2–D Modelling of Long Period Variations of Galactic Cosmic Ray Intensity

M Siluszyk[1], **K Iskra**[1] **and M Alania**[1,2]

[1] Institute of Mathematics and Physics, Siedlce University, Poland
[2] Ivan Javakhishvili Tbilisi State University, Nodia Institute of Geophysics, Tbilisi, Georgia

E–mail: marek.siluszyk@uph.edu.pl

Abstract.
A new two–dimensional (2–D) time dependent model describing long-period variations of the Galactic Cosmic Ray (GCR) intensity has been developed. New approximations for the changes of the magnitude B of the Interplanetary Magnetic Field (IMF), the tilt angle δ of the Heliospheric Neutral Sheet (HNS) and drift effects of the GCR particles have been included into the model. Moreover, temporal changes of the exponent γ expressing the power law – rigidity dependence of the amplitudes of the 11–year variation of the GCR intensity have been added. We show that changes of the expected GCR particle density precedes changes of the GCR intensity measured by the Moscow Neutron (MN) monitor by about 18 months. So ~18 months can be taken as an effective delay time between the expected intensity caused by the combined influence of the changes of the parameters implemented in the time-dependent 2–D model and the GCR intensity measured by neutron monitors during the 21 cycle of solar activity.

1. Introduction and Motivation

Generally, to model a propagation of the GCR in the heliosphere is rather complicated problem. Difficulties are related to an accurate implementation of the temporal changes of parameters (among them obtained from direct measurements) into the transport equation that determines fundamental processes in the heliosphere and causes a modulation of GCR. An additional difficulty is related with the selection of the length of the modelling time interval, due to the existence of a varying delay time τ between the changes of the GCR intensity, on the one hand, and various parameters characterizing electro-magnetic conditions in the heliosphere, on the other. We present in figure 1 the temporal changes of the monthly smoothed sunspot number SSN (top panel), the GCR intensity *I(CR)* observed by the Moscow neutron monitor (middle panel), and the exponent γ of the rigidity spectrum of the GCR intensity variations [1] for the period of 1968–2012 (bottom panel). In table 1 we present the maximal invers correlation coefficients corresponding to the delay times τ for the pair of *I(CR)* and SSN and for the pair of *I(CR)* and the exponent γ. The *SSN* is a relative index describing the level of solar activity, but it is not in a straight line usable as a quantitative parameter to implement it in the transport equation of the GCR propagation. However, bearing in mind that the magnitude B of the IMF is proportional to the SSN and an inverse relationship between I(CR) and B (“CR-B”) is valid everywhere inside the termination shock [2], one can make use of correlation between the parameter SSN and I(CR) as an indication of the GCR modulation.
For the positive (A>0) polarity period 1968-1976, delay time τ is 2 months, and for the period 1990-2002 that is zero. Hence, there is a clear polarity dependence of delay times τ for various 11–year cycles of solar activity. Delay time τ between SSN and I(CR) for periods 1977–1987 (A>0 (4 years) and A<0 (7 years)) is 10 months and 14 months for the negative (A<0) polarity periods 2003-2012. However, we think that this problem needs a more careful study. Delay times τ between I(CR) and exponent γ for all 11–year cycles of solar activity is near zero; that was expected, as far γ expresses the rigidity dependence of the amplitudes of the long-period variations found directly from neutron monitors data for given period of consideration. Thus, it must be noted that a caution about an existence of difficulties related to the selection of a length of the modelling time interval because of dynamical changes of delay time τ, e.g., between SSN and I(CR) is not

groundless; to model the GCR propagation for any time interval needs a careful preliminary study of the dynamical changes of delay time τ for the chosen time interval, and what is an extremely important, in model should be implemented different parameters with corresponding different delay time τ.

Table 1. Correlation coefficients r and delay time τ (in months) between the parameter SSN and I(CR) and between the changes of I(CR) and parameter γ for period 1968 to 2012.

