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Improvement of rotated comb decimation filter magnitude characteristic using sharpening technique

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Decreasing of sampling rate in a digital form is called decimation. This process may introduce aliasing, i.e. non desired replicas of the main spectrum of the decimated signal. That is why we need the filter before decimation. This filter is called the decimation or antialiasing filter. The most popular decimation filter is comb filter which is usually used in the first stage of decimation. However it has a high passband droop and low attenuations in the folding bands. Different methods have been proposed to improve comb magnitude characteristic. The methods based on simple zero rotations have been proposed recently. In this paper we use the sharpening technique to improve both the passband and the stopband of comb filter with the rotated zeros. The method is compared with methods based on the rotation of zeros of comb filter.

1. Introduction

The decreasing of the sampling rate in digital form is called decimation. This process may introduce the aliasing effects that deteriorate the signal and have to be eliminated by the decimation filter, as shown in Figure 1, where M is the decimation factor [1].

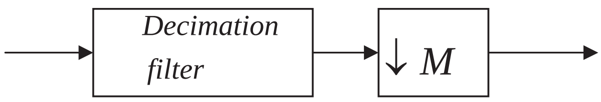


Figure 1. Decimation.

The decimation is usually accomplished using a cascade of two or more stages. Because the filter in the first stage must be very simple, it is usually used comb filter, which does not require the multipliers. The magnitude response of comb filter must be flat in the passband to avoid the distortion of the decimated signal. Additionally, the comb magnitude response must have high attenuations in the so called folding bands, (bands around the zeros of comb filter). However, the comb magnitude response exhibits a high passband droop and the folding bands are narrow and have low attenuations. Different methods have been proposed to improve comb magnitude characteristic in the passband [2-7], in the folding bands [8-9] and in both, the passband and the folding bands [10-11]. Recently, the simple zero-rotation of comb filter has been introduced in [12] to improve the alias rejections in the folding bands, for even decimation factors. However, by introducing the zero rotation, the passband droop is increased. The main goal of this work is to apply the sharpening technique to decrease comb passband droop and further improve the alias rejection in comb filters with the simple zero rotation.

The rest of the paper is organized in the following way. Next section briefly describes the simple zero rotation of comb filters. Section 3 presents the sharpening technique. The proposed method is described in Section 4 and illustrated with examples. Section 5 describes a two-stage structure, where the sharpening is applied only at the second stage.

2. Simple zero-rotation of comb filters [12]

The system function of a comb filter can be expressed, either in the recursive form,

$$H(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^{K}, \tag{1}$$

or in recursive form,

$$H^{K}(z) = \left[\frac{1}{M}\sum_{k=0}^{M-1} z^{-k}\right]^{K}, \tag{2}$$

where M is a decimation factor and K is order of the filter and presents the number of the cascaded comb filters.

The magnitude response of the filter is in the sinx/x form :

$$\left|H(e^{j\omega})\right|^{K} = \left|\frac{\sin(\omega M/2)}{M\sin(\omega/2)}\right|^{K}. \tag{3}$$

[1] The rotation in [12] is obtained by subtracting a positive constant a at the point of symmetry of the comb impulse response. Taking into account that we consider here comb filters with the even length, in order to obtain odd length of the comb filter we apply the convolution of two even length comb filters. The resulting point of symmetry is at the point M-1. By omitting the normalisation constant, the rotation term is given as:

$$H_r(z) = \left[\sum_{k=0}^{M-1} z^{-k}\right]^{2} - az^{-(M-1)},. \tag{4}$$

The Rouche's theorem is used to prove that the rotated zeros are on unit circle. The system function of the rotated comb is the cascade of K_1 combs and the rotated term:

$$H_R(z) = H^{K_1}(z)H_r(z). \tag{5}$$

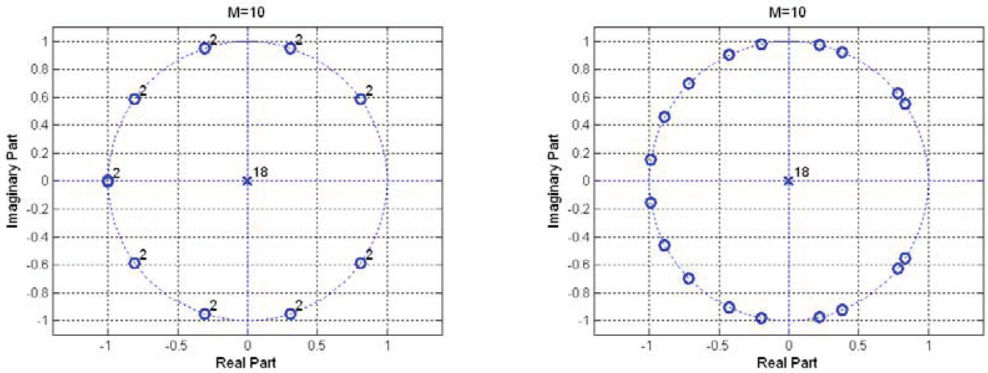
Example 1:
Consider M=10 and a=1/2. The rotated term is given as:

