

Wave function of the Universe, preferred reference frame effects and metric signature transition

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Abstract. Gravitational model of non-minimally coupled Brans Dicke (BD) scalar field ϕ with dynamical unit time-like four vector field is used to study flat Robertson Walker (RW) cosmology in the presence of variable cosmological parameter $V(\phi) = \Lambda\phi$. Aim of the paper is to seek cosmological models which exhibit metric signature transition. The problem is studied in both classical and quantum cosmological approach with large values of BD parameter $\omega \gg 1$. Scale factor of RW metric is obtained as $R(t) = 6\sqrt{\frac{3}{\Lambda}} \cosh\left(\frac{t}{4}\sqrt{\frac{\Lambda}{3}}\right)$ which describes nonsingular inflationary universe in Lorentzian signature sector. Euclidean signature sector of our solution describes a re-collapsing universe and is obtained from analytic continuation of the Lorentzian sector by exchanging $t \rightarrow it$ as $R(t) = 6\sqrt{\frac{3}{\Lambda}} \cos\left(\frac{t}{4}\sqrt{\frac{\Lambda}{3}}\right)$. Dynamical vector field together with the BD scalar field are treated as fluid with time dependent barotropic index. They have regular (dark) matter dominance in the Euclidean (Lorentzian) sector. We solved Wheeler De Witt (WD) quantum wave equation of the cosmological system. Assuming a discrete non-zero ADM mass $M_j = 4\sqrt{2}(2j+1)\sqrt{\frac{\Lambda}{3}}$ with $j = 0, 1, 2, \dots$, we obtained solutions of the WD equation as simple harmonic quantum Oscillator eigen functionals described by Hermite polynomials. Absolute values of these eigen functionals have nonzero values on the hypersurface $R = 6\sqrt{\frac{3}{\Lambda}}$ in which metric field has signature degeneracy. Our eigen functionals describe nonzero probability of the space time with Lorentzian (Euclidean) signature for $R > 6\sqrt{\frac{3}{\Lambda}}$ ($R < 6\sqrt{\frac{3}{\Lambda}}$). Maximal probability corresponds to the ground state $j = 0$.

1. Introduction

Lorentz invariance is a fundamental requirement of the standard model of particle physics, verified to high precision by many tests [1], whereas, string theory predicts that we may live in a universe with non-commutative coordinates [2] which leads to a violation of Lorentz invariance [3]. Furthermore, astrophysical observations suggest the presence of high energy cosmic rays above the Greisen-Zatsepin-Kuzmin cutoff [4] which may be explained by a breaking down of Lorentz invariance. More generally, Lorentz invariance violation seems to be related to unknown physical effects of space time at the Planck scale ($l_p = (16\pi G)^{-1/2}$ with $c = \hbar = 1$). A straightforward method of implementing local Lorentz violation in a gravitational setting is to introduce a tensor field with a non-vanishing expectation value, and then couple this tensor to gravity or matter fields. The simplest proposal of this approach is to consider a single time-like vector field N_μ with a fixed norm [5]. Physical interpretation of such a vector field denotes to gravitational effects of preferred reference frames. Regarding general covariance principle, N_μ should be taken as a dynamical field. There have been also elsewhere, cosmological studies in the presence of vector fields without the fixed norm. Lorentz invariance violation directly causes the change of the metric signature of space time. The motivation for the metric signature transition



is usually caused by the extension of real functions to corresponding functions with complex variables via trivial Wick rotations in a complex plane. According to the work presented by Barbero et al [5], the authors attempt was previously to generalize the BD gravity action [6,7] by transforming its background metric as $g_{\mu\nu} \rightarrow g_{\mu\nu} + 2N_\mu N_\nu$ where N_μ is assumed to be a dynamical unit time like vector field. Further, dust, radiation and inflationary cosmological models were obtained.

In this lecture we first present our scalar-vector-tensor gravity [6,7] in the presence of variable cosmological parameter. Then we seek flat RW cosmological models which exhibit metric signature transition from Lorentzian $(-,+,+,+)$ to Euclidean $(+,+,+,+)$ sectors in both classical and quantum approaches. Last section contains the concluding remark.

