional, unsteady, incompressible and the fluid is assumed to be Newtonian fluid. The governing equations corresponding with this study consist of continuity, momentum and mass transfer equations. The dimensionless conservative governing equations can be written as:

$$r\frac{\partial w}{\partial z}+\frac{\partial(ur)}{\partial r}=0, \tag{2}$$

$$\frac{\partial w}{\partial t}+\frac{\partial(wu)}{\partial r}+\frac{\partial w^2}{\partial z}+\frac{(wu)}{r}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{M^2}{\mathrm{Re}}\frac{(mu-w)}{(1+m^2)}, \tag{3}$$

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial r}+\frac{\partial(wu)}{\partial z}+\frac{u^2}{r}=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)-\frac{M^2}{\mathrm{Re}}\frac{(u+mw)}{(1+m^2)}, \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=\frac{1}{\mathrm{Re}\cdot\mathrm{Sc}}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right) \tag{5}$$

where u and w are the radial and axial velocity components along the r- and z-axes respectively, p is pressure, r is radial axis, C is the mass concentration, $\mathrm{Re}=U_0 r_0 \rho/\mu$ is Reynolds number, $M=B_0 r_0(\sigma/\mu)^{\frac{1}{2}}$ is the Hartmann number and used as the dimensionless parameter of the magnetic field, m is Hall parameter and $Sc=\mu/\rho D$ is the Schmidt number with ρ is density of blood, μ is dynamic viscosity, D is the coefficient of diffusion, U_0 is mean velocity and r_0 is the normal radius in the arterial segment.

2.3. Boundary and initial conditions

Along the symmetry axis of the artery, the radial flow is zero. Therefore, the normal component of the radial velocity and the axial velocity gradient and the mass concentration gradient of the blood along the axis are vanished which means that the shear stress does not exist. These may be stated mathematically as

$$\text{at } r=0, \quad \frac{\partial w(r,z,t)}{\partial r}=u(r,z,t)=\frac{\partial C(r,z,t)}{\partial r}=0. \tag{6}$$

The radial velocity at arterial wall may be assumed to be equal to the changes of radius of the arterial segment in the constricted artery. Hence the velocity boundary conditions on the arterial wall obey the no-slip condition and can be written as

$$\text{at } r = R, \quad w(r,z,t) = C(r,z,t) = 0\,, \quad u(r,z,t) = \frac{\partial R}{\partial t}. \tag{7}$$

The inlet velocity (when z = 0) conditions are assumed to be fully developed with parabolic velocity profile corresponding to Hagen-Poiseuille flow through a long circular tube. The mass concentration is assumed to be constant. It is provided with evidence by study of Mustapha et al. [11] as

$$\text{at } z = 0, \text{ for } M = 0\,, \quad w(r,z,t) = 2(1-x^2), u(r,z,t) = 0\,, C(r,z,t) = 1\,, \tag{8}$$

$$\text{at } z = 0, \text{ for } M \neq 0\,, \quad w(r,z,t) = 2\left(\frac{I_0(MR)}{I_0(MR)-1}\right)\left(1-\left(\frac{I_0(Mr)}{I_0(MR)}\right)\right), u(r,z,t) = 0\,, C(r,z,t) = 1. \tag{9}$$

At the outlet (when $z = L$), where L is the finite length of the arterial segment, the velocity gradients and mass concentration gradient are taken to be traction-free conditions and can be stated as

$$\text{at } z = L, \quad \frac{\partial w(r,z,t)}{\partial z} = \frac{\partial u(r,z,t)}{\partial z} = \frac{\partial C(r,z,t)}{\partial z} = 0. \tag{10}$$

It is also assumed that no flow takes place when the system is at rest (t = 0) except at the inlet,

$$\text{at } t = 0, \text{ for } z > 0, \quad w(r,z,0) = u(r,z,0) = p(r,z,0) = C(r,z,0) = 0\,. \tag{11}$$

3. Marker and Cell (MAC) method

The governing equations (2-5) together with initial and boundary conditions (6-11) are solved numerically using the finite difference scheme known as MAC method. This method is based on the central difference approximations for the uniform spatial derivatives and the forward difference formula for time derivatives. The discretized form for the continuity equation is

$$x_{lj}R_{li}^{k}\left(\frac{w_{i,j}^{k}-w_{i-1,j}^{k}}{\Delta z}\right)-\left(x_{lj}\right)^{2}\left(\frac{\partial R}{\partial z}\right)_{li}^{k}\left(\frac{w_{at}-w_{ab}}{\Delta x}\right)+\frac{x_j u_{i,j}^{k}-x_{j-1}u_{i,j-1}^{k}}{\Delta x}=0 \tag{12}$$

with z_{ij} and x_{ij} are represents the respective coordinates of the cell centre while z_i and x_j represents the cell at the top right corner of the $(i,j)^{\text{th}}$ control volume.

