

# The filamentation of the laser beam as a *labyrinth* instability

**Florin Spineanu and Madalina Vlad**

National Institute of Laser, Plasma and Radiation Physics, Magurele, 077125 Bucharest,  
Romania

E-mail: [florin.spineanu@free.fr](mailto:florin.spineanu@free.fr)

**Abstract.** At incident powers much higher than the threshold for filamentation a pulse from a high-power laser generates in the transversal plane a complex structure. It consists of randomly meandering stripes defining connected regions where the field intensity is high; and, the complementary regions dominated by diffusive plasma with defocusing property. The pattern is similar to an ensemble of clusters of various extensions. We provide evidence that there is a correlation between this filamentation and the *labyrinth* instability in reaction-diffusion systems. Besides the similarity of the spatial organization in the two cases, we show that the differential equations that describe these two dynamical processes lead to effects that can be mutually mapped. For the laser beam at high power the Non-linear Schrodinger Equation in a regime of strong self-focusing and ionization of the air leads to multiple filamentation and the structure of clusters. Under the effect of the *labyrinth* instability a model of activator-inhibitor leads to a similar pattern. The origin of this connection must be found in the fact that both optical turbulence and the activator-inhibitor dynamics have the nature of competition between two phases of the same system.

## 1. Introduction

The propagation of the beam of a high power laser is an interaction with the atmospheric medium that involves the local activation of the polarization and of other processes that depends nonlinearly on the incident field. The first consequence is the tendency of self-suppression of the propagation, caused by the self-focusing of the beam. In an analytic description this is manifested by the existence of a singularity of the electric field of the wave, which arises in finite time [1]. Effectively, the characteristic trajectories of the propagation converge toward a caustic. The propagation (as a beam) has no meaning beyond this point.

In reality, the dynamics is more complex, including wave diffraction and dispersion of the group velocity, which act to prevent the singularity [2], [3]. More important, the self-focusing increases the intensity ( $\sim |\mathbf{E}|^2$ ) of the incident field to a level that affects the gaseous medium: ionization of the neutral atoms generates a plasma which in turn has an effect of defocalization. The process toward singularity is stopped and a balance between the two opposite tendencies become possible. A regime of quasi-equilibrium is established with the structure of the beam in the transversal plane becoming inhomogeneous: symmetrically centered on the axis there is a channel of high beam intensity, having a diameter of the order of  $100 \mu$ , surrounded by a much larger (diameter of the order of several millimeters) cylindrical region where the intensity is much lower. In this latter region most of the energy is located. This structure is a filament. The



fluctuation around the equilibrium, for powers in the beam higher than a threshold  $P_{in} > P_{cr}$  allows the propagation on large distances, that can reach kilometers.

For powers that are a large factor (of the order of tens of units) greater than the critical power  $P_{cr}$  the picture changes. Essentially the symmetrical structure of the propagation, mentioned above, becomes azimuthally unstable [4]. The azimuthal perturbations evolve under weak mutual interaction and become sources of local filamentation, which dispose of sufficient power ( $> P_{cr}$ ) to propagate individually. In this multi-filament structure the self-focalization followed by multiple ionization and defocalization produced by the plasma take place for each filament. The interaction leads to the random nucleation of filaments [5], then self-focusing followed by defocusing, everything leading to a random alternation of filaments with short time of existence in a field of small amplitude where the zones of plasma are dynamically redistributed in space [6], [7]. This regime has been called *optical turbulence* [8]. From measurements and numerical simulations it results that the plane transversal to the beam direction is organized in randomly meandering connected stripes (channels) where the field has higher intensity (in this region there are the filaments too) alternating and limited by, - similarly connected regions of plasma. This picture is clearly seen in the Figures of the work by Ettoumi et al. [9].

