Journal of Physics: Conference Series **633** (2015) 012128 doi:10.1088/1742-6596/633/1/012128

Numerical solution of viscous and viscoelastic fluids flow through the branching channel by finite volume scheme

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Abstract. This work deals with the numerical modelling of steady flows of incompressible viscous and viscoelastic fluids through the three dimensional channel with T-junction. The fundamental system of equations is the system of generalized Navier-Stokes equations for incompressible fluids. This system is based on the system of balance laws of mass and momentum for incompressible fluids. Two different mathematical models for the stress tensor are used for simulation of Newtonian and Oldroyd-B fluids flow. Numerical solution of the described models is based on cetral finite volume method using explicit Runge-Kutta time integration.

1. Introduction

Branching of pipes occurs in many technical or biological applications. The use of proper mathematical model is important to get the correct results. In the practice, the numerical simulations are crucial for the solution of a large number of technical problems.

In this work the incompressible laminar viscous and viscoelastic fluids flow are numerically simulated. The mathematical model is based on the system of balance laws of mass and momentum for incompressible fluids. For the different choice of fluids model the different model of the stress tensor is used. For viscous flows the simple *Newtonian model* is considered. In the case of viscoelastic fluids, the simplest viscoelastic model can be used. This model is denoted as *Maxwell model*. By combination of these two models (Newtonian and Maxwell models) the behaviour of mixture of viscous and viscoelastic fluids can be described. This model is called *Oldroyd-B model*. Both models (Newtonian and Oldroyd-B) could be generalized. Viscosity function is defined according to the cross model.

In prewious work we studied the numerical simulation of generalized Newtonian and Oldroyd-B fluids flow in two dimensional branching channel, [4]. In this work we extended our problem to the numerical modelling of tested fluids flow in three dimensional branching channel with T-junction.

2. Mathematical model

The governing system of equations is the system of generalized Navier-Stokes equations, see [6]. This system consists of the continuity equation and the momentum equation

$$\operatorname{div} \boldsymbol{u} = 0 \tag{1}$$


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$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u}.\nabla)\boldsymbol{u} = -\nabla P + \text{div } \mathbf{T}, \tag{2}$$

where P is the pressure, ρ is the constant density, $\boldsymbol{u}$ is the velocity vector. The symbol $\mathbf{T}$ represents the stress tensor.

The stress tensor $\mathbf{T}$ in the system can be decomposed to the viscous $\mathbf{T}_s$ and viscoelastic part $\mathbf{T}_e$ ($\mathbf{T} = \mathbf{T}_s + \mathbf{T}_e$). Both tensors are defined by corresponding rheological models, Newtonian and Maxwell, see e.g. [1], [2], [4]

$$\mathbf{T}_s = 2\mu_s\mathbf{D}, \qquad \mathbf{T}_e + \lambda_1\frac{\delta \mathbf{T}_e}{\delta t} = 2\mu_e\mathbf{D}, \tag{3}$$

where λ_1 is the relaxation time. By combination of these two mathematical models the behaviour of mixture of viscous and viscoelastic fluids can be described. This model is called Oldroyd-B and it's used together with Newtonian model for numerical simulation of tested fluids. The symbol $\frac{\delta}{\delta t}$ used in the viscoelastic part of the stress tensor denotes the upper convected derivative [2], [4].

For the numerical modelling of the generalized Newtonian and Oldroyd-B fluids flow it is necessary to generalize the mathematical models. In this case the viscosity function is defined by cross model [7]

$$\mu(\dot\gamma) = \mu_\infty + \frac{\mu_0 - \mu_\infty}{(1 + (\lambda\dot\gamma)^b)^a}, \quad \dot\gamma = 2\sqrt{\frac{1}{2}\text{tr } \mathbf{D}^2}, \tag{4}$$

with special parameters $\mu_0 = 1.6 \cdot 10^{-1} Pa.s, \mu_\infty = 3.6 \cdot 10^{-3} Pa.s, a = 1.23, b = 0.64, \lambda = 8.2s$. For Newtonian and Oldroyd-B flow, the viscosity is kept constant and equal to μ_∞.