No.	Periods	r(SSN ; I(CR))	$\tau(I(CR), SSN)$	r(γ ; I(CR))	$\tau(I(CR), \gamma)$
I	1968-1976	-0.87 ± 0.01	2 ± 1	-0.94 ± 0.01	1 ± 1
II	1976-1987	-0.87 ± 0.01	10 ± 1	-0.71 ± 0.01	0 ± 1
III	1990-2002	-0.93 ± 0.01	0 ± 1	-0.76 ± 0.01	0 ± 1
IV	2003-2012	-0.88 ± 0.01	14 ± 1	-0.58 ± 0.02	1 ± 1

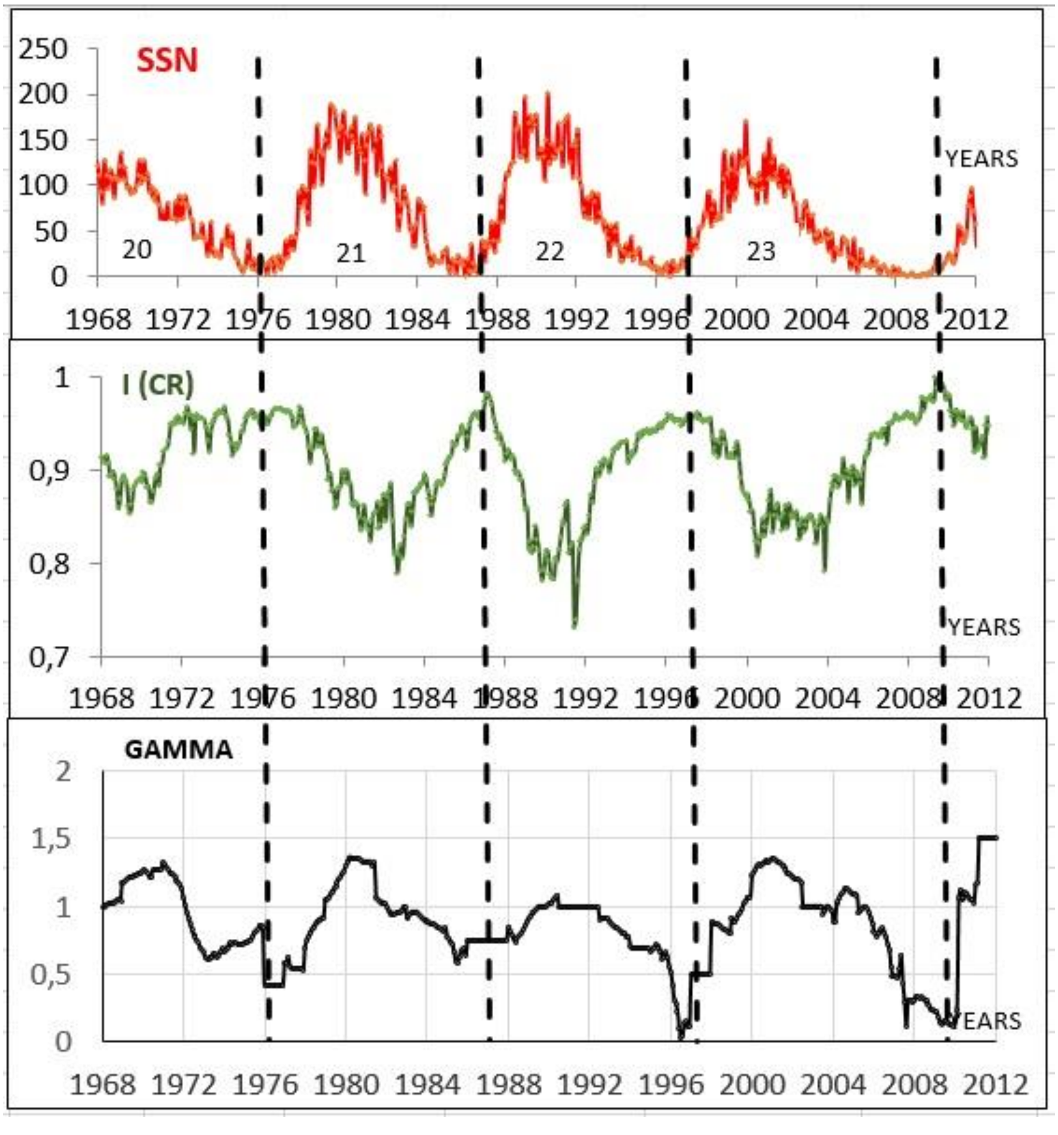

Figure 1. Temporal changes of the monthly smoothed *SSN* – sunspot number (top panel), I(CR) intensity observed by the Moscow NM (middle panel) and rigidity exponent $\gamma(t)$ of the GCR intensity variations (bottom panel) for the period of 1968–2012.

