# On the determination of source spectrum of ultra high energy cosmic rays

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**Abstract.** The calculations of source spectra of extragalactic cosmic rays with ultra high energies are made based on the solution of inverse problem for a system of transport equations that describes the propagation of protons and nuclei with energies $10^{18}$ to $10^{21}$ eV in the expanding Universe filled with the background radiation. It is assumed that protons and Iron nuclei are the most abundant species in the sources. The data from the Auger experiment is used to illustrate the method.

## 1. Introduction

The origin of cosmic rays with energies $E > 10^{18}$ eV observed at the Earth is associated with the most active extragalactic objects - active galactic nuclei, gamma-ray burst progenitors, newborn rapidly rotating pulsars, interactive galaxies etc., see review [1]. The present knowledge about the highest energy cosmic rays was mainly acquired from the High Resolution Fly's Eye Experiment (HiRes), Pierre Auger Observatory (Auger), Telescope Array experiment (TA), and from the Yakutsk EAS array. The transport of cosmic ray nuclei in the intergalactic medium is accompanied by the transformation of energy spectrum and elemental composition due to interactions with background electromagnetic radiation that fills the expanding Universe.

The solution of direct problem when the calculations follow cosmic ray propagation from the source to the observer is commonly used for the determination of source spectrum and composition, e. g. [2]. A power law source spectrum with some maximum energy is usually assumed. The source characteristics are determined by the trial-and-error method when the calculations can be repeated for the refined source spectrum after comparison with observations. In the present work, we inverse the procedure and determine the source properties starting from the spectrum observed at the Earth without ad hoc assumptions about the shape of source spectrum. A simple case of the source composition that includes only protons and Iron nuclei is considered and the analytical approximation of the data from the Auger experiment is used.

## 2. Calculations by the inverse problem method

We use the following transport equation for cosmic ray protons and nuclei interacting with the background electromagnetic radiation in the expanding Universe (see [3] for detail):

$$-H(z)(1+z)\frac{\partial}{\partial z}\left(\frac{F(A,\varepsilon,z)}{(1+z)^3}\right) -$$

$$-\frac{\partial}{\partial \varepsilon}\left(\varepsilon\left(\frac{H(z)}{(1+z)^3}+\frac{1}{\tau(A,\varepsilon,z)}\right)F(A,\varepsilon,z)\right)+\nu(A,\varepsilon,z)F(A,\varepsilon,z)$$
$$=\sum_{i=1,2...}\nu(A+i\rightarrow A,\varepsilon,z)F(A+i,\varepsilon,z)+q(A,\varepsilon)(1+z)^m. \quad (1)$$

The system of eqs.(1) for all kinds of nuclei with different mass numbers $A$ from Iron to Hydrogen should be solved simultaneously. The energy per nucleon $\varepsilon = E/A$ is used here because it is approximately conserved in a process of nuclear photodisintegration, $F(A,\varepsilon,z)$ is the corresponding cosmic-ray distribution function, $z$ is the redshift, $q(A,\varepsilon)$ is the density of cosmic-ray sources at the present epoch $z = 0$, $m$ characterizes the source evolution (the evolution is absent for $m = 0$), $\tau(A,\varepsilon,z)$ is the characteristic time of energy loss by the production of $e^-e^+$ pairs and pions, $\nu(A,\varepsilon,z)$ is the frequency of nuclear photodisintegration, the sum in the right side of eq. (1) describes the contribution of secondary nuclei produced by the photodisintegration of heavier nuclei, $H(z) = H_0((1+z)^3\Omega_m + \Omega_\Lambda)^{1/2}$ is the Hubble parameter in a flat universe with the matter density $\Omega_m(= 0.3)$ and the $\Lambda$-term $\Omega_\Lambda(= 0.7)$.

The solution of eqs. (1) at the Earth can be presented as

$$F(A,\varepsilon,z=0)=\sum_{A'}\int d\varepsilon' G(A,\varepsilon;A',\varepsilon')q(A',\varepsilon'), \quad (2)$$

where the Green function $G$ is introduced.

The spectrum on the total energy $E$ at the Earth is given by the equation

$$N(A,E,z=0)=A^{-1}\sum_{A'}\int d\varepsilon' G(A,E/A;A',\varepsilon')q(A',\varepsilon'), \quad (3)$$

In a process of numerical solution of the inverse problem, eqs. (2,3) are presented in a matrix form and inverted to find the source functions $q(A,\varepsilon)$ for different types of nuclei. This procedure is described at length in [4]. The numerical solution of cosmic-ray transport equations follows the finite differences method. The variables are $z$ and $\log(E/A)$.

The all-particle energy spectrum of cosmic rays $J(E)$ obtained from Auger observations [5] is used in our calculations. The following analytical approximation of the spectrum is accepted (see also Figure 1):

$$J(E) \propto E^{-3.23}, E < 5\times 10^{18}\mathrm{eV};$$
$$J(E) \propto E^{-2.63}\times[1+exp(log(E/10^{19.63}\mathrm{eV})/0.15)]^{-1}\times$$
$$exp(-(E/(1.5\times 10^{20}eV))^4), E > 5\times 10^{18}\mathrm{eV}. \quad (4)$$

This formula is similar to the equation suggested by the Auger team but contains $exp(-(E/1.5\times 10^{20}eV)^4)$ factor of cosmic ray flux suppression at energies $\geq 1.5\times 10^{20}$ eV.