## 2. Hypothesis

The qualitative aspect of the pictures suggests the following association:

*the shape and the dynamics of the meandering structures (connected ramifications of two types) in the transversal plane to the direction of propagation are identical with structures that result from the reaction-diffusion dynamics of 2D media characterized by the competition of two components: activator and inhibitor.*

This suggests to examine the possible common nature of the dynamics associated with

- optical turbulence of the regime of multiple filamentation
- the activator-inhibitor competition in nonlinear media

The simple qualitative comparison of the image of the transversal plane, and respectively of the two-dimensional domain of a system activator-inhibitor suggests that they may have a common nature (compare Figs. 1 and 2). Indeed, the activator-inhibitor is universal and there would be no surprise to be found in particular circumstances, as optical turbulence. The essential content of the two phenomena is common: it is a competition between two components, with one having auto-catalytic development and the other acting to limit and eventually to suppress this effect.

## 3. The equation of the envelope

The propagation of the laser beam exhibits various regimes. For a power greater than the critical threshold  $P_{in} > P_{cr}$  the filament (central channel and the surrounding energy bath) self-focuses up to the limit that activates the opposing reaction of ionization followed by energization of the electrons: multiphoton ionization and inverse brehmstrahlung energy transfer. At much higher powers the same regime only persists for a finite time (or length of propagation) followed by multi-filamentation and the random dynamics produced by the modulational instability [2], [3].

The electric field is represented in a multiple space-time analysis by separating the slow evolution of the envelope  $A(x, y, z, t)$ , as  $E = A(x, y, z, t) \chi(t)$  where  $\chi(t) = \exp(-i\omega_0 t)$  is the fast wave factor. The nonlinear polarization of the air, together with processes of interaction

with the plasma created by ionization, lead to the equation

$$\begin{aligned} \frac{\partial A}{\partial z} = & \frac{i}{2k_0} \Delta_{\perp} A - \frac{i}{2} \left( \frac{d^2 k}{d\omega^2} \right) \frac{\partial^2 A}{\partial t^2} + i\omega n_2 |A|^2 A \\ & - \frac{\sigma}{2} \rho A - \frac{i}{2} \sigma \omega \tau \rho A - \frac{\beta^{(K)}}{2\sqrt{K}} |A|^{2K-2} A \end{aligned}$$

where  $k_0$  is the central wavenumber of the beam and the terms represent: the diffraction, the group-velocity dispersion (GDV), the Kerr nonlinearity of the polarization, the transfer of energy from the beam to the electrons of the plasma ( $\sigma$  is the cross section of the inverse brehmstrahlung effect) the rate of generation of the electrons ( $\tau$  is the inverse of the collision frequency) and the multi-photon ionization. A separate equation, for the density of electrons  $\rho$  is explained below. The equation above, which is a modified Non-linear Schrodinger equation, is integrated numerically in various regimes of beam power. For  $P_{in} \gg P_{cr}$  the result shows that from a rather homogeneous transversal state it occurs along propagation the nucleation of filaments. The first to be lost is the azimuthal symmetry followed by the quasi-independent evolution of the perturbations to definite filaments, by concentration of the photon energy from the surrounding medium. The self-focalization leads to episodic extinction and further re-nucleation of filaments [10]. This is the regime of *optical turbulence*. The equilibrium becomes a dynamical state placed at marginal stability. There is a competition between two opposite tendencies, and this is manifested as random fluctuations in close proximity of a statistical equilibrium. For part of the propagation, the fluctuations of the spatial distributions of the two fields  $A$  and  $\rho$  have correlations that do not exhibit any intrinsic scale, a situation that is characteristic to criticality. At longer distances from the source, a sharp transition occur and the transversal structure is broken into clusters of finite size [9].

#### 4. The common analytical structure of the optical turbulence and of the activator-inhibitor dynamics

The transition from quasi-homogeneity in the transversal plane to a structure consisting of connected stripes where the fluence is high separated by similar branched channels of low fluence is similar to the fingering instability in a reaction-diffusion system. In the latter case the interface separates two distinct, competing, phases. In the high fluence region there is nucleation of filaments. The analogous effect in the activator-inhibitor system is the formation of “spotty-spiky” solutions [11].