Our aim in this paper is: (1) to construct time dependent 2–D model of the 11–year variations of the GCR intensity and algorithm solved by difference scheme method in C# programming language, (2) to consider 11–year variation of the GCR intensity during the period 1976–1987 by the monthly smoothed data of the Moscow NM, in such way there are excluded fluctuations of the GCR intensity shorter than one month, (3) to implement in the Parker's transport equation the parameters characterizing temporal changes (for the case of constant solar wind velocity *U=400 km/s*) of the magnitude *B* of the IMF, tilt angle δ of the HNS and changes of drift effect of the GCR particles. We assume that drift effects have a maximum value normalized to 100% in the minimum epoch of solar activity (drift dominated epoch), and 20% in the maximum epoch (almost diffusion dominated period), and (4) we also implement in the model changes of the exponent γ (for the first time) of the rigidity *R* spectrum of the long-period variations of the GCR intensity. The importance of the latter is demonstrated in [3], [4] and [5], where it is shown that in the case of almost constant solar wind velocity, a central role in the formation of the 11–year variation of the GCR intensity can be ascribed to the changes of the diffusion coefficient versus the solar activity. So, changes of the character of the rigidity dependent diffusion remain an essential source of the 11–year variation of the GCR intensity, playing a vital role in the formation of the rigidity dependence of the amplitudes of the GCR intensity variations.

2. **Model of** the **11–year variation of GCR: 1976 – 1987.**

For a modelling we take the period of 1976–1987 (11–year cycle #21). Cycle #21 was chosen because of, (1) almost a symmetric changes of the monthly smooting of the GCR intensity were observed for ascending (A>0) and descending (A<0) epochs of the 11–year cycle of solar activity, and (2) during the cycle #21 the largest number (~7 events) of clearly expressed step like changes of the GCR intensity were observed comparing to any other 11–year cycle during last 50 years. These step like changes of the GCR intensity can be related to the Forbush decreases (Fds) [5]. However the roles of Merged Interaction Regions (MIRs) and Global Merged Interaction Regions (GMIRs) [6] as barriers for the propagation of the GCR particles in the heliosphere cannot be excluded [6] and [7]. Moreover in cycle #21 a pure inverse correlation between the temporal changes of the rigidity *R* spectrum exponent γ and of the exponent ν_y of the power law spectral density of the B_y component of the IMF turbulence is observed. Our calculations [3] show that the turbulence of the IMF have a Gaussian distribution and the GCR particles propagation can be considered as normal diffusion for whole 11-year cycle #21; of course we accept that delay times τ between changes of the GCR intensity, on the one hand, and various parameters characterizing electro-magnetic conditions in the heliosphere remain constant during the solar cycle #21, on the other.

To model the 11–year variations of GCRs we use Parker's non–stationary transport equation [8] and [9]

$$\frac{\partial N}{\partial \tau} = -\left(U + \langle v_d \rangle\right) \cdot \nabla N + \nabla \cdot \left(K_{ij}^{S} \cdot \nabla N\right) + \frac{1}{3}(\nabla \cdot U)\frac{\partial N}{\partial \ln R} \tag{1}$$

where *N* and *R* are the omnidirectional distribution function and rigidity of the GCR particles, respectively; τ –the time, U –the solar wind velocity and v_d is the drift velocity. We set up the dimensionless density $f = \frac{N}{N_0}$, the time $t = \frac{\tau - \tau_0}{\tau_S - \tau_0}$ and the distance $r = \frac{\rho}{\rho_0}$; N_0 is the density in the Local Interstellar Medium (LISM) accepted as $N_0 = 4\pi I_0$, where the intensity I_0 in the LISM has the form: $I_0 = 21.1T^{-2.8} / \left(1 + 5.85T^{-1.22} + 1.18T^{-2.54}\right)$ in [10], [11]; T is kinetic energy in GeV ($T = \sqrt{R^2 \cdot e^2 + 0.938[GeV]^2} - 0.938[GeV]$, e is an elementary electric charges, ρ and ρ_0 are the radial distance and size of the modulation region; τ_0 – is the characteristic time corresponding to the changes in the heliosphere for the certain class of the GCR variation; we considered in model $\tau_0 = 1976$ and $\tau_S = 1987$.