$$H_r(z) = \left[\sum_{k=0}^{9} z^{-k}\right]^{2} - 0.5z^{-9}. \tag{6}$$

The system function of the corresponding rotated comb is:

$$H_R(z) = \left[\frac{1}{10}\frac{1-z^{-10}}{1-z^{-1}}\right]^{K_1}\left[\left[\sum_{k=0}^{9} z^{-k}\right]^{2} - 0.5z^{-9}\right]. \tag{7}$$

Figure 2a presents the z-plane plot of the corresponding comb with K=2. Similarly, Figure 2b shows the corresponding z-plane plot for the rotated term (6). Note, that the zeros of the rotated term, are rotated from their original positions, shown in Fig.2a. As a result, the widths of the folding bands are increased. The overall magnitude responses and the passband zooms of the rotated comb (5) for K_1=2 and the corresponding comb with K=4, are shown in Figure 3.



a. Comb. b. Rotated term.

Figure 2. Zeros of comb and rotated term.

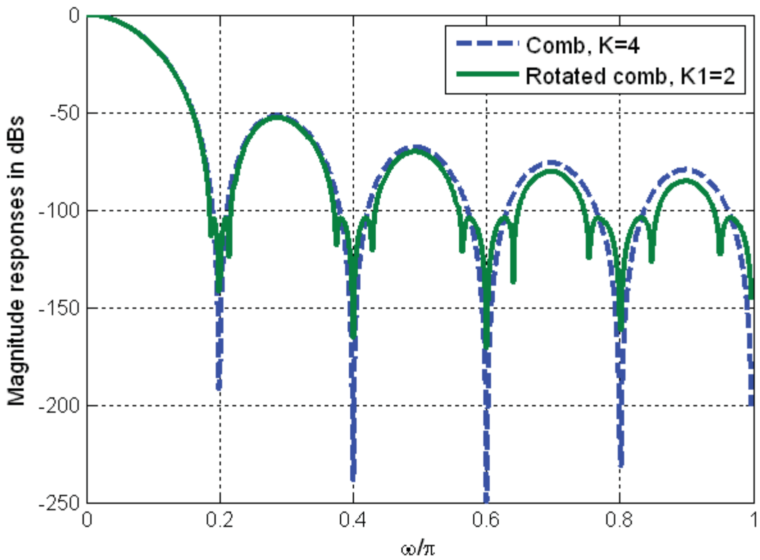


Figure 3. Magnitude responses of rotated comb and corresponding comb.

3. Sharpening technique

The sharpening technique [13] simultaneously improves the passband and the stopband of symmetrical prototype filter H, using a polynomial relationship of the form $H_{sh} = f(H)$ between the amplitudes of the sharpened H_{sh} and the prototype filters H. The improvement in the magnitude response near the passband edge H=1, or near the stopband edge H=0, depends on the order of tangencies m and n of the polynomial relationship at H=1, or at H=0, respectively.

The polynomial relationship in terms of m and n is given [13]

$$H_{sh} = H^{n+1} \sum_{s=0}^{m} \frac{(n+s)!}{n!\,s!}(1-H)^{s} = H^{n+1} \sum_{s=0}^{m} C(n+s,s)(1-H)^{s}, \qquad (8)$$

where $C(n+s,s)$ is the binomial coefficient.

The most simple polynomial for the passband improvement, (m=1, n=0) is in the form $H_{sh1}=2H-H^2$.

Similarly, the most simple polynomial for improvement of passband and stopband is given as $H_{sh2}=3H^2-2H^3$.

Example 2:

We apply both sharpening polynomials to comb filter with M=10 and K=2. Figure 4 shows the corresponding magnitude responses.

