

Movement of a vortex filament near oscillating pinning centers in the hard superconductor

D Apushkinskaya¹, E Apushkinskiy² and M Astrov³

¹ Department of Mathematics, Saarland University, Saarbrücken, Germany

² St. Petersburg Polytechnical University, St. Petersburg, Russia

³ D.V. Efremov Research Institute of Electrophysical Apparatus, St. Petersburg, Russia

E-mail: darya@math.uni-sb.de

Abstract. A problem on a vortex behavior near oscillating pinning centers in the type III superconductors (SCs) is considered. The movement of a vortex filament is described by the parabolic free boundary problem, where an a priori unknown free boundary is the set of points, at which a transition from superconducting to normal state and vice versa occurs. A number of statements on the qualitative properties of a phase boundary superconductor - nonsuperconductor are considered.

1. Introduction

As is known, the real type II superconductors (the so called hard or type III SCs) involve different defects of a crystal lattice, and the magnetic field penetrates into a superconductor in the form of separate vortices interacting with these defects. The arising forces (known as pinning forces) form a state of vortex matter with special electro-acoustic properties. The vortices have different lengths provided by the intricate configuration of the superconducting grains. The vortices may be pinned to a crystal lattice in cites of its defect. These are regions with broken superconductivity due to the changes in the carrier-lattice interaction, that results in the deterioration of conditions for superconducting pair production (low density of superconducting pairs at defects). Since the crystal defects are distributed inside a grain in a complicated manner and the magnetic field penetrates into the sample regions without superconducting pairs only, the configuration of each magnetic filament (vortex) is also quite complicated (Fig. 1).

Determination of the vortex interaction with a concrete pinning center (elementary pinning force) is a hard problem because of the complicated microstructure of a real sample. Direct experimental investigations of the elementary pinning forces require very complicated instrumentation as well as experimenter skills [1]. Theoretical investigations of the elementary pinning forces are made, mainly, in the linear approximation and far from all the defect types [2]. It is well-known that hard SCs are a strongly nonlinear medium. So, the works, where elementary pinning force analysis is carried out without substantial limitations related with a kind of interaction between vortex and crystal lattice defects, are of a great interest.

2. Theory

We consider the vortex oscillations excited by an external ac field action [3] in the absence of any vortex jumps between the pinning centers. The surface exposed to the vortex magnetic moment



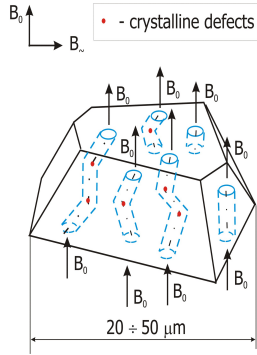


Figure 1. A superconductor powder grain in the external dc magnetic field \mathbf{B}_0

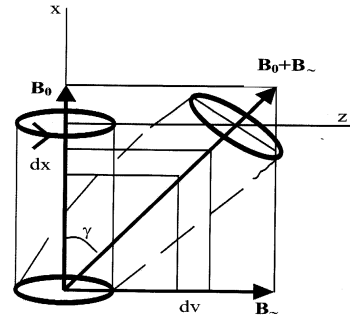


Figure 2. Vortex oscillation caused by ac field \mathbf{B}_\sim

should oscillate following the total external magnetic field (Fig. 2). Simultaneously, the vortex, which forms it, should also oscillate around the equilibrium position. The vortex axis deviates from the equilibrium position by $v(x, t)$. In geometry terms, $\tan \gamma = v_x = B_0^{-1} B_\sim(t)$ (Fig. 2) that determines a formula for the vortex deviation at boundaries.