The size of the modulation region ρ_0 equals 100AU, and the solar wind velocity $U = 400 km \cdot s^{-1}$ is used throughout the heliosphere. The equation (1) for the dimensionless variables *f*, *R* and *t* in the 2–D spherical coordinate system (r, θ) can be written, as:

$$A_0 \frac{\partial f}{\partial t} = A_1 \frac{\partial^2 f}{\partial r^2} + A_2 \frac{\partial^2 f}{\partial \theta^2} + A_3 \frac{\partial^2 f}{\partial r \partial \theta} + A_4 \frac{\partial f}{\partial r} + A_5 \frac{\partial f}{\partial \theta} + A_6 f + A_7 \frac{\partial f}{\partial R} \quad (2)$$

where $A_0 = \frac{\rho_0^2}{\tau_0 K_{II}}$ and the coefficients $A_1, A_2, ..., A_7$ are functions of the spherical coordinates (r, θ), the rigidity *R* of the GCR particles and the time *t*. The anisotropic diffusion tensor of GCR $K_{ij} = K_{ij}^{(S)} + K_{ij}^{(A)}$ consists of the symmetric $K_{ij}^{(S)}$ and $K_{ij}^{(A)}$ – antisymmetric parts. We implement a drift velocity of the GCR particles in the model as, $< v_{D,i} > = \frac{\partial K_{ij}^{(A)}}{\partial x_j}$ [12]. This expression is equivalent to the standard formula for $< v_D >$ [13]. The heliospheric magnetic field vector $\vec{B}$ is given as in [14] and [15]:

$$\vec{B} = \left(1 - 2H(\theta - \theta')\right)\left(B_r \vec{e}_r\right) \quad (3)$$

where *H* is the Heaviside step function changing the sign of the global magnetic field in each hemisphere θ' corresponds to the heliolatitudinal position of the HNS and e_r is the unite vector directed along the component B_r of the 2D Parker's field [16]. Parker's spiral heliospheric magnetic field is implemented through the angle $\psi = \arctan\left(\Omega r \sin\theta \cdot U^{-1}\right)$ in the anisotropic diffusion tensor for the GCR particles where ψ is the angle between magnetic field lines and radial direction in the equatorial plane.

As an ad hoc assumption, a quasi linear theory (QLT) [21] formally arising from hard-sphere scattering (or the billiard ball diffusion) is considered as a more reasonable and simple tool for describing a propagation of GCR in heliosphere. The QLT is not generally the best approximation [17], [18], [19] and [20] for describing a propagation of large energy range of GCR particles, but it works well in the energy range to which neutron monitors and muon telescopes respond (for rigidities R>(10–15)GV) [22]. So, as an ad hoc assumption we employ the ratios of the perpendicular $K_\perp$ and drift K_d diffusion coefficients to the parallel $K_\parallel$ diffusion coefficient $\beta = \frac{K_\perp}{K_{II}}$ and $\beta_1 = \frac{K_d}{K_{II}}$ for the cosmic ray particles of rigidities R≥ 10GV, as follows,

$$\beta = \frac{1}{1 + \omega^2 \tau_1^{\,2}}, \qquad \beta_1 = \frac{\omega \tau_1}{1 + \omega^2 \tau_1^{\,2}}, \quad (4)$$

where for rigidities R= 10GV is accepted that $\omega\tau_1$=3, i.e. $\beta = \frac{K_\perp}{K_{II}} = 0.1$ at the earth orbit. Then, changes of $\omega\tau_1$ is determined by the Parker's spiral magnetic field in the whole heliosphere.

A parallel diffusion coefficient used in the model is expressed, as:

$$K_{II} = K_0 K(r) K(t) K(R, \gamma(t)), \quad (5)$$

where $K_0 = 1.9 \times 10^{19} cm^2/s$, $K(r) = 1 + 50r$; a function *K(t)* is introduced to make a consistent change of the diffusion coefficient K_{II} throughout the 11–year cycle of solar activity. The expression $K(t) = 2.3 \cdot \exp(4t)$ implemented in equation (1), is presented in figure 2. $K(R,\gamma(t))$ contributes to the changes of the parallel diffusion coefficient K_{II} due to dependence on the GCR particles rigidity *R*. In the QLT this dependence is expressed as $K(R,\gamma(t)) = R^{\gamma(t)}$ which is valid for rigidities R>10 GV [21], [22], [23] and [24]. The analytical expression of the $\gamma(t)$ (figure 3) implemented in the model is shown in the figure caption.