Meanwhile, the discretized form of the axial momentum equation can be written into the form

$$\frac{w_{i,j}^{k+1}-w_{i,j}^{k}}{\Delta t}=\left(\frac{p_{i,j}^{k}-p_{i+1,j}^{k}}{\Delta z}\right)+\frac{x_{lj}}{R_i^{k}}\left(\frac{\partial R}{\partial z}\right)_{i}^{k}\left(\frac{p_t-p_b}{\Delta x}\right)+\left(w_{me}\right)_{i,j}^{k} \tag{13}$$

Finally, the discretized form of the radial momentum equation can be put in the form

$$\frac{u_{i,j}^{k+1}-u_{i,j}^{k}}{\Delta t}=\frac{1}{R_{li}^{k}}\left(\frac{p_{i,j}^{k}-p_{i,j+1}^{k}}{\Delta x}\right)+\left(u_{me}\right)_{i,j}^{k} \tag{14}$$

where all the terms are given in [9], in details.

On the other hand, the discretization of the convection-diffusion equation for mass transfer can be written as

$$\frac{C_{i,j}^{k+1}-C_{i,j}^{k}}{\Delta t}=\frac{x_j}{R_i^k}\left(\frac{\partial R}{\partial t}\right)_i^k\left(\frac{\partial C}{\partial x}\right)_{i,j}^k-\frac{u_{i,j}^k}{R_i^k}\left(\frac{\partial C}{\partial x}\right)_{i,j}^k-w_{i,j}^k\left(\left(\frac{\partial C}{\partial z}\right)_{i,j}^k-\frac{x_{lj}}{R_i^k}\left(\frac{\partial R}{\partial z}\right)_i^k\left(\frac{\partial C}{\partial x}\right)_{i,j}^k\right)$$
$$-\frac{1}{\mathrm{Re}\cdot Sc\cdot R_i^{k^2}}\left[\left\{1+\left(x_j\left(\frac{\partial R}{\partial z}\right)_i^k\right)^2\right\}\left(\frac{\partial^2 C}{\partial x^2}\right)_{i.j}^k\right. \tag{15}$$
$$\left.+\left\{\frac{1}{x_j}+2x_j\left(\left(\frac{\partial R}{\partial z}\right)_i^k\right)^2-x_jR_i^k\left(\frac{\partial^2 R}{\partial z^2}\right)_{i,j}^k\right\}\left(\frac{\partial C}{\partial x}\right)_{i,j}^k+\left(R_i^k\right)^2\left(\frac{\partial^2 C}{\partial z^2}\right)_{i,j}^k\right]$$

with

$$\frac{\partial C}{\partial x}=\frac{C_{i,j+1}^k-C_{i,j-1}^k}{2\Delta x},\quad \frac{\partial C}{\partial z}=\frac{C_{i+1,j}^k-C_{i-1,j}^k}{2\Delta z},\quad \frac{\partial^2 C}{\partial x^2}=\frac{C_{i,j+1}^k-2C_{i,j}^k+C_{i,j-1}^k}{\left(\Delta x\right)^2},\quad \frac{\partial^2 C}{\partial z^2}=\frac{C_{i+1,j}^k-2C_{i,j}^k+C_{i-1,j}^k}{\left(\Delta z\right)^2}$$

4. Results and discussion

The governing equations (12-15) corresponding with the boundary and initial conditions have been solved numerically using finite difference method which is known as Marker-and-Cell (MAC) method. Results for present paper are obtained for Re = 450, Sc = 5, M = 1, 2, 5, 10, 20, and m = 1, 2, 5, 10. The geometry of stenosis considered in this study is cosine-shaped stenosis. From figure 1, it is shown that there are two regions represent the non-stenotic region or normal artery and stenotic region. For z = 0 until 1 and z = 3 until 5 are known as non-stenotic regions while the stenotic region located at z = 1 until 2 where the critical height of the stenosis occur at z = 0.7241 with δ = 0.276.

Figure 2 illustrates the behavior of the axial velocity profiles for different magnetic field at z = 3.5 and Re = 450. It can be seen that the shape of the velocity profiles varies depending on the intensity of the magnetic field applied. As magnetic field increases, the axial velocity becomes flatter and the velocity gradient becomes steeper towards the wall surface of the stenosed artery. Meanwhile, the magnitude of the maximum velocity decreases as the magnetic field increases. This is because the magnetic field leads to a rising drag-like force known as Lorentz force. This force has tendency to reduce the fluid velocity in the boundary layer.