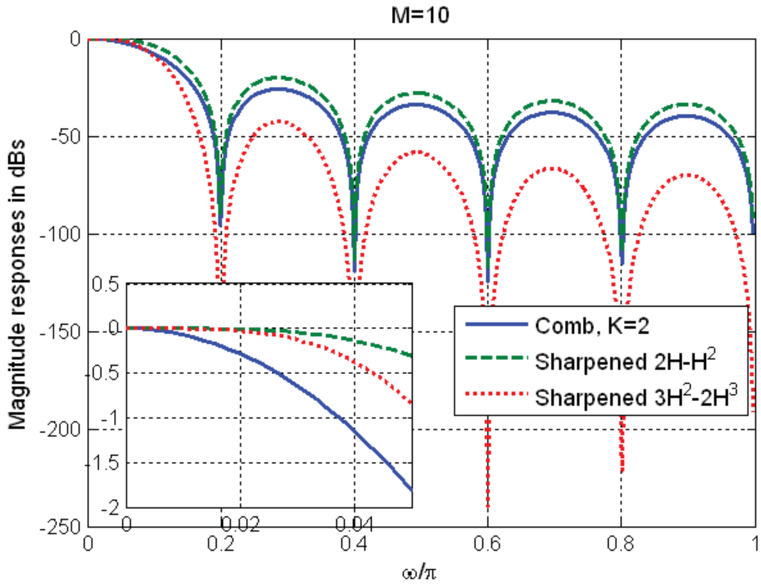


Figure 4. Magnitude responses of sharpened combs and the prototype comb.

4. Sharpened rotated comb

We propose to use simple sharpening polynomials $H_{sh1}=2H-H^2$, and $H_{sh2}=3H^2-2H3$, to improve the magnitude response of the rotated comb.

The sharpened rotated combs are defined by,

$$H_{sh1}(z) = 2H_R(z) - H_R^2(z)\,, \tag{9}$$

$$H_{sh2}(z) = 3H_R(z) - 2H_R^3(z)\,. \tag{10}$$

where $H_R(z)$ is given in (5).

4.1. Comparison with comb

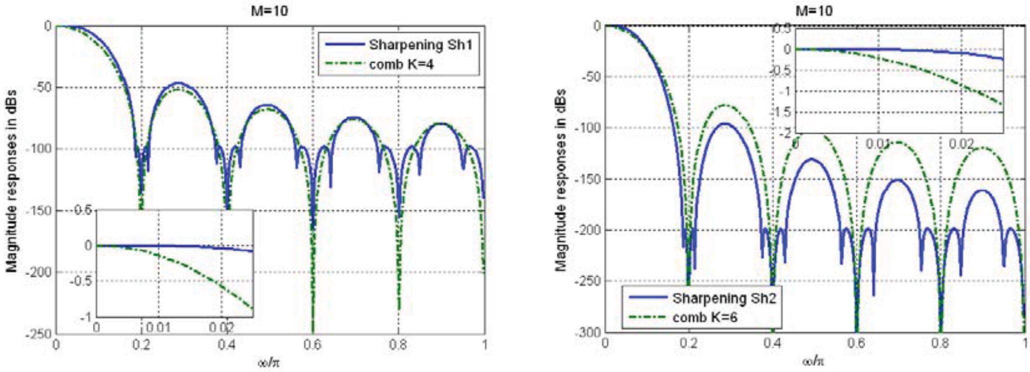
Example 3:

In this example we first compare magnitude responses of the sharpened rotated comb from Example 1, using the sharpening polynomial (9) with the magnitude response of comb with K=4. Figure 5a shows the corresponding magnitude responses along with the passband zooms. Similarly, Figure 5b compares the magnitude responses of sharpened rotated comb (10) and comb filter.

4.2. Comparison with rotated comb

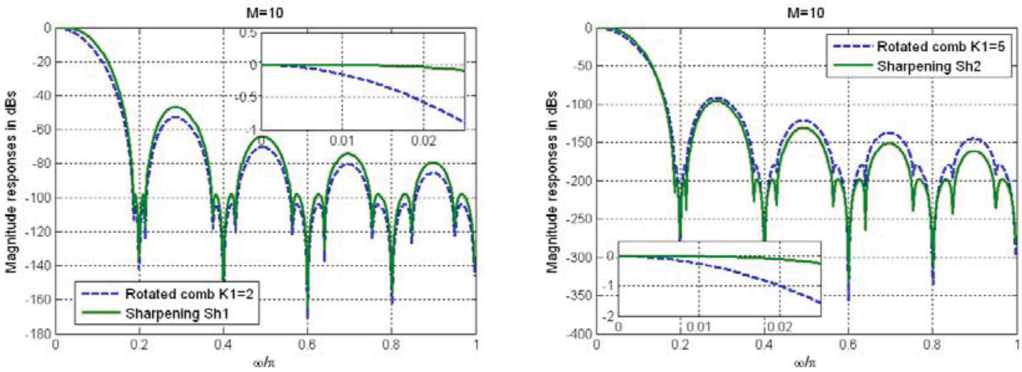
Example 4:

We compare sharpened rotated combs with the original rotated combs. To this end, Figures 6a and 6b show magnitude responses of the sharpened rotated combs and the original combs taking the sharpening polynomials (9) and (10).



a. Sharpening polynomial (9). b. Sharpening polynomial (10).