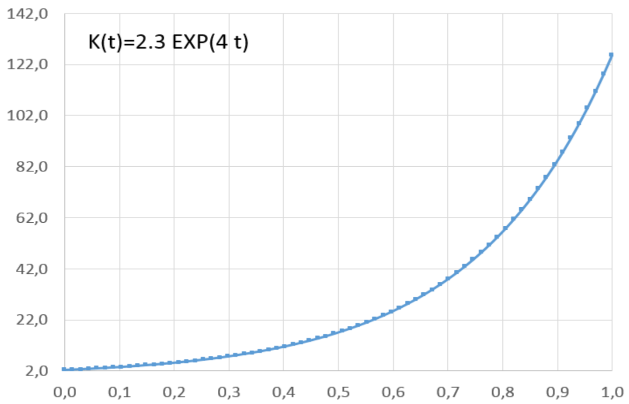

Figure 2. Temporal changes of the normalized parallel coefficient *K(t)* used in modeling.

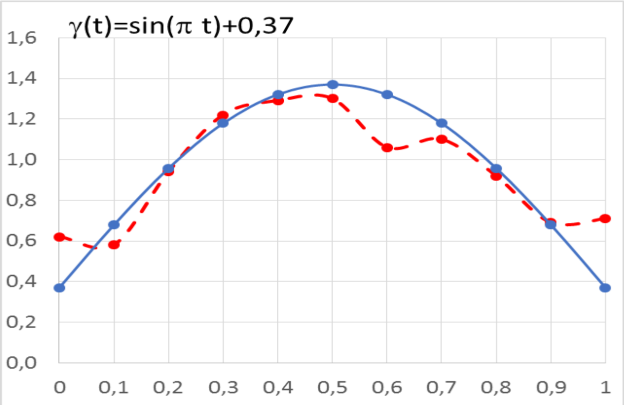

Figure 3. Changes in the exponent *γ(t)* in the period: 1976–1987 normalized to <0;1>. The 1–year averages (dashed red line) and the trigonometrical approximation (solid blue line) are included in the model.

The neutral sheet drift was taken into account according to the boundary condition method [26]. The delta function at the HNS is a consequence of the abrupt change in sign of the IMF. Changes of the magnitude B of the IMF (dashed line) in the considered period and its approximation (solid line) are shown in figure 4.

We assume that the drift effect *D(t)* has maximum value (100%) in the minima epochs, 1976–1977 and 1986–1987, and it is scaled down to 20% in maximum epoch of solar activity (figure 5). The temporal changes of the observed tilt angle δ of the HNS in the range of ~4^0 up to ~70^0 (dashed red line) and its approximation $\delta(t) = 408.6 \cdot t^3 - 759 \cdot t^2 + 360.7 \cdot t - 1.4$ (solid blue line) are presented In figure 6. In addition, in the model the waviness of the HNS is implemented, using the formula of [25] $\theta' = \frac{\pi}{2} + \delta(t)\sin\left(\frac{r\Omega}{U}\right)$.

The equation (2) was transformed to the algebraic system of equations using the implicit finite difference scheme, and then solved by the Gauss–Seidel iteration method (e.g. [26]) using the following boundary conditions:

$$f\big|_{r=100AU} = 1, \quad \frac{\partial f}{\partial r}\bigg|_{r=0} = 0, \quad \frac{\partial f}{\partial \theta}\bigg|_{\theta=0} = \frac{\partial f}{\partial \theta}\bigg|_{\theta=\pi} = 0, \tag{6}$$

as well as the initial conditions $f\big|_{R=100GV} = 1$ and $f(r,\theta,R_k,t)\big|_{t=0} = f(r,\theta,R_k)$ [26]. We start the calculations from the inner radius *(r=0)* of the spherical system.

The solutions for each k-layer of rigidity R_k where $(R_k=100, 90, 80, \ldots, 10$ *GV)* for the stationary case are considered as an initial conditions for the non-stationary case for the given rigidity *R* and at time *t*=0.