Let us write the equation of motion for a vortex displacement v for volume element ΔV

$$-G_d \mathbf{v}_t - \mathbf{F}_{rest} + [\mathbf{j}_{RF} \mathbf{B}_0] \Delta V + \mathbf{F}_T = m_f \mathbf{v}_{tt}, \quad (1)$$

similar to that presented in [4]. The forces entering in (1) are:

- $G_d \mathbf{v}_t$ is the force of viscous decay of the vortex motion, which manifests itself only at vortex filament edges due to radiation and at the pinning sites;
- \mathbf{F}_{rest} is the retrieving force conditioned by the interaction with other fluxoids and the voltage induced by the randomly distributed separate pinning centers;
- $[\mathbf{j}_{RF} \mathbf{B}_0] \Delta V$ is the Ampere force; The induction $B_\sim(x)$ at sample boundary is determined by the external action and it decreases while penetrating deep into the sample. Hence, $v_{xx} = B_0^{-1} (B_\sim)_x$ and $[\mathbf{j}_{RF} \mathbf{B}_0] = \mathbf{k} \mu_0^{-1} B_0^2 v_{xx} = \mathbf{k} C_{44} v_{xx}$, where $C_{44} = \mu_0^{-1} B_0^2$ is introduced for the vortex lattice modulus of elasticity.
- \mathbf{F}_T is the random force of thermal deviation, which is considered to be zero;
- $m_f \mathbf{v}_{tt}$ is the force of inertia, which is usually small near the grain boundary in the external ac magnetic field, because the vortex inertia mass m_f of its core is very small.

If the external action is quite strong, one can neglect m_f in Eq. (1). This assumption is valid only at boundary of the type II SC or in the vicinity of the pinning centers inside SC. \mathbf{F}_{rest} is zero in the equilibrium state; for the nonequilibrium state it can be written as

$$F_{rest} = \alpha_L v dV + f(x, t) \chi_{\{v>0\}}. \quad (2)$$

Here α_L is the Labusch parameter, while the values of $f(x, t)$ are determined by the pinning center distribution, the entanglement of the vortices, the anisotropic properties of SC as well as by the transport current if the latter exists. The function $\chi_{\{v>0\}}$ denotes the characteristic function of a set $\{v > 0\}$, i.e., $\chi_{\{v>0\}}(x, t) = 1$ if $v(x, t) > 0$ and $\chi_{\{v>0\}}(x, t) = 0$ otherwise.

Considering the aforesaid, Eq. (2) at the sample boundaries and in the vicinity of pinning centers under the strong external action takes the form

$$C_{44} dV v_{xx} - G_d v_t - \alpha_L v dV = f(x, t) \chi_{\{v>0\}}. \quad (3)$$

Since the vortex deviations from the equilibrium state in the positive and negative directions are absolutely symmetric, we restrict our analysis by a case $v \geq 0$.

The model equation (3) can be treated as the parabolic obstacle problem

$$a(x, t)v_{xx} + b(x, t)v_x + c(x, t)v - v_t = f(x, t)\chi_{\{v>0\}}, \quad (x, t) \in Q_R, \quad (4)$$

$$v(x, t) \geq 0 \quad \text{in} \quad Q_R := \{(x, t) \in \mathbb{R}^2 : |x| < R, k|t| < R^2\}. \quad (5)$$

For coefficients a, b, c and function f we assume that

- (i) a and f are nondegenerate in Q_R , i.e., there is $\delta_0 > 0$ such that $a(x, t) > \delta_0$ and $f(x, t) > \delta_0$ for any $(x, t) \in Q_R$;
- (ii) a, b, c and f belong to $C^{\alpha, \alpha/2}(Q_R)$ for some $\alpha \in (0, 1)$.

Note, $\{v > 0\}$ is a priori unknown subset of Q_R . We denote by Γ the intersection of Q_R with the boundary of the set $\{v = 0\}$. Observe that an a priori unknown interface Γ describes a set of points in the plane (x, t) where transition from superconducting to normal state or vice versa is realized. Indeed, suppose that a vortex is initially fixed and its core has a stationary boundary separating the superconducting area outside a vortex and normal region inside it. After beginning of the external action on the vortex, some deviation v of the core boundary from its stationary state has place. The separation point starts moving from the sample boundary or from the pinning centers. So, on the plane (x, t) the trajectory of separation point is given by $x = g(t)$ determining the interface between superconducting and normal areas.