Figure 5. Magnitude responses of comb, and sharpened rotated combs.



a. Sharpening polynomial (9). b. Sharpening polynomial (10).

Figure 6. Magnitude responses of rotated comb, and sharpened rotated comb.

5. Two-stage sharpened compensated filter

In order to decrease the complexity, consider that the decimation factor can be factorized in the form $M=M_1M_2$, where M_2 is even number. In this case we can split decimation into two stages, where in the first stage is the comb filter,

$$H_1(z)=\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{K_{11}}, \tag{11}$$

decimated by M_1, while in the second stage is the comb filter,

$$H_2(z)=\left[\frac{1}{M_2}\frac{1-z^{-M_2}}{1-z^{-1}}\right]^{K_{22}}, \tag{12}$$

decimated by M_2.

In that way, we can apply the rotation only to the second stage, and thus to apply sharpening only to the rotated comb at that stage:

$$H_{2sh1}(z)=2H_{2R}(z)-H_{2R}^{2}(z) \tag{13}$$

$$H_{2sh2}(z)=3H_{2R}(z)-2H_{2R}^{3}(z), \tag{14}$$

where

$$H_{2R}(z) = H_2^{K_{22}}(z)H_{2r}(z) \ , \tag{15}$$

$$H_{2r}(z) = \left[\sum_{k=0}^{M_2-1} z^{-k}\right]^2 - az^{-(M_2-1)} \ . \tag{16}$$

The corresponding structure is given in Figure 7.

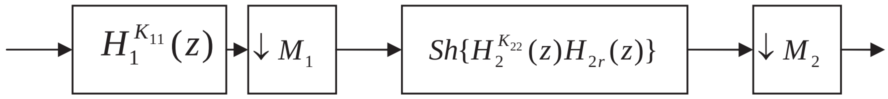


Figure 7. Two-stage sharpening structure.

Next example illustrates the characteristics of two-stage structure.
Example 6
We consider a two-stage structure with M_1=3 and M_2=4. Figure 8 shows the magnitude response of the two stage sharpening structure, and the corresponding comb with K=6.In the first stage is the comb filter with the parameter K_{11}=6, while the second sharpening polynomial is applied to the rotated comb $H_2(z)$, with K_{22}=2. Note that the two-stage structure provides more attenuations in the folding bands, and less passband droop.

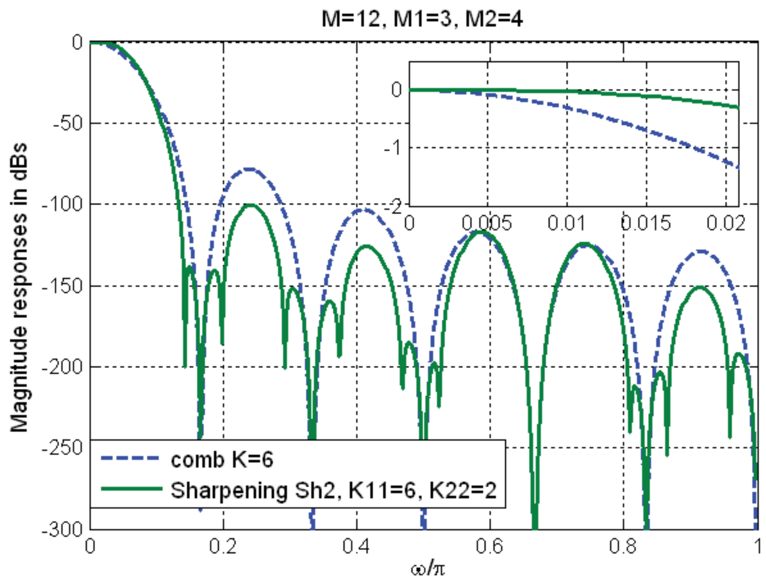


Figure 8. Magnitude responses of two-stage sharpened structure and the comb.

6. Concluding remarks

This paper presents the application of sharpening technique for the improvement of magnitude response of a simple rotated comb. Two simple sharpening polynomials have been considered. In that way the improvements in both, passband and stopband, in comparisons with the original and rotated combs, have been achieved. In order to decrease the complexity, introduced by the sharpening technique, a novel two-stage structure is also proposed, in which the zero rotation and the corresponding sharpening are applied only at the second stage.

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