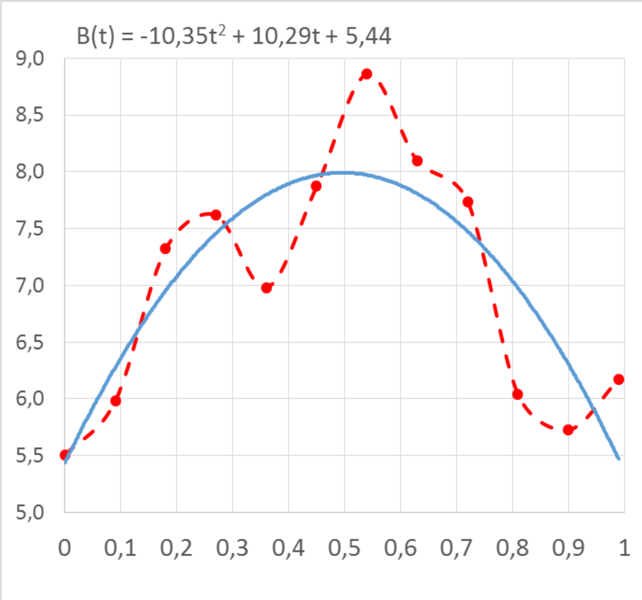

Figure 4. Magnitude B of the IMF in the period between 1976 and 1987 normalized to <0;1>. The yearly averages of data from SPIDR (dashed red line)

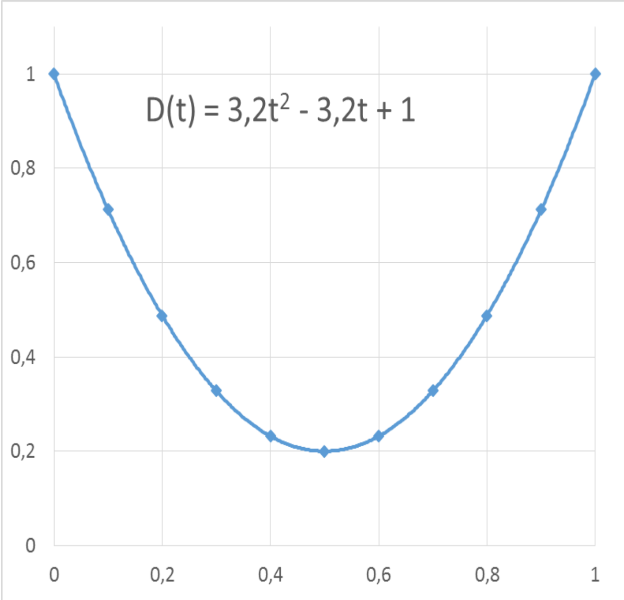

Figure 5. Changes of the drift ratio D(t) normalized to the minimum epochs implemented in the model is shown by polynomial approximation.

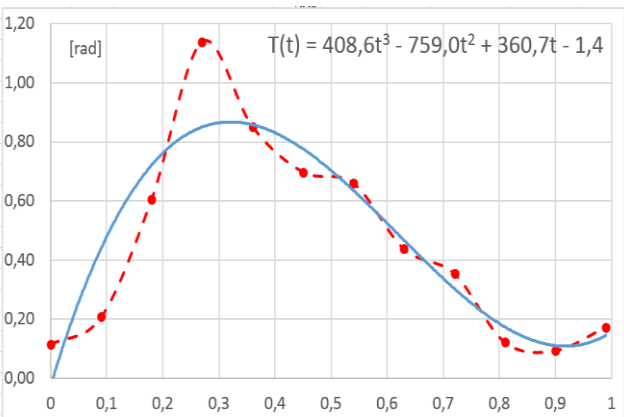

Figure 6. Yearly averages of the HNS tilt angle in the period 1976 and 1987 (dashed red line) and the approximation included in the model (solid blue line) normalized to < 0;1 >.

The results of the numerical solution of equation (2) for rigidity R=10 GV (dashed blue line), monthly averaged changes of the GCR intensity (solid red line) observed by the Moscow neutron monitor and its second order polynomial approximation (dotted red line) are demonstrated in figure 7. Figure 7 shows that the changes of the expected (dashed line) GCR particles density precede the smoothed by the second order polynomial approximation (dotted line) of the GCR intensity. To make clearer the existence of delay time between them we present in figure 8 the results of the numerical solution of equation (2) for rigidity R =10 GV (dashed line) shifted for 18 months with respect to the second order polynomial approximation (dotted line) of monthly averaged changes of the GCR intensity observed by the Moscow neutron monitor.

Our analysis shows that the delay time is ~18 months [27]. So, ~18 months can be accepted as an effective delay time caused by the combined influence of all parameters implemented in the 2–D model for the period 1977–1987.

