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The effect of thermal losses on traveling waves for in-situ combustion in porous medium

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Abstract. We study a model for the injection of air into an underground porous medium that contains a solid fuel. In our previous works the model was simplified and all wave sequences for the Riemann problem solution were obtained without taking into account thermal losses to the surrounding rock. In this work the first step is made to understand the effect of heat losses, which are important especially in laboratory experiments. In order to simplify the proof of the existence and uniqueness of the traveling wave solution, we disregard diffusion effects and the dependence of gas density on temperature.

1. Introduction

Air injection with in-situ combustion offers several potential technical and economic advantages that may include faster oil production, reduced operational costs and increased CO_2 content with decreased oil viscosity. Despite other difficulties related to engineering and chemical modeling, solving the equations for such models is a challenge.

This paper is part of long term research project the purpose of which is to identify waves that arise in one-dimensional models of combustion in porous media, and to understand how the waves fit together in solutions of Riemann problems; see [1, 2, 3, 4, 5, 6], and references therein. The paper is motivated by a model for the injection of air into a porous medium that contains oil so viscous that it can be considered a solid fuel. The model was proposed in [7] and further studied in [1]. This model was simplified in [8] in order to (i) reproduce the variety of phenomena observed when air is injected into a porous medium containing a solid fuel, yet (ii) to be simple enough to permit a rigorous investigation.

In this paper we prove the existence of a traveling wave solution corresponding to the combustion wave for a simple in-situ combustion model. This model is more general then the previously considered one [8] in three aspects. First it considers more physically realistic thermal capacity of the medium, leading to distinct thermal and gas velocities. Second, we utilize a more correct form of Arrhenius law, allowing chemical reaction to happen at any temperatures, see also [9]. Third, we take into account the effect of thermal losses, which is important for laboratory experiments. We solve possible Riemann problems and classify the resulting solutions depending on the presence of slow, fast or resonance combustion waves. We emphasize that the existence and uniqueness proofs for the results presented in this paper are technically simple and should be accessible to students. For other approaches addressing combustion with heat losses see [10].

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A model for combustion is presented in Section 2. It consists of three balance laws for energy, oxygen, and fuel. We use a reaction rate described by Arrhenius law combined with Law of Mass Action. The combustion waves studied in this paper have been called "reaction-trailing smolder waves" [11] and "coflow (or forward) filtration combustion waves" [12] in the context of more realistic models of air injection into a porous medium. We formulate the main results of the paper in Section 3. Due to lack of space we present only schematically the proofs of the existence and uniqueness of the combustion traveling waves in Section 4. The complete proof will be published soon in a companion paper with some numerical examples showing the phase portrait of the traveling wave.

2. Combustion model

We consider one-dimensional flow due to air injection into a porous medium. We use notation and assumptions from [1]. We neglect changes of porosity during the reaction and gas expansibility under temperature increase. We assume that the temperature of solid and gas is the same (local thermal equilibrium). This work is concerned with heat losses, which we consider to depend linearly on the temperature difference with the prevailing temperature, see e.g., [7]. We also assume that pressure variations are small compared to the prevailing pressure. In order to prove the existence of traveling waves minimizing technical difficulties we neglect thermal diffusion effects. In dimensionless form the governing system of equations is written as follows [8]:

$$\frac{\partial \theta}{\partial t} + v_\theta \frac{\partial \theta}{\partial x} = -\beta\theta + \rho Y \Phi, \tag{1}$$

$$\frac{\partial Y}{\partial t} + v_Y \frac{\partial Y}{\partial x} = -\mu_Y \rho Y \Phi, \tag{2}$$

$$\frac{\partial \rho}{\partial t} = -\rho Y \Phi, \tag{3}$$

$$\Phi = \exp\left(\frac{-\mathcal{E}}{\theta + \theta_0}\right), \tag{4}$$

where the dependent variables are temperature θ, oxygen fraction Y and fuel ρ. Here v_θ and v_Y are dimensionless thermal and oxygen wave speeds; β is the constant thermal loss coefficient; $\bar{\lambda}$ represents the dimensionless thermal diffusion coefficient; μ_Y represents the dimensionless quantity of oxygen consumed during the reaction; $\mathcal{E}$ is the scaled activation energy and θ_0 is the scaled reservoir temperature. The oxygen Y is a component of the gas moving with velocity $v_Y > 0$. The heat θ is transported with velocity v_θ. We are of course interested in solutions with $\rho \geq 0$ and $Y \geq 0$ everywhere. We consider (1)–(3) on $0 < x < \infty$, $t \geq 0$, with the (constant) boundary conditions

$$(\theta, \rho, Y)(0) = (\theta^L, \rho^L, Y^L), \qquad (\theta, \rho, Y)(\infty) = (\theta^R, \rho^R, Y^R). \tag{5}$$

We assume that the reaction does not occur at the boundaries, i.e., the reaction terms in (1)–(3) vanish. Differently from [6, 8], here we consider the correct Arrhenius law (4) and thus there are only two reasons for the reaction terms to vanish: (i) fuel control (FC) – the reaction ceases due to lack of fuel, $\rho = 0$; (ii) oxygen control (OC) – the reaction ceases due to lack of oxygen, $Y = 0$. In the next section we study the solution of the Riemann problem for system (1)–(4).

3. Wave sequences

In this section we follow [6, 8] and denote by $(\theta^-, \rho^-, Y^-) \xrightarrow{v} (\theta^+, \rho^+, Y^+)$ a wave of velocity v that connects (θ^-, ρ^-, Y^-) at the left to (θ^+, ρ^+, Y^+) at the right. At the end states of the wave, the reaction terms in (1)–(3) vanish. States at which the reaction terms vanish can be classified as FC or OC. The type of the state indicates exactly which conditions hold at that state. For simplicity we consider that $\rho = 1$ in OC state and $Y = 1$ in FC state.