3. Numerical solution

The mathematical models described above are solved numericaly the artificial compressibility approach combined with the finite-volume discretization. The artificial compressibility method [3], [4], [6] is used to obtain equation for pressure. It means that the continuity equation is completed by a pressure time derivative term $\frac{\partial p}{\beta^2 \partial t}$, where β is positive parameter, making the inviscid part of the system of equations hyperbolic. The parameter β in this work is chosen equal to the maximum inlet velocity. This value ensures good convergence to steady state but is not large enough to make the transient solution accurate in time. Therefore it is suitable for steady flows only. The discretization is done by a cell-centered finite-volume method with hexahedral finite volumes. The system including the modified continuity equation and the momentum equations can be written

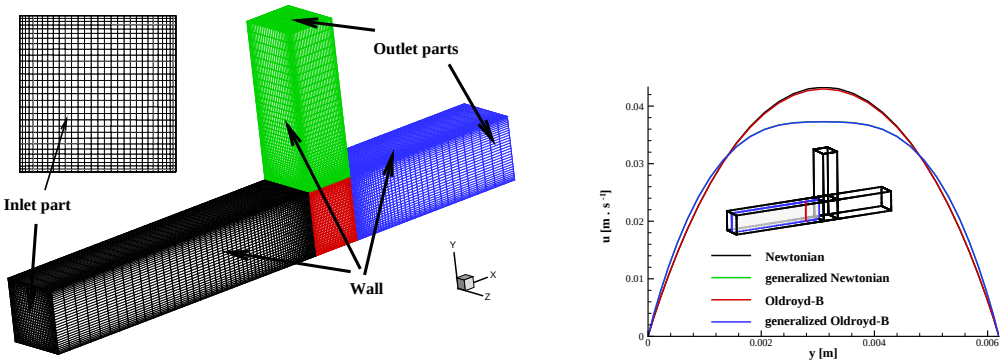
$$\tilde R_\beta W_t + F^c_x + G^c_y + H^c_z = F^v_x + G^v_y + H^v_z + S, \quad \tilde R_\beta = \text{diag}(\frac{1}{\beta^2}, 1, \cdots, 1), \tag{5}$$

where W is vector of unknowns, $W = (p, u, v, w, t_{e1}, \ldots, t_{e6})$, by superscripts c and v the inviscid and the viscous fluxes are denoted. The symbol S denotes the source term. Eq. (5) is discretized in space by the finite volume method and the arising system of ODEs is integrated in time by the explicit multistage Runge–Kutta scheme ([4], [5]).

The flow is modelled in a bounded computational domain where a boundary is divided into three mutually disjoint parts: a solid wall, an outlet and an inlet. At the inlet Dirichlet boundary condition for velocity vector and for the stress tensor is used and for the pressure homogeneous Neumann boundary condition is used. At the outlet parts the pressure value is prescribed and for the velocity vector and the stress tensor homogeneous Neumann boundary condition is used. The no-slip boundary condition for the velocity vector is used on the wall. For the pressure and stress tensor homogeneous Neumann boundary condition is considered.

4. Numerical Results

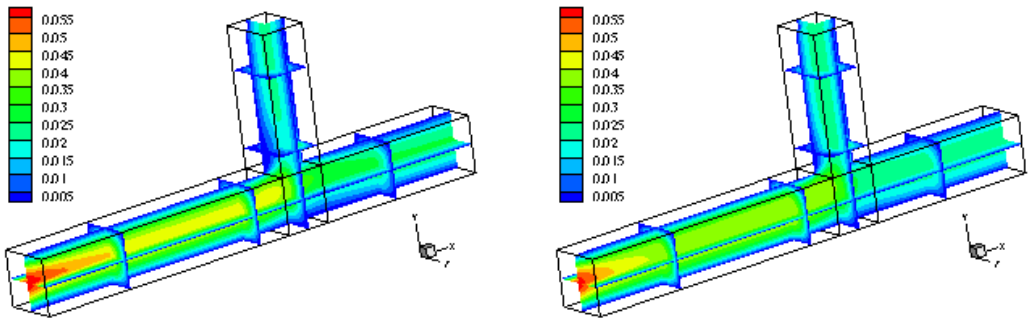
This section deals with the comparison of the numerical results of generalized Newtonian and generalized Oldroyd-B fluids flow. Numerical tests are performed in an idealized branched channel with the square cross-section. Fig. 1 (left) shows the shape of the tested domain. The computational domain is discretized using a structured, wall fitted mesh with hexahedral cells. The domain is divided to 4 blocks with 153 000 cells.