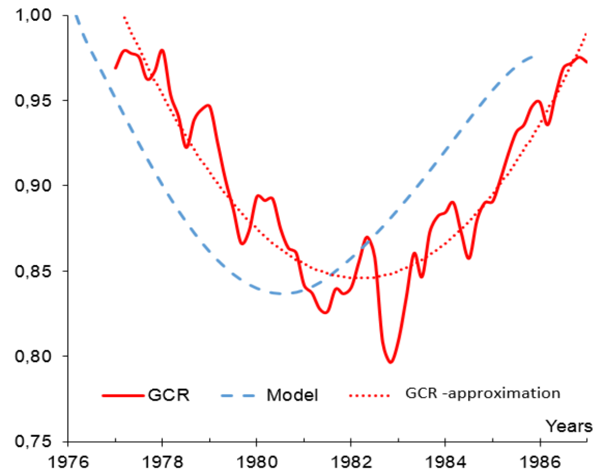

Figure 7. Changes of the amplitude of the 11–year variation of the GCR normalized intensity (red solid line) and approximated observed changes of the GCR intensity (red dotted line). Model – the solution of the equation (1) for the rigidity of 10 GV (blue dashed line)

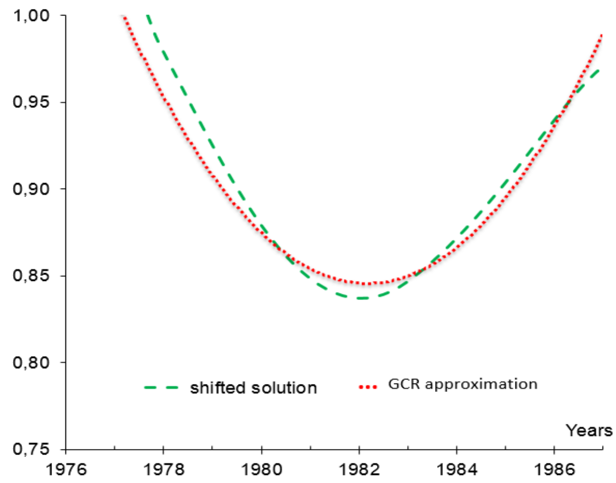

Figure 8 Shifted solution of the equation (1) by 18 months (dashed green line) with respect to the approximated observed changes of the GCR intensity (red dotted line).

Recently Bobik et al.,[28] developed a 2-D stochastic MC code for GCR particles propagation across the heliosphere, and have received an acceptable agreement between modeling results of antiproton to proton ratios (aP/P) and observations by BESS and PAMELA. Also, they estimate two specific times - (1) tsw ~ 14 months, time needed by solar wind to expend from the outer corona up to 100AU with a speed 400 km/s, and (2) a) time τev ~ 1month and b) time τev ~ few days needed for stochastic evolution of particles in heliosphere with energies of 10 GV and 200 MeV, respectively. Besides, in paper is touched upon an important problem dealing with GCR modulation, namely, a necessity to consider a whole heliosphere as a consisting from small regions with different electromagnetic conditions, e.g., determined by monthly averaged parameters. Similar study has been performed in various papers, among them in [29], where was taken into account that the free path of GCR scattering is controlled by the real distribution of the Sun's coronal green line intensity (CGLI).

Conclusions

1. A new 2–D time dependent model of the 11–year variation was developed. This model implements the parameters characterizing the temporal changes of the magnitude *B* of the IMF, tilt angle δ of the HNS for the period of 1976–1987 of cycle #21 and the changes of drift coefficient of the GCR particles versus the solar activity. The drift coefficient of the GCR particles has a maximum value ~ 100% in the minimum epoch (drift dominated period), and 20% in the maximum epoch (almost diffusion dominated period).
2. In the model temporal changes of the rigidity spectrum exponent γ, characterizing a rigidity dependence of amplitudes of the 11–year variations of the GCR intensity, were implemented for the first time.

3. The temporal changes of the parameters implemented in the 2−D model have different delay times with respect to the temporal changes of the smoothed experimental data of the GCR intensity observed by Moscow neutron monitor. We show that an acceptable compatibility is kept for the period of 1976−1987 (solar cycle #21), when the minimum of the expected temporal changes of the GCR particles density is shifted by 18 months with respect to the minimum of the temporal changes of the smoothed experimental data of the GCR intensity.
4. We conclude that a delay time ~18 months can be accepted as an effective delay time caused by the combined influence of all parameters implemented in the 2−D model. Generally, a direct implementation of the delay time τ in modeling is a important problem needing additional study.

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