2. Brans-Dicke scalar-vector-tensor Robertson Walker cosmology

Let us take the scalar-vector-tensor-gravity action $I_{total} = I_{BD} + I_\Lambda + I_N$ [6,7] where the first term is well known BD scalar tensor gravity [8] as $I_{BD} = \frac{1}{16\pi} \int dx^4 \sqrt{g} \left\{ \phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right\}$, the second term denotes the action of self interacting BD field (called as ‘variable cosmological parameter’) such that $I_\Lambda = \frac{\Lambda}{16\pi} \int dx^4 \sqrt{g} \phi^n$, in which Λ, n are real constant parameters and the last term given by $I_N = \frac{1}{16\pi} \int dx^4 \sqrt{g} \left\{ \zeta(x^\nu) (g^{\mu\nu} N_\mu N_\nu + 1) + 2\phi F_{\mu\nu} F^{\mu\nu} - \phi N_\mu N^\nu (2F^{\mu\lambda} \Omega_{\nu\lambda} + F^{\mu\lambda} F_{\nu\lambda} + \Omega^{\mu\lambda} \Omega_{\nu\lambda} - 2R_\mu^\nu + \frac{2\omega}{\phi^2} \nabla_\mu \phi \nabla^\nu \phi) \right\}$ with $F_{\mu\nu} = 2(\nabla_\mu N_\nu - \nabla_\nu N_\mu)$, and $\Omega_{\mu\nu} = 2(\nabla_\mu N_\nu + \nabla_\nu N_\mu)$, describes action of unit time like dynamical four vector field $N_\mu(x^\nu)$. Non-minimally vector field N_μ is called usually as four velocity of preferred reference frame. The action I_{total} is written in units $c = \hbar = 1$ with Lorentzian signature $(-,+,+,+)$. The undetermined Lagrange multiplier $\zeta(x^\nu)$ controls that N_μ to be an unit time-like vector field. Self-interacting coupling constant Λ has dimension as $(length)^{2n-4}$. Absolute value of determinant of the metric $g_{\mu\nu}$ is defined by g . BD field ϕ describes inverse of variable Newton’s gravitational coupling parameter and its dimension is $(length)^{-2}$ in units $c = \hbar = 1$. Present limits of ω based on time-delay experiments [9,10,11] require $\omega \geq 4 \times 10^4$ and general relativistic approach I_{BD} is obtained with $\omega \rightarrow \infty$ [12,13,14]. As an example we set RW line element which with Lorentzian signature $(-,+,+,+)$ is given from point of view of free falling comoving observer as $ds^2 = -dt^2 + R^2(t)\{dx^2 + dy^2 + dz^2\}$ where $R(t)$ is spatially part scale factor of space time. In this case one can obtain explicitly time dependent fields $\zeta(t)$, $\phi(t)$ and $R(t)$ such as follows: $\phi(t) = \phi_0 \exp[8/(2\omega - 5)(\Omega t)]$ and $R(t) = R_0 \cosh(\Omega t)$ (details of the mathematical calculations are given in ref. [15]), where $\phi_0 = \phi(0)$, $R_0 = R(0)$ and we defined $\Omega = \frac{3(2\omega-5)}{4\sqrt{(6\omega-3-2\sqrt{6})(6\omega-3+2\sqrt{6})}} \sqrt{\frac{\Lambda}{3}}$. The parameter Ω has real value under the conditions $\frac{1}{2} - \frac{\sqrt{6}}{3} < \omega < \frac{1}{2} + \frac{\sqrt{6}}{3}$ with $\Lambda < 0$ or $\frac{1}{2} - \frac{\sqrt{6}}{3} > \omega > \frac{1}{2} + \frac{\sqrt{6}}{3}$ with $\Lambda > 0$. Our obtained scale factor describes non-singular inflationary expanding flat universe. With the above obtained solutions the undetermined Lagrange multiplier $\zeta(t)$, fluid density $\rho(t)$ and corresponding pressure $p(t)$ become respectively $\zeta(t) = \left\{ \frac{(6\omega-31-4\sqrt{46})(6\omega-31+4\sqrt{46})}{(2\omega-5)(6\omega-3-2\sqrt{6})(6\omega-3+2\sqrt{6})} \right\} \Lambda \phi_0 \exp(\Omega t)$, $\rho(t) = 3\Omega^2 \tanh^2 \Omega t$, and $p(t) = -\Omega^2 \{2 + \tanh^2 \Omega t\}$ where we set initial condition as $\rho(0) = 0$. One can obtain equation of state as $\eta(t) = \frac{p(t)}{\rho(t)} = -\frac{1}{3} - \frac{2}{3} \coth^2 \Omega t$ which describes dark matter dominant [16] of the fluid from point of view of free falling comoving frame. It is applicable to choose a preferred reference frame with its corresponding time coordinate T related to the cosmological comoving time t as $T(t) = (\Omega t)^{\frac{2}{3}}$. From the point of view of the latter preferred reference frame the RW line element can be rewritten as $ds^2 = R_0^2 d\bar{s}^2$ in which $d\bar{s}^2 = -T dT^2 + \cosh^2(T^{3/2})\{dx^2 + dy^2 + dz^2\}$ and we set $R_0 = \frac{3}{2\Omega}$. We see that hypersurfaces $t = \text{constant}$ with $T > 0$ correspond to Lorentzian signature $(-,+,+,+)$ of the space time metric. Its analytic continuation is obtained with hypersurfaces $it = \text{constant}$ which correspond to Euclidean signature $(+,+,+,+)$ with negative times $T < 0$. Hence metric components have degeneracy on the signature transition