3. Discussion

3.1. Qualitative properties of v

- (S1) The problem (4)-(5) has a unique solution v under suitable initial and boundary data [5].
- (S2) Functions v and v_x are continuous and the set $\{v = 0\}$ is closed [6, 7].
- (S3) Locally, v has bounded second derivatives in space and first derivative in time [8].
- (S4) v_{xx} is discontinuous across Γ .
- (S5) v_t is continuous for almost every t . More precisely, $v_t = 0$ in the interior of the set $\{v = 0\}$. If $v(x^0, t^0) > 0$, the standard parabolic estimates imply that v_t is continuous near (x^0, t^0) . So, the only difficulty is the regularity of v_t on Γ . We emphasize that the continuity of v_t cannot be obtained everywhere in t . The relevant counterexample can be found in [8].
- (S6) If, in addition, $v_t \geq 0$, then v_t is continuous for all t and satisfies $v_t = 0$ on Γ . The assumption, that v_t is nonnegative, can be established in some special cases.
- (S7) v is nondegenerate on the closure of $\{v > 0\}$, i.e., there is an absolute constant c such that

$$\sup_{Q_\rho^-(x^0, t^0)} v \geq c\rho^2, \quad c > 0, \quad (6)$$

where $Q_\rho^-(x^0, t^0) := \{(x, t) \in \mathbb{R}^2 : |x - x^0| < \rho, t^0 - t < \rho^2\}$ and ρ is small enough [8]. Ineq. (6) prevents Γ from being flat with the set $\{v = 0\}$ below (such case is shown on Fig. 3).

3.2. Properties of the phase boundary Γ

- (FB1) Ineq. (6) guarantees that Γ cannot appear or disappear suddenly, or is not "blurred".
- (FB2) There is a set $\Gamma^* \subset \Gamma$ such that the Lebesgue measure of the set $\Gamma \setminus \Gamma^*$ is equal to zero. The set Γ^* is locally contained in a $C^{1/2}$ -graph as a function of t [9].

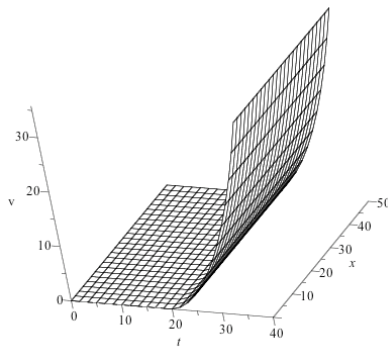


Figure 3. Impossible case of Γ dynamics

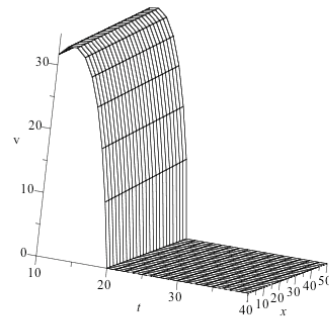


Figure 4. Possible case of Γ dynamics

3.3. Interpretation

- Properties (S2) and (S3) guarantees that the propagation of superconductivity along a sample proceeds with the speed bounded for almost all t . The latter means that the Lebesgue measure of the set t , where v_t is unbounded, is equal to zero. Formation or destruction of superconducting carriers occur precisely in such moments.
- It follows from (S7) that the formation of superconducting carriers cannot occur simultaneously on the spatial interval. The opposite case, i.e., simultaneous destruction of carriers on the spatial interval, is allowed (Fig. 4).
- Due (S2), a vortex remains elastic at all x and t . It was verified experimentally [1].
- Γ describes the set of points, where transition from SC to normal state or vice versa is realized. In view of (FB2), for almost every t the set Γ can be locally defined by the equation $x = g(t)$ with bounded continuous function g satisfying the inequality $|g(t)| \leq C\sqrt{t}$ with an absolute constant $C > 0$. Thus, in the local coordinate system for almost all t we have a growth estimate for the function describing Γ .

4. Conclusions

Based on analysis of the model we prove finiteness of propagation speed for the boundary between normal and superconducting (SC) states and show that

- The force of decay of the oscillating vortex is continuous for almost all t ;
- A transition from normal to SC state is impossible on the spatial interval. The opposite case, i.e., a transition from SC to normal state on the spatial interval is allowed;
- The boundary between normal and SC states cannot appear or disappear suddenly.

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