The evolution of the envelope amplitude  $A$  is associated with that of the density  $\rho$  of the electrons of the plasma generated by ionization at focalization

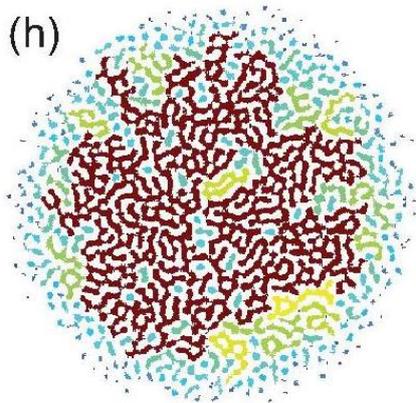
$$\frac{\partial \rho}{\partial t} = D_{\rho} \Delta_{\perp} \rho + \frac{\beta^{(K)}}{K \hbar \omega_0} |A|^{2K} \left( 1 - \frac{\rho}{\rho_{at}} \right)$$

Besides the last term that describes multi-photon ionization, we have introduced a new term, absent in the standard treatments, of diffusion of the electron density. We simplify the writing of the two equation (introducing coefficients  $a$ ,  $b$ ,  $\zeta$  and  $\xi$ )

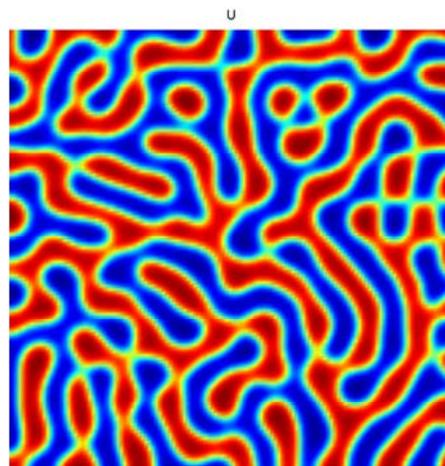
$$\begin{aligned} \frac{2k_0}{i} \frac{\partial \psi}{\partial z} &= \Delta_{\perp} \psi - a |\psi|^2 \psi + b \rho \psi \\ \frac{\partial \rho}{\partial t} &= D_{\rho} \Delta_{\perp} \rho - \zeta \rho |\psi|^{2K} + \xi |\psi|^{2K} \end{aligned}$$

We first note that in the first equation above the term in the left and the first term in the right side, if alone, would give a set of multiple (Gaussian-like) bumps, disposed periodically on a

line in the transversal plane. This may actually be seen as a local limit of a circular contour where spots of high intensity (core of filaments) exists (as confirmed by experiments [12]). This is very similar to what is found for activator-inhibitor systems, where the spot solutions are also periodic [13].



**Figure 1.** Structure of clusters in the transversal plane. This is subfigure (h) of Fig.1 from Ref [9]. (Courtesy W. Ettoumi)



**Figure 2.** Labyrinth pattern for the Cahn-Hilliard activator-inhibitor system.

We note the similarity between the system of equation from where it results the *optical turbulence* and the analytical structure of the model FitzHugh-Nagumo, which exhibits the *labyrinth instability* [14]

$$\begin{aligned}\frac{\partial u}{\partial t} &= \varepsilon^2 \Delta u + f(u) - v \\ \frac{\partial v}{\partial t} &= \Delta v - \delta \gamma v + \delta u\end{aligned}$$

where  $f(u) = u(1-u)(u-u_\kappa)$  for  $u_\kappa \in (1, 1/2)$ . The regime in which this dynamics consists of competing phases that occupy labyrinthic, mutually excluded, connected channels (like clusters) needs a fast inhibitor ( $v$ ) diffusion.

The analogous behavior of the fields  $(\psi, \rho)$  and respectively  $(u, v)$  can be found to other models of the type *activator-inhibitor*, like Gierer-Meinhardt, Gray-Scott, Cahn-Hilliard [15]. The picture of meandering ramifications in plane is similar and has at the origin the competition between the two physical fields [14], [16].

The activator-inhibitor systems have been shown to have a dynamics that is constructed on a deep level of order, the “shadow system” [17]. The existence of this system is a common property of several reaction-diffusion models and explains why they have similar behavior: labyrinthic interfaces, spot-like solutions, their spatially regular distribution (polygonal), their attraction and coalescence.