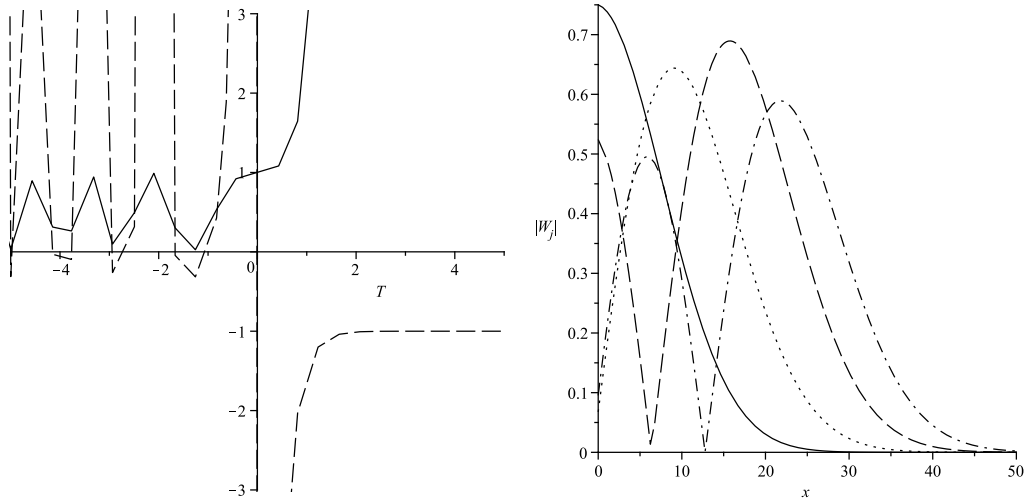


Figure 1. Solid line in left diagram describes Lorentzian (Euclidean) sector scale factor of RW space time for times $T > 0(T < 0)$. Dash line in left diagram describes barotropic index $\eta(T)$ of Lorentzian (Euclidean) sector for times $T > 0(T < 0)$. Right diagram describes variation of absolute value of the WD eigen functionals $|W_j(x)|$ against scale factor x for $j = 0, 1, 2, 3$ with solid, dot, dash and dash-dot lines respectively.

hypersurface $T = 0$ (see LHS in the figure 1). Diagram of the barotropic index $\eta(T)$ is plotted against T in LHS of the figure 1 with dash line. It shows that the scalar-vector matter source treats as regular (dark) matter in the Euclidean (Lorentzian) sector of the space time. In the quantum level we obtain below that the Lorentzian sector of space times may be tunneled quantum mechanically to the corresponding Euclidean sector and vice versa. Using minisuperspace approach of canonical quantum cosmology one can obtain corresponding Wheeler DeWitt probability wave equation of the system as $\left\{ \frac{(6\omega^2 - 81\omega - 27)}{4(2\omega + 3)^2} \frac{\delta^2}{\delta R^2} + \frac{3(7\omega + 12)}{(2\omega + 3)^2} \frac{\phi}{R} \frac{\delta}{\delta R} \frac{\delta}{\delta \phi} - \frac{(2\omega + 21)}{2(2\omega + 3)^2} \frac{\phi^2}{R^2} \frac{\delta^2}{\delta \phi^2} - \phi R(M + \Lambda \phi R^3) \right\} W(R, \phi) = 0$ (details of mathematical calculations are given in ref. [15]). In general relativistic approach $\omega \rightarrow +\infty$ our classical solution leads to $\lim_{\omega \rightarrow +\infty} \phi(t) = \phi_0 = \text{constant}$ and $\lim_{\omega \rightarrow +\infty} R(t) = R_0 \cosh\left(\frac{t}{4}\sqrt{\frac{\Lambda}{3}}\right)$ where $R_0 = \frac{3}{2\Omega} = 6\sqrt{\frac{3}{\Lambda}}$ and $\phi_0 = \frac{1}{16\pi G}$. The above WD wave equation has not a simple solution with small ω , but we can obtain some physical interpretation from solution of its general relativistic approach $\omega \rightarrow +\infty$. In the latter approach the WD wave equation becomes $W''_{\infty} + U(x)W_{\infty}(x) \cong 0$ where we defined $U(x) = -x(a + bx^3)$, $a = \frac{108M}{\pi G \Lambda} \sqrt{\frac{3}{\Lambda}}$, $b = \frac{1944}{\pi^2 G^2 \Lambda^2}$, $x = \frac{R}{R_0}$ and over-prime ' denotes differentiation with respect to the minisuperspace variable x . Subscript of the WD wave solution W_{∞} denotes to $\omega \rightarrow +\infty$. We point that R_0 is scale factor of space time where the metric has signature degeneracy. Solving $U'(x) = 0$ we obtain minimum point of the potential $U(x)$ as $x_m = -\frac{1}{2} \left(\frac{\pi G M}{3} \sqrt{\frac{\Lambda}{3}} \right)^{1/3}$ for which corresponding Taylor series expansion becomes $U(x) \cong \frac{81}{2} \left(\frac{M}{\Lambda \sqrt{\pi G}} \right)^{4/3} - \frac{1944}{\Lambda} \left(\frac{M}{\pi^2 G^2 \Lambda} \right)^{2/3} (x - x_m)^2 + \dots$. Defining $x - x_m = \sigma y$ where $\sigma = \left(\frac{\Lambda}{1944} \right)^{1/4} \left(\frac{\pi^2 G^2 \Lambda}{M} \right)^{1/6}$ the WD wave equation becomes $W''(y) + (\varepsilon - y^2)W(y) = 0$ where we eliminated subscript ∞ and defined $\varepsilon = \frac{M}{4\sqrt{2}} \sqrt{\frac{3}{\Lambda}}$. This equation describes simple harmonic quantum Oscillator and its solutions are described in terms of Hermite polynomials $H_j(y) = (-1)^j e^{y^2} \frac{d^j}{dy^j} e^{-y^2}$ as