(a) structure of the domain (b) axial velocity profile

Figure 1. Structure of the tested domain (left) and axial velocity profile of tested fluids (right).

As initial condition the following model parameters are used: $L_0 = 0.0031m$, $\mu_e = 0.0004Pa.s$, $\mu_s = 0.0036Pa.s$, $\lambda_1 = 0.06s$, $U_0 = 0.0615m.s^{-1}$, $\rho = 1050kg.m^{-3}$. At the inlet

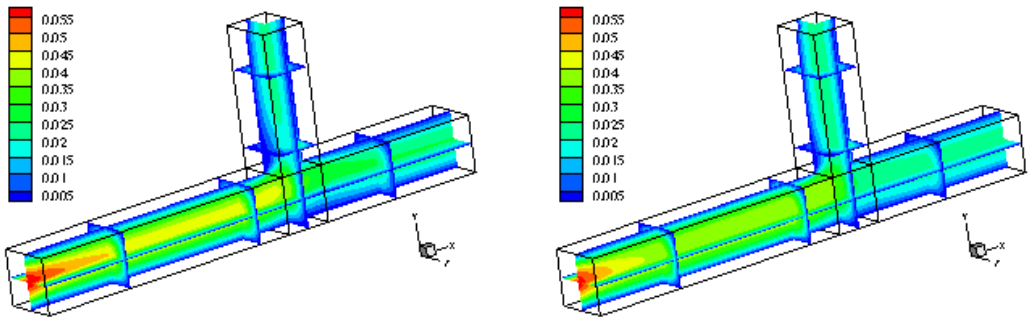


(a) Newtonian (b) generalized Newtonian

Figure 2. Velocity isolines of steady flows for generalized Newtonian fluids.

the Dirichlet boundary conditions for velocity are used, the parabolic profile with maximum velocity value U_0. At the outlet the constant pressure values are prescribed (0.0005 Pa (main channel) and 0.00025 Pa (branch)). In Fig. 1 the axial velocity profile for tested types of fluids close to the branching is shown. The lines for Newtonian and Oldroyd-B fluids are similar to the parabolic line, as was assumed. From this velocity profile is clear that the shear thinning fluids attain lower maximum velocity in the central part of the channel (close to the axis of symmetry) which is compensated by the increase of local velocity in the boundary layer close to the wall.

In Figs. 2 and 3 the velocity isolines and the cuts through the main channel and the small branch are shown.



(a) Oldroyd-B (b) generalized Oldroyd-B

Figure 3. Velocity isolines of steady flows for generalized Oldroyd-B fluids.

It can be observed from these that the size of separation region for generalized Newtonian and generalized Oldroyd-B fluids is smaller than for Newtonian and Oldroyd-B fluids.

5. Conclusion

In this paper a finite volume solver for incompressible laminar viscous and viscoelastic flows in the branching channel with T-junction and circle cross section was described. Newtonian and Oldroyd-B fluids models were generalized by the cross model for numerical solution of generalized Newtonian and Oldroyd-B fluids flow. The explicit Runge-Kutta method was considered for time integrating. The numerical results obtained by this method were presented.

Numerical results were compared. It has been shown that in this type of the channel the numerical results for Newtonian and Oldroyd-B fluids are similar. From the presented velocity profile is clear that the shear thinning fluids (generalized Newtonian and Oldroyd-B fluids) attain lower maximum velocity in the central part of the channel (close to the axis of symmetry) which is compensated by the increase of local velocity in the boundary layer close to the wall.

The future work will be occupy with the numerical simulation of viscous and viscoelastic fluids flow in the branching channel with circle cross-section and with the comparison numerical results of both type of channels.

Acknowledgments

This work was supported by the grant SGS13/174/OHK2/3T/12.

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