## 5. Results that become accessible by the mapping between optical turbulence and the *activator-inhibitor* dynamics

The analogy between the two systems may be useful. This is because the class of *activator-inhibitor* systems has been investigated mathematically and disposes of precise description of its

various solutions. We expect to transfer some of these results to the model of optical turbulence, especially for the regimes where it has been examined experimentally or by numerical simulation

- (i) The solutions of the Gierer-Meinhardt (GM) system can be, for the case of rapid diffusion of the *inhibitor*, localized (“spot-like”) [11]; this corresponds to the filaments observed in the high-fluence region.
- (ii) in some conditions, the GM solutions form groups (clusters) with regular spatial disposition, eventually polygonal. Correspondingly, in a laser beam it has already been found regular spacing of the multiple filaments. It seems supported theoretically by the application of the notion of *Chaplygin gas with anomalous polytropic* [18].
- (iii) the solutions of GM, found to be grouped in clusters, have been proved to be unstable and a number of “spots” disappear after an evolution in time, being replaced by a single central bump solution. This looks to be the analogous case to the coalescence of filaments and re-formation of a single central filament.

A problem raised by this mapping: which of the diffusion-reaction systems that have a behavior of the type *activator-inhibitor* can be identified as the equivalent of the modified NSEq in the regime of multiple-filamentation? The response appears to not be constraining, because at fast inhibitor they have the property of being manifestation of a *shadow system* which means a common type of behavior. However we must confine therefore to those characteristics that are common and can be made to correspond to the multi-filamentation.

A specific property is the proliferation of interfaces caused in the activator-inhibitor system, by the *labyrinth* instability and in the laser field, by the competition between high fluence clustered (branched) spatial regions (where filaments can nucleate as spots) and zones of plasma with defocalization effect which keeps control on the local expansion of the first phase.

An interesting aspect that can result from a comparative investigation performed on the two systems is the effective interaction between filaments. This is based on the connection between the NSEq (in its extended form for beam propagation) and the Complex Ginzburg-Landau (CGL) equation [19]. The exact solution of the CGL equation is a soliton with an oscillating tail. If there would be no interaction then the sum of two such functions would also be a solution too. Replacing this sum in the expression of the energy, it should result a sum of two times the individual energy of a single solution. Or, this is not so, showing that besides the individual energies we have a term of interaction. This depends parametrically on the positions of the centers of the two solitons. When the relative distance between the centers is varied, the supplementary term decreases (if there is attraction) or increases (if there is repulsion). The method is identical with the one used to find the interaction between vortices of the Abelian-Higgs superconduction model [20] at non-self-duality. However the energy of interaction between solitons of the CGL equation is found to be exponentially small, which means that the coalescence of filaments is slow. Or, the mapping to an *activator-inhibitor* system helps to reformulate the problem: indeed there are solutions consisting of several localized bumps with a space distribution which is regularly periodic. This solution is unstable and the final state consists of a central spike [21].

## 6. Conclusion

The filamentation generated during the propagation of the pulse of a high power laser has many regimes and in particular the optical turbulence. It is the formation in the transversal plane of a system of randomly meandering ramifications where the incident field is high, separated, and limited by -, a similar region dominated by defocusing plasma. This structure is dynamical and in addition in the high intensity zone new filaments nucleate. They are transient and end up by coalescing into a single channel of propagation. This regime can be mapped onto the

activator-inhibitor dynamics of a nonlinear reaction-diffusion system. Reformulated in the new framework, some problems of beam propagation can be simpler.

An important objective of further investigation is how is reflected in the activator-inhibitor model the inverse phase transition that suppresses progressively the long range correlations in the beam field, *i.e.* breaks up the large scale clusters.

## Acknowledgments

The authors thank to W. Ettoumi for permitting to reproduce a figure from the paper [9].

This work is partially supported by the Contract PN 09 39 01 01.