On the other hand, the dimensionless axial velocity for different Hall parameter is illustrated as in figure 3. It is shown that as Hall parameter increases, the axial velocity will increase. Theoretically, the influence of the magnetic field may cause the positive and negative ions that bounce back and forth between the sides of the vessel and induce a Hall current. The combination of the electromotive force, altered ionic pattern, and currents may potentially cause the blood vessel dilation which corresponds to the increase in blood flow.

Figure 4 shows the dimensionless mass concentration of LDL at different z for a fixed Schmidt number, Sc = 5. The results illustrate that at normal artery (at z = 1), the mass concentration is higher than at the critical height of the stenosis (at z = 3.5). This is because the arterial wall is thicker at the stenotic region, thus the resistance to LDL is higher than on the non-stenotic region. Hence, the mass concentration decreases toward the stenosed artery region.

Figure 5 shows the behavior of dimensionless mass concentration for different Hall parameter. It is shown that as the value of Hall parameter increases, the mass concentration will decrease. We can also see that the mass concentration fluctuates in most radial part of the artery and it becomes flatter near the wall surface of the artery. As for various values of magnetic field, M, there is insignificant result for dimensionless mass concentration.

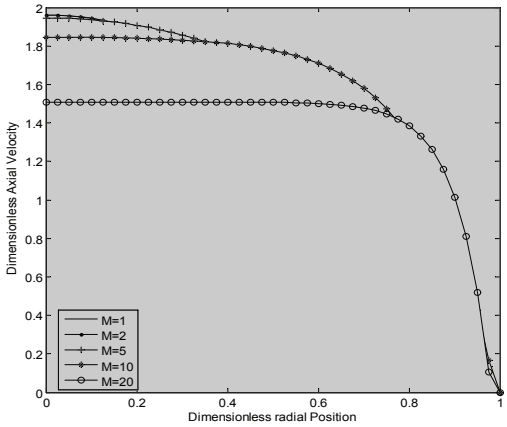

Figure 2. Dimensionless axial velocity for different magnetic field, M.

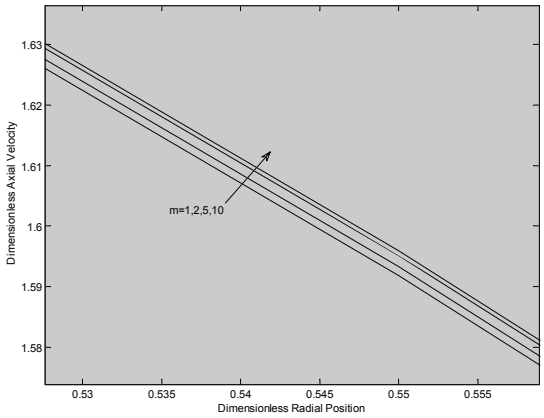

Figure 3. Dimensionless axial velocity for different Hall parameter, *m*.

5. Conclusions

The problem of blood flow coupled with mass transfer through an arterial stenosis with Hall current effect has been studied numerically in this paper. The effects of magnetic field, M and Hall parameter, *m*, on the axial velocity and mass concentration has been examined in details in this paper. As conclusion, the present study shows that how the magnetic field plays a role that reduces the velocity of blood but at the same time, the presence of Hall current increases the velocity. However, the effect from magnetic field is much more apparent compared to the Hall effect. With the large enough intensity of applied magnetic field, it is believed that it can streamlining the flow of blood around the body. Thus, it still needs further investigation to conclude the adequate of the intensity applied magnetic field in order to have any such effects.

Furthermore, it is evidence from the results that the LDL concentration is lower in the stenotic region. This finding shows that the possibility of the LDL adheres at the upstream of stenosis, thus contributes to the enlargement of stenosis. As for different Hall parameter, the concentration having a fluctuate profile near the axis, and slowly approaches zero at the wall.

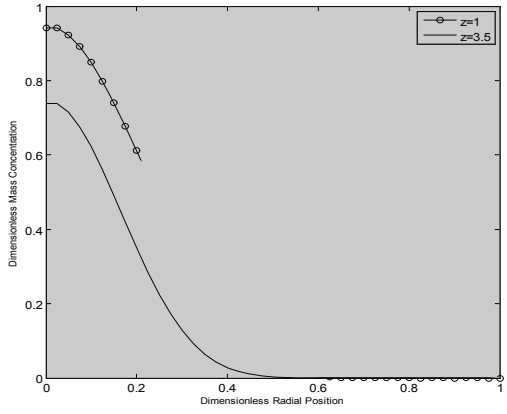

Figure 4. Dimensionless mass concentration at different *z*.

