Euclidean Complex Relativistic Mechanics: A New Special Relativity Theory

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Keywords: electromagnetic tensor, Euclidean Request, fine structure of atomic spectrum, human senses, linear spacetime transformation, Lorentz Transformation, Relativistic Doppler Shift, Special relativity, Thomas Precession.

PACS: 02.10.Ud, 02.40.Dr, 03.30.+p, 03.50.De, 03.65.-w, 32.10.Fn

Abstract. Relativity Theory (RT) was fundamental for the development of Quantum Mechanics (QMs). Special Relativity (SR), as is applied until now, cancels the transitive attribute in parallelism, when three observers are related, because Lorentz Boost (LB) is not closed transformation. In this presentation, considering Linear Spacetime Transformation (LSTT), we demand the maintenance of Minkowski Spacetime Interval (S^2). In addition, we demand this LSTT to be closed, so there is no need for axes rotation. The solution is the Vossos Matrix (Λ_{B}) containing real and imaginary numbers. As a result, space becomes complex, but time remains real. Thus, the transitive attribute in parallelism, which is equivalent to the Euclidean Request (ER), is also valid for moving observers. Choosing real spacetime for the unmoved observer (O), all the natural sizes are real, too. Using Vossos Transformation (VT) for moving observers, the four-vectors' zeroth component (such as energy) is real, in contrast with spatial components that are complex, but their norm is real. It is proved that moving (relative to O) human O′ meter length, according to Lorentz Boost (LB). In addition, we find Rotation Matrix Vossos-Lorentz (R_{BL}) that turns natural sizes' complex components to real. We also prove that Speed of Light in Vacuum (c) is invariant, when complex components are used and VT is closed for three sequential observers. After, we find out the connection between two moving (relative to O) observers: $X'' = \Lambda_{\mathrm{LO''(O)}} \Lambda_{\mathrm{LO(O')}} X'$, using Lorentz Matrix ($\Lambda_{\mathrm{L}}$). We applied this theory, finding relations between natural sizes, that are the same as these extracted by Classic Relativity (CR), when two observers are related (i.e. relativistic Doppler shift is the same). But, the results are different, when more than two observers are related. VT of Electromagnetic Tensor ($F^{\mu\nu}$), leads to Complex Electromagnetic Fields (CEMFs) for a moving observer. When the unmoved observer O and a moving observer O′ are related, O measures the same Real Electromagnetic Fields (REMFs) as those are given, using LB, but O′ measure CEMFs with the same formula. Complex Electromagnetic Tensor (F'_{B}) turns to Real Electromagnetic Tensor (F') using the formula $F' = \tilde{R}_{\mathrm{BL}\,O'(O)}\ F'_{\mathrm{B}}\ \tilde{R}_{\mathrm{BL}\,O(O')}$. When there are two moving (relative to O) observers O′ and O′′, their real electromagnetic tensors are related, using the form $F'' = \Lambda_{\mathrm{LO''(O)}} \Lambda_{\mathrm{LO(O')}} F' [\Lambda_{\mathrm{LO''(O)}} \Lambda_{\mathrm{LO(O')}}]^{\mathrm{T}}$. In addition, we prove that

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the relation between two moving (relative to O) observers, when they use Real Coordinates (RCs), causes a real rotation between their frames. The turn is opposite to the turn of Thomas and has a little different measure, when the velocities are small. We apply these to the Uniform Circular Motion (UCM) and to the Hydrogen Atom (H), considering that the Proton (p) is the unmoved observer O and the laboratory observer O΄ has infinitesimal velocity. Using Perturbation Theory (PT) we calculate the position of the fine structure peaks of the atomic spectrum. The result is better not only than this extracted using P. Dirac Theory, but also than that extracted using L. H. Thomas Method.

1. Introduction
SR as is applied until now uses LB. But it is known that LB is not closed transformation. In contrast, Lorentz Transformation (LT) (product of Rotation and LB) is closed [1]. Thus, if three observers O, O΄ and O΄΄ are related, where O΄ has parallel axes not only to O axes, but also to O΄΄ axes, then the axes of observers O and O΄΄ are not parallel. This cancels the transitive attribute in parallelism, when more than two observers are related. This option leads to successful results, such as Thomas Precession (TP) which explains the fine structure of atomic spectra. But this happens only if we take successive observers O΄ and O΄΄ with the Thomas order [2]. If we reverse the order of this sequence, it yields a result with 200% relative error. In this presentation, we prove that there is closed LSTT which maintains the S^2 for every inertial observer. Thus, the transitive attribute in parallelism is valid and the axes rotation that happens in case that RC are used, when more than two observers are related, is only an equivalent phenomenon. The full proofs of the following equations there are at the referred papers in Greek [3,4,5] and in English [6].