## References

- [1] Marburger J 1975 *Progress in Quantum Electronics* **4**, Part 1 35 – 110 ISSN 0079-6727 URL <http://www.sciencedirect.com/science/article/pii/0079672775900038>
- [2] Couairon A and Mysyrowicz A 2007 *Physics Reports* **441** 47 – 189 ISSN 0370-1573 URL <http://www.sciencedirect.com/science/article/pii/S037015730700021X>
- [3] Bergé L 1998 *Physics Reports* **303** 259 – 370 ISSN 0370-1573 URL <http://www.sciencedirect.com/science/article/pii/S0370157397000926>
- [4] Skupin S, Peschel U, Etrich C, Leine L, Michaelis D and Lederer F 2002 *Opt. Lett.* **27** 1812–1814 URL <http://ol.osa.org/abstract.cfm?URI=ol-27-20-1812>
- [5] Kandidov V P, Kosareva O G, Tamarov M P, Brodeur A and Chin S L 1999 *Quantum Electronics* **29**(10) 911–915
- [6] Mlejnek M, Wright E M and Moloney J V 1998 *Opt. Lett.* **23** 382–384 URL <http://ol.osa.org/abstract.cfm?URI=ol-23-5-382>
- [7] Méchain G, Couairon A, Franco M, Prade B and Mysyrowicz A 2004 *Phys. Rev. Lett.* **93**(3) 035003 URL <http://link.aps.org/doi/10.1103/PhysRevLett.93.035003>
- [8] Mlejnek M, Kolesik M, Moloney J V and Wright E M 1999 *Phys. Rev. Lett.* **83**(15) 2938–2941 URL <http://link.aps.org/doi/10.1103/PhysRevLett.83.2938>
- [9] Ettoumi W, Kasparian J and Wolf J P 2015 *Phys. Rev. Lett.* **114**(6) 063903 URL <http://link.aps.org/doi/10.1103/PhysRevLett.114.063903>
- [10] Tzortzakis S, Bergé L, Couairon A, Franco M, Prade B and Mysyrowicz A 2001 *Phys. Rev. Lett.* **86**(24) 5470–5473 URL <http://link.aps.org/doi/10.1103/PhysRevLett.86.5470>
- [11] del Pino M, Kowalczyk M and Wei J 2003 *Ann. I. H. Poincaré* **20** 53–85
- [12] Méchain G, CDAmico, André Y B, Tzortzakis S, Franco M, Prade B, Mysyrowicz A, Couairon A, Salmon E and Sauerbrey R 2005 *Optics Communications* **247** 171 – 180 ISSN 0030-4018 URL <http://www.sciencedirect.com/science/article/pii/S0030401804011812>
- [13] Wei J and Winter M 2008 *Journal of Mathematical Biology* **57** 53–89 ISSN 0303-6812 URL <http://dx.doi.org/10.1007/s00285-007-0146-y>
- [14] Goldstein R E, Muraki D J and Petrich D M 1996 *Phys. Rev. E* **53**(4) 3933–3957 URL <http://link.aps.org/doi/10.1103/PhysRevE.53.3933>
- [15] Desai R C and Kapral R 2009 *Dynamics of self-organized and self-assembled structures* (Cambridge University Press) ISBN 978-0-511-71966-0
- [16] Roland C and Desai R C 1990 *Phys. Rev. B* **42**(10) 6658–6669 URL <http://link.aps.org/doi/10.1103/PhysRevB.42.6658>
- [17] Li F and Ni W M 2009 *Journal of Differential Equations* **247** 1762 – 1776 ISSN 0022-0396 URL <http://www.sciencedirect.com/science/article/pii/S0022039609001806>
- [18] Trubnikov B and Zhdanov S 1987 *Physics Reports* **155** 137 – 230 ISSN 0370-1573 URL <http://www.sciencedirect.com/science/article/pii/0370157387901232>
- [19] Malomed B A 1991 *Phys. Rev. A* **44**(10) 6954–6957 URL <http://link.aps.org/doi/10.1103/PhysRevA.44.6954>
- [20] Jacobs L and Rebbi C 1979 *Phys. Rev. B* **19**(9) 4486–4494 URL <http://link.aps.org/doi/10.1103/PhysRevB.19.4486>
- [21] Alikakos N D and Fusco G 1998 *Archive for Rational Mechanics and Analysis* **141** 1–61 ISSN 0003-9527 URL <http://dx.doi.org/10.1007/s002050050072>