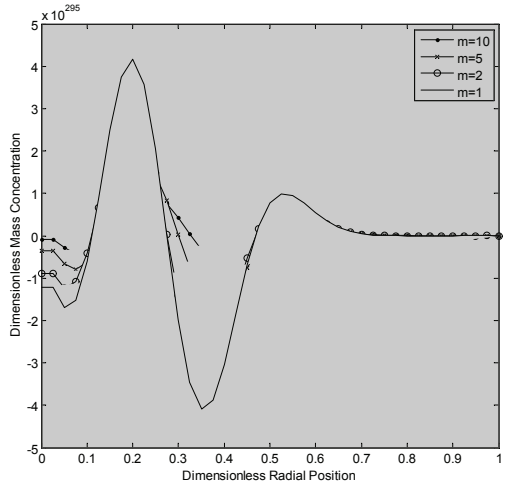

Figure 5. Dimensionless mass concentration for different Hall parameter, *m*.

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References

[1] World Health Organization (WHO) 2011 *Global Atlas on Cardiovascular Disease Prevention and Control* ed S Mendis, P Puska and B Norrving (Geneva: WHO, World Heart Federation, and World Stroke Organization)
[2] Tan Y B and Mustapha N 2013 Numerical modeling of blood flow in irregular stenosed artery with the effects of gravity *Jurnal Teknologi* **62** 57-64
[3] Abdullah I, Amin N and Hayat T 2011 Magnetohydrodynamic effects on blood flow through an irregular stenosis *Int. J. Num. Meth. in Fluids* **67** 1624-36
[4] Chakravarty S, Datta A and Mandal P K 1996 Effect of body acceleration on unsteady flow of blood past a time-dependent arterial stenosis *Mathl. Comput. Modelling* **24** 57-74
[5] Chakravarty S and Sannigrahi A K 1999 A nonlinear mathematical model of blood flow in a constricted artery experiencing body acceleration *Mathl. Comput. Modelling* **29** 9-25
[6] Chakravarty S, Mandal P K and Mandal A 2000 Mathematical model of pulsatile blood flow in a distensible aortic bifurcation subject to body acceleration *Int. J. Engineering Sci.* **38** 215-38
[7] Kaazempur-Mofrad M R, Wada S, Myers J G and Ethier C R 2005 Mass transport and fluid flow in stenotic arteries: axisymmetric and asymmetric models *Int. J. Heat and Mass Transfer* **48** 4510–17
[8] Sun N, Wood N B, Hughes A D, Thom S A M and Xu X Y 2006 Fluid-wall modelling of mass transfer in an axisymmetric stenosis: effects of shear-dependent transport properties *Annals of Biomedical Engineering* **34** 1119–28
[9] Abdullah I, Talib A H and Sabri I 2013 Numerical simulation on convection-diffusion in an overlapping stenosed artery *Malaysian J. Fundamental and Applied Sciences* **9** 156-60
[10] Hayat T, Awais M and Alsaedi A 2012 Newtonian heating and magnetohydrodynamic effects in flow of a Jeffery fluid over a radially stretching surface *Int. J. Physical Sciences* **7** 2838-44
[11] Mustapha N, Amin N, Chakravarty S and Mandal P K 2009 Unsteady magnetohydrodynamic blood flow through irregular multi-stenosed arteries *Comp. in Biology and Medicine* **39** 896-906
[12] Abo-Eldahab E M, El-Aziz M A, Salem A M and Jaber K K 2007 Hall current effect on MHD mixed convection flow from an inclined continuosly stretching surface with blowing/suction and internal heat generation/absorption *Applied Mathematical Modeling* **31** 1829-46
[13] Shit G C 2010 Hall effects on MHD free convective flow and mass transfer over a stretching sheet *Int. J. Applied Mathematics and Mechanics* **5** 22-38
[14] Ahmed N, Kalits H and Barua D P 2010 Unsteady MHD free convective flow pass a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source *Int. J. Engineering Science and Technology* **2** 55-74
[15] Tashtoush B and Magableh A 2008 Magnetic field effect on heat transfer and fluid flow characteristics of blood flow in multi-stenosis arteries *Int. J. Heat and Mass Transfer* **44** 297–304
[16] Prakash O, Singh S P, Kumar D and Dwivedi Y K 2011 A study of effects of heat source on MHD blood flow through bifurcated arteries *AIP Advances* **1** 042128-1-7
[17] Mekheimer K S and Kot M A E 2008 Influence of magnetic field and Hall currents on blood flow through a stenotic artery *Appl. Math. Mech.* **29** 1093–104