2. The Matrix Λ_B
Initially, we find out Λ_B that expresses the LSTT which we postulate to a) be closed, b) be invariant to the rotation and c) maintain the spacetime interval S^2. For simplicity reasons, from now on wherever it is written i, is meant ±i. The result is VT: $X' = \Lambda_{B(\beta)} X$. So, the four-vectors of two whoever inertial observers O and O΄ are related with *Vossos Matrix*:

$$\Lambda_{B(\beta)} = \gamma \begin{bmatrix} 1 & -\beta_x & -\beta_y & -\beta_z \\ -\beta_x & 1 & \mathrm{i}\beta_z & -\mathrm{i}\beta_y \\ -\beta_y & -\mathrm{i}\beta_z & 1 & \mathrm{i}\beta_x \\ -\beta_z & \mathrm{i}\beta_y & -\mathrm{i}\beta_x & 1 \end{bmatrix} \tag{1}$$

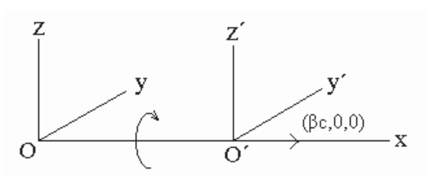

Figure 1. Two coordinate systems: Oxyz and O΄$x΄y΄z΄$ which is moved with velocity (βc, 0, 0) relative to Oxyz.

The typical Vossos Matrix along x-axis (Figure 1) is

$$\Lambda_{B\,\tau\upsilon\pi} = \gamma \begin{bmatrix} 1 & -\beta & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{i}\beta \\ 0 & 0 & -\mathrm{i}\beta & 1 \end{bmatrix} \tag{2}$$

Using Minkowski four-vector, the corresponding typical *Vossos Matrix* in Minkowski space becomes

$$\Lambda^{M}_{B\,\tau\upsilon\pi} = \gamma \begin{bmatrix} 1 & -\mathrm{i}\beta & 0 & 0 \\ \mathrm{i}\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{i}\beta \\ 0 & 0 & -\mathrm{i}\beta & 1 \end{bmatrix} \tag{3}$$

which is a rotation matrix. We observe that the same happens to (c$t΄$, $x΄$) and ($z΄$, $y΄$). We observe that

$$\mathrm{A}_{(\beta)}\,\delta = \left[-\vec{\beta} \times \vec{\delta}\right] = \left[\vec{\delta} \times \vec{\beta}\right] \tag{4}$$

where

$$\beta = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix},\ \delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix},\ \mathrm{A}_{(\beta)} = \begin{bmatrix} 0 & \beta_z & -\beta_y \\ -\beta_z & 0 & \beta_x \\ \beta_y & -\beta_x & 0 \end{bmatrix} \tag{5}$$

Thus

$$\Lambda_{\mathrm{B}(\beta)} = \gamma \begin{bmatrix} 1 & -\beta^T \\ -\beta & \mathrm{I} + \mathrm{i}\,\mathrm{A}_{(\beta)} \end{bmatrix} \tag{6}$$

3. Measure of Time and Length

As Λ_B contains some elements which are imaginary numbers, we conclude that the space-time of one moving observer is complex. Thus, we put an index M or B in complex natural sizes. We have already made the option that O measures real space-time. This simplifies the problem, because the followings are proven: a) Time is real for every observer, b) The unmoved observer measures real velocity, c) Every observer measures square of velocity which is real and positive, d) Every observer measures γ-factor which is real and positive, for particles moved with $\upsilon < c$, e) The unmoved observer measures natural sizes that, can be suitably determined to be real. VT may be written as

$$X' = \Lambda_{\mathrm{B}(\beta)} X \tag{7}$$

Using vectors VT becomes

$$\mathrm{c}t' = \gamma(\mathrm{c}t - \vec{\beta}\cdot\vec{x}),\ \vec{x}'_M = \gamma\left(\vec{x} - \vec{\beta}\,\mathrm{c}t\right) - \mathrm{i}\gamma\vec{\beta}\times\vec{x} \tag{8}$$

$$\Rightarrow \mathrm{c\,d}t' = \gamma\,(\mathrm{c\,d}t - \vec{\beta}\cdot d\vec{x}),\ d\vec{x}'_M = \gamma\left(d\vec{x} - \vec{\beta}cdt\right) - i\gamma\vec{\beta}\times d\vec{x} \tag{9}$$

3.1. Λ_B Properties – Inverse VT

$$\Lambda_{\mathrm{B}(0)} = \mathrm{I} \tag{10}$$

$$\Lambda^{-1}_{\mathrm{B}(\beta)} = \Lambda_{\mathrm{B}(-\beta)} \tag{11}$$

$$\det \Lambda_{\mathrm{B}(\beta)} = 1 \tag{12}$$

So, VT is closed subgroup of proper LT, under the operation

$$\mathrm{g}_1 * \mathrm{g}_2 = (\Lambda_{\mathrm{B}2}\Lambda_{\mathrm{B}1},\ \Lambda_{\mathrm{B}2}\mathrm{b}_1 + \mathrm{b}_2) \tag{13}$$

where $b^{\mu}_{1\mathrm{M}}$ is μ-CC measured by O′, if O measures $x^{\nu} = 0$ and $b^{\mu}_{2\mathrm{M}}$ is μ-CC measured by O′′, if O′ measures $x'^{\nu}_{\mathrm{M}} = 0$ [7].