Using the Auger data on energy dependence of the mean logarithm of the atomic mass number $\langle lnA\rangle$ calculated in the EPOS-LHC model of particle interactions in the atmosphere [5], we accept the following approximation

$$\langle lnA\rangle = 0.5 + 4.2\times(E/10^{20}\mathrm{eV})^{0.6}. \quad (5)$$

The calculated source spectra of extragalactic sources that reproduce the all-particle cosmic ray spectrum observed at the Earth (4) are illustrated in Figures 2, 4.

Figure 2 shows the spectra calculated on the assumption that accelerated protons and Iron nuclei have similar spectra on magnetic rigidity $E/Z$. Different curves correspond to the different

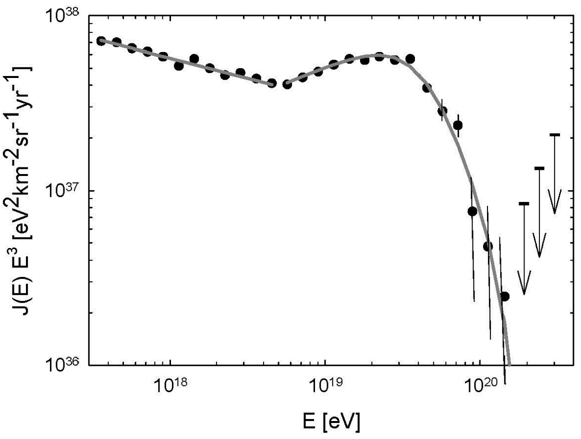


**Figure 1.** Analytical approximation (4) used in the present calculations to describe the Auger data are shown by solid line together with Auger data [5].

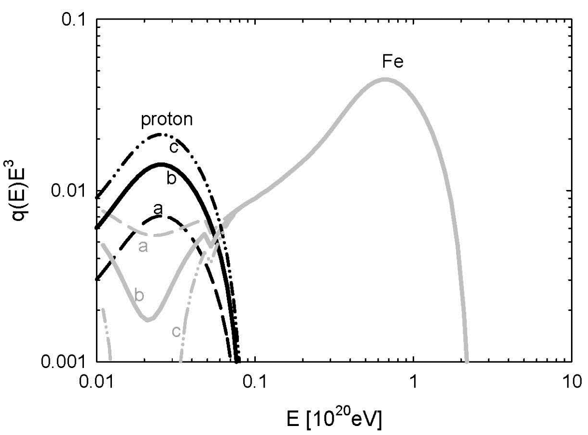


**Figure 2.** Calculated source spectra in arbitrary units based on the Auger data for different Iron-to-proton source ratios labelled (a) for $S_{Fe}/S_p = 2 \times 10^{-2}$, (b) for $S_{Fe}/S_p = 10^{-2}$, and (c) for $S_{Fe}/S_p = 6.7 \times 10^{-3}$. Black lines for proton source; gray lines for Iron source. Proton and Iron source spectra are assumed to have the same dependence on magnetic rigidity.

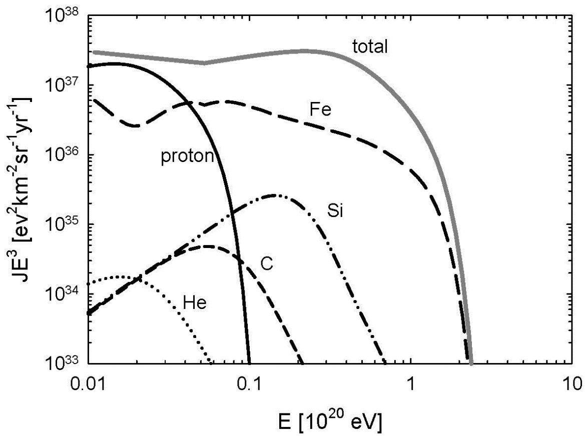


**Figure 3.** Calculated spectra of different types of nuclei for Iron-to-proton source ratio $S_{Fe}/S_p = 10^{-2}$ and the total observed Auger cosmic ray spectrum at the Earth.

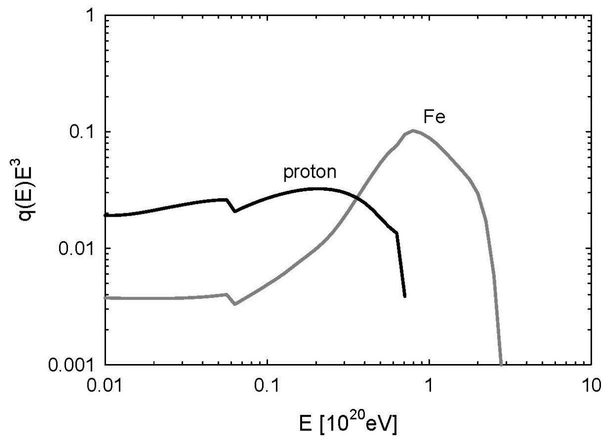


**Figure 4.** Calculated source spectra of protons and Iron based on Auger data on cosmic ray spectrum and $\langle lnA \rangle$ function.

assumptions on the proton and Iron abundance at the source. The calculated maximum energy which particles reach in a process of acceleration is approximately equal to $4 \times 10^{18} Z$ eV. The obtained shape of the source spectrum differs from the power-law. The calculated elemental composition of cosmic rays at the Earth for the case $S_{Fe}/S_p = 10^{-2}$ is presented in Figure 3.

Figure 4 present the proton and Iron source spectra calculated without assumption that they are similar functions of magnetic rigidity. Instead, the observed dependence of $\langle lnA \rangle$ on energy (5) is imposed. It is evident that the source energy spectra have different dependence on rigidity in this case and the Iron spectrum is not of the power-law form.

The results of our research and conclusions are presented in more detail in [4].

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