$W_j(y) = \left(\frac{1}{2^j j! \sqrt{\pi}}\right)^{\frac{1}{2}} \exp(-\frac{y^2}{2}) H_j(y)$ with eigen values $\varepsilon_j = (2j + 1)$, $j = 0, 1, 2, \dots$. In terms x our obtained WD eigen functions become $W_j(x) = \left(\frac{1}{2^j j! \sqrt{\pi}}\right)^{\frac{1}{2}} \exp\{-\frac{\sigma_j^2(x-x_m^j)^2}{2}\} H_j(\sigma_j(x-x_m^j))$ in which $x_m^j = -\left(\frac{\pi\sqrt{2}(2j+1)\Lambda G}{18}\right)^{\frac{1}{3}}$, $\sigma_j = \left(\frac{1}{1944}\right)^{\frac{1}{4}} \left(\frac{\pi^2 \Lambda^2 G^2}{4(2j+1)} \sqrt{\frac{3}{2}}\right)^{\frac{1}{6}}$ and discrete ADM mass becomes $M_j = 4\sqrt{2}(2j+1)\sqrt{\frac{\Lambda}{3}}$. Their superposition leads to general wave solution of quantum universe of our model as $W(x) = \sum_{j=0}^{\infty} P_j W_j(x)$ where P_j is probability amplitude of the universe which should stay on the eigen state j with ADM eigen mass (energy) M_j . However our knowledge about the universe is not enough and so we can not determine the coefficients P_j , because LHS in the above summation ' $W(x)$ ' is unknown. But we can obtain some physical statements about the quantum tunneling of metric signature via eigen states $W_j(x)$: Our classical solution $x(T) = \frac{R}{R_0} = \cosh(T^{3/2})$ predicts a metric signature transition on the hypersurface $T = 0$ corresponding to $x = 1$. Lorentzian sector of space time corresponds to $x > 1$ but its Euclidean sector corresponds to $0 < x < 1$ (see LHS of the figure 1). The metric equation has signature degeneracy on the hypersurface $x = 1$ corresponding to the time $T = 0$ in the classical regime. $W_j(0 < x < 1)$ ($W_j(x > 1)$) describe the probability of Euclidean (Lorentzian) signature of the space time. Nonzero values $W_j(x = 1) \neq 0$ describe the probability of metric signature quantum tunneling which takes maximal value when the universe exhibits its ground state $j = 0$ (see RHS of the figure 1 where we set $\Lambda G \pi = 1$).

3. Summary and concluding remarks

Flat RW space time is studied by using the BD scalar-vector tensor gravity and variable cosmological parameter. A nonsingular inflationary cosmological model is obtained which exhibits metric signature transition from Lorentzian to Euclidean topology. Unit time like dynamical vector field and BD scalar field are treated as regular (dark) fluid in Euclidean (Lorentzian) sector of flat RW space time with time dependent barotropic index. WD wave equation of the cosmological system is solved by assuming nonzero ADM mass. Its solutions are obtained as simple harmonic quantum Oscillator eigen functionals. Maximal probability of signature quantum tunneling is obtained when the quantum Oscillating RW universe lives in its ground state. As future work the author intends to investigate the dynamical effects of preferred reference frames on black holes and anisotropic-inhomogeneous Bianchi cosmological metrics .

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