3.2. Lorentz Boost

Human realize real spacetime, through senses. In contrast, according to VT for a moving observer, time is real, but space is complex. This conflict is solved as following: Let have a rod O′Σ, perpendicular to x-axis moving along it, with velocity (βc, 0, 0) relative to Oxyz (Figure 2). Observer O project point Σ (edge or the rod), perpendicularly to y' and z'-axis. Observer O′ consider as RC the length of the resultant rods O′A και O′B. It emerges that for O′, RCs of point Σ are:

$$y' = \left|\left(\mathrm{O}'\vec{\mathrm{A}}\right)'\right| = y, \quad z' = \left|\left(O'\vec{B}\right)'\right| = z$$

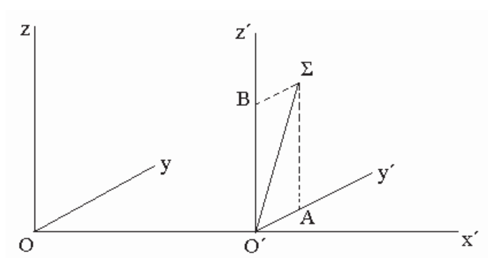

Figure 2. Moving rod O′Σ, perpendicular to x-axis moving along it, with velocity (βc, 0, 0) relative to Oxyz

We observe that using RCs, only the transformation from O to O′, happens according to Proper LT: $X' = \Lambda_{\mathrm{L}\tau\upsilon\pi(\beta)} X$.

4. Electromagnetism, Vossos Precession and Fine Structure of Atomic Spectrums

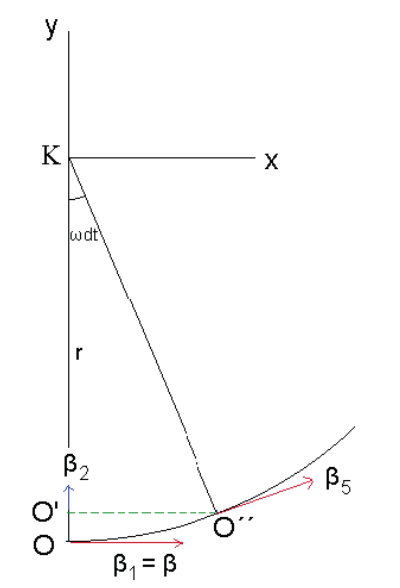

Figure 3. UCM with radius *r*. When t = 0, the particle has velocity β_1 c = β c, while t = d*t*, the particle has velocity β_5 c (relative to O). Inertial observer O′ is moved with velocity β_2 c = α d*t* (relative to O).

We applied VT to electromagnetism and we find out that every classic physical law keep its form. After, we prove that VT is in order with QMs. Because QMs produce real eigenvalues, we work with three frames using RCs (Figure 3). It is proven that O′ measures for O′′ *Vossos Real Angular Velocity*

$$\vec{\omega}'_{\mathrm{B}} = -\frac{\gamma}{(\gamma+1)\mathrm{c}^2}\vec{\upsilon}\times\vec{\alpha} \tag{14}$$

We observe that it emerges angular velocity γ times smaller value than L. H. Thomas Method.

According to QMs electron (e) motion around nucleus has not definite path. But, for a very short time interval the phenomenon may be approached with UCM. So, the above results may be applied, where particle is e and point K is the position of nucleus. In laboratory frame O′, the magnetic energy of an atom with reduced mass *m* is

$$U' = -\left(\frac{g''}{2}\frac{m}{m_e} - \frac{1}{\gamma(\gamma+1)}\right)\frac{e}{m^2c^2}\frac{1}{r}\frac{dV}{dr}\vec{L}\cdot\vec{S}'' \tag{15}$$

In case of H, the energy difference between 2 $^2P_{3/2}$ and 2 $^2P_{1/2}$ is

$$\Delta\overline{U}' = \frac{1}{256}\left(\frac{g''}{2}\frac{m}{m_e} - \frac{1}{\gamma(\gamma+1)}\right)\frac{me^8}{c^2h^4\varepsilon_0{}^4} \tag{16}$$

and is calculated [8]: Δf = 10956.9±9.5 MHz
The corresponding experimental data [9] is

Δf_{exp}= 10969.127±0.095 MHz

The calculation has relative error E_r = - 0.00111(87) ≈ - 0.111% less than P. Dirac and L. H. Thomas Methods.

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