3.1. Contact waves

In the non-combustion waves supported by system (1)–(3) the source terms vanish. The characteristic eigenvalues and corresponding eigenfunctions of the resulting hyperbolic system are (see [8] for details) $\lambda_\theta = v_\theta$ and $(1,0,0)^T$, $\lambda_Y = v_Y$ and $(0,1,0)^T$, $\lambda_f = 0$ and $(0,0,1)^T$. We can see that the Riemann problem possesses three non-combustion waves. As the characteristic speeds are constant the waves correspond to contact discontinuities. The latter separate moving spatial intervals in which the reaction does not occur (since (θ, ρ, Y) is constant).

3.2. Combustion waves

As in [8], system (1)–(4) possesses a traveling combustion wave. We formulate the main result below and give a sketch of the proof in Section 4.

Theorem 3.1. *The system* (1)–(3) *possesses a unique combustion wave in the following cases*

(i) If $v_Y < (\mu_Y + 1)v_\theta$ then a unique slow combustion wave with speed $v < v_\theta$ exists.

(ii) If $v_Y > (\mu_Y + 1)v_\theta$ then a unique fast combustion wave with speed $v > v_\theta$ exists.

(iii) If $v_Y = (\mu_Y + 1)v_\theta$ then there exists a unique combustion wave if and only if: either $\mathcal{E} < 4\theta_0$ or there are exactly three values of $\theta > 0$ satisfying Eq. $4\beta\theta \exp(\mathcal{E}/(\theta + \theta_0)) = 1$.

In all cases the combustion wave is of type $FC \xrightarrow{v} OC$, where its speed is given by v in Eq. (7).

3.3. Solutions of the Riemann problem

For the existence of a wave sequence describing Riemann solution obviously it is has to start as one equilibrium state (FC or OC) and finish at another equilibrium state (FC or OC). The waves speeds in the sequence appear in increasing order from left to right. This fact together with the results concerning contact and combustion waves described above lead to three possibilities for wave sequences corresponding respectively to three cases described in Theorem 3.1.

(i) If the sequence contains a slow combustion wave $FC \xrightarrow{v} OC \xrightarrow{v_\theta} OC$.

(ii) If the sequence contains a fast combustion wave $FC \xrightarrow{v_\theta} FC \xrightarrow{v} OC$.

(iii) If the sequence is composed of a single resonance combustion wave $FC \xrightarrow{v} OC$.

In Fig. 1 we plot the wave sequences separated by constant states for the cases (i) (left figure), (ii) (center figure) and (iii) right figure.

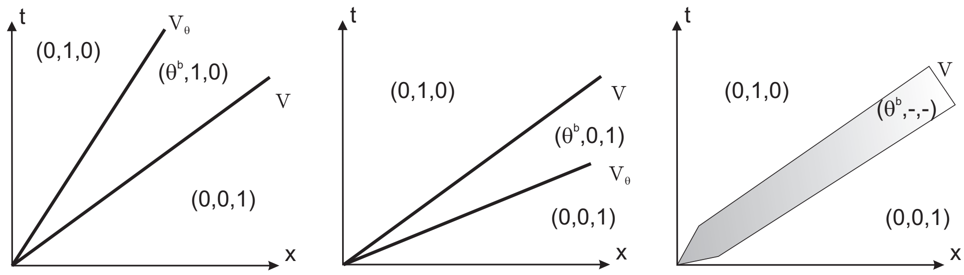

Figure 1. Wave sequences in Riemann solution separated by constant states: (i) (left figure), (ii) (center figure) and (iii) right figure. θ^b is the temperature of the combustion wave.

4. Existence and uniqueness of the combustion traveling waves

We rewrite the system (1)–(4) in traveling coordinates $(x, t) \to (\xi = x - vt, t)$, where v is positive velocity of the traveling wave. We integrate the resulting system of ODEs once and substitute the boundary conditions (5), obtaining

$$d_\xi \rho = \rho(1-\rho)\frac{\Phi}{v}, \qquad d_\xi \theta = \frac{\rho(1-\rho)\Phi - \beta\theta}{v_\theta - v}. \tag{6}$$

After some manipulations, similar to those done in [8], we obtain the combustion wave speed v and the oxygen concentration as function of fuel concentration

$$v = v_Y/(\mu_Y + 1), \quad Y = 1 - \rho. \tag{7}$$

Following [8], waves with velocities $v > v_\theta$ and $v < v_\theta$ are called *fast* and *slow combustion waves* respectively. The case when $v_\theta = v$ is known as resonance condition for the combustion wave, see [2, 12, 13] and references therein.

The proof of Theorem 3.1 is extensive and can not be presented here due to limitation in space, it will be published later in a companion paper. The existence and uniqueness of the traveling wave solution for the system (1)–(4) is equivalent to the existence and uniqueness of an heteroclinic orbit of system (6) connecting two equilibria, which are hyperbolic. The local analysis based on the Stable Manifold Theorem is performed to understand the behavior of stable and unstable manifolds in the vicinity of these equilibria and verify the necessary conditions for the existence of such an heteroclinic orbit. The rest of the proof is based on the geometrical analysis of the flux defined by Eqs. (6).

In the model (1)–(3) without thermal losses the traveling wave equations under the resonance condition $v = v_\theta$ yields in a degenerate traveling wave without combustion. The same situation happens for the model studied in [8]. Taking into account thermal losses allows the appearance of a different type of solution, which is interesting from the physical point of view.

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