On the method of the GCR partial intensities related to the main physical processes of solar modulation

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Abstract. We discuss in more details the earlier suggested method to decompose the calculated GCR intensity into the partial intensities connected with the main physical processes (the diffusion, convection, adiabatic cooling and magnetic drift) of the solar modulation. The expressions for the partial intensities connected with the convection and adiabatic cooling are improved and some results are illustrated.

1. Introduction

This work deals with some additional calculations that can be made in the course of the usual numerical solution of the boundary problem for the GCR intensity. The aim of these calculations is to improve our understanding of behavior of this intensity in the heliosphere.

But first, why are we not fully satisfied with the conventional approach and look for some additional means? Of course, all physics of modeling the GCR modulation lies in the Transport Partial differential Equation (TPE) with the boundary and "initial" conditions for the intensity and in the models of the TPE coefficients used. So one solves the boundary problem, compares the results with the observations and makes the conclusions about the degree of understanding of the modulation process. However, observations of the GCR intensity and heliospheric characteristics are very incomplete; besides, even in the simplest case the calculated GCR differential intensity $J(\vec{r}_0, T_0, t_0)$ for the position $\vec{r}_0$, energy T_0 and time t_0 is the result of complex interplay between several main physical processes (the diffusion, convection, drift, adiabatic cooling) for all higher energies ($T > T_0$) and in previous moments ($t < t_0$) in the whole heliosphere. And the details of this interplay are still unrevealed in the calculations, only its result. But to understand this interplay, in particular, the contribution of different processes to the calculated intensity, is very interesting and important.

In [1, 2] we suggested the method to decompose the calculated GCR intensity into the partial intensities connected with the main physical processes. In [3] the method was applied to understand the mechanisms forming the time profiles of the GCR intensity near the Earth during the last three periods of low solar activity. In this paper, first, we shall briefly discuss the usual process of modeling the GCR intensity and specify which additional calculations could be made and how they can be used for our purpose. Second, the expressions used earlier for the partial intensities connected with the convection and adiabatic cooling will be improved. Then some results will be illustrated and discussed.

2. Modeling the GCR intensity

2.1. The boundary–value problem

The TPE for the cosmic ray intensity J in its conventional form was derived by Parker [4, 5], Krymsky [6], Jokipii et al. [7] (if we list few names) in the first twenty years of the theoretical study of the GCR modulation. Usually it deals with the distribution function $f(\vec{r},p,t) = J(\vec{r},T,t)/p^2$, where p is the momentum of particles. For the stationary case the TPE balances the spatial divergences of the diffusion, drift and convection fluxes and of that due to adiabatic cooling in the momentum space:

$$-\frac{\partial f}{\partial t} = \underbrace{-\nabla(K\nabla f)}_{\text{diffusion}} + \underbrace{\vec{V}^{sw}\nabla f - \frac{\nabla\vec{V}^{sw}}{3}p\frac{\partial f}{\partial p}}_{\text{convection+adiabatic loss}} + \underbrace{\vec{V}^{dr}\nabla f}_{\text{drift}} = 0, \tag{1}$$

where $\vec{V}^{sw}$, $\vec{V}^{dr}$ and K are the solar wind and magnetic drift velocities, and diffusion tensor, respectively. Note that we combined the second and third terms as the sum for the convection and adiabatic cooling processes because each of these terms by itself is not equal to the divergence of the convection and adiabatic fluxes, respectively (see subsection 2.4). For the case of axial symmetry in a spherical system of coordinates the boundary–value problem, i. e., the transport equation with boundary and "initial" conditions, appears as follows

$$\underbrace{-\frac{1}{r^2}\frac{\partial}{\partial r}\left(K_{rr}r^2\frac{\partial f}{\partial r}\right) - \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(K_{\vartheta\vartheta}\sin\vartheta\frac{\partial f}{\partial\vartheta}\right)}_{\text{diffusion}} +$$

$$\underbrace{V_r^{sw}\frac{\partial f}{\partial r} - \frac{\nabla\vec{V}^{sw}}{3}p\frac{\partial f}{\partial p}}_{\text{convection+adiabatic loss}} + \underbrace{V_r^{dr}\frac{\partial f}{\partial r} + \frac{1}{r}V_\vartheta^{dr}\frac{\partial f}{\partial\vartheta}}_{\text{drift}} = 0 \tag{2}$$

$$\left.\frac{\partial f}{\partial r}\right|_{r=r_{min}} = 0, \qquad f|_{r=r_{max}} = f_{nm}(p), \qquad \left.\frac{\partial f}{\partial\vartheta}\right|_{\vartheta=0,\pi} = 0 \tag{3}$$

$$f|_{p=p_{max}} = f_{nm}(p_{max}), \tag{4}$$

where $f_{nm}(p)$ is the nonmodulated distribution function corresponding to nonmodulated GCR intensity $J_{nm}(T)$ and r_{min}, r_{max}, p_{max} are the radii of the inner and outer boundaries of the modulation region and the momentum above which there is no modulation. Here for us it is important that only adiabatic loss of the energy is considered, without any acceleration (e. g., at the termination shock of the solar wind or in the heliosheath). Then with respect to the momentum or energy it is the Cauchy problem with the "initial condition" (4).

In the following we discuss the numerical solution of the boundary–value problem (2-4), some additional calculations and their results. Note that nowadays there are research groups dealing with much more sophisticated 3D and nonstationary models of the heliosphere and electromagnetic field – charged particle interaction (e. g., [8]). However we believe that the main features of the long-term (11– and 22–year) GCR intensity variations in the inner and intermediate heliosphere ($r < 70$ AU) can be described and understood using much simpler 2D and quasi–stationary tools like our boundary–value problem (2-4) with rather simple TPE coefficients. Besides, we are interested in two types of tasks. The first task is the study of the space–energy distribution of the GCR intensity with some specific set of the main parameters characteristic for the given phase of solar cycle (e. g., the sunspot minimum) and different heliospheric magnetic field (HMF) polarity (the sign of the HMF radial component B_r in the N–hemisphere, $A = \pm 1, 0$). The second task is the study of the time profile of the GCR intensity for some spatial locations and energies. In this paper we deal mainly with the first task.

2.2. Solving the boundary–value problem

To specify which additional calculations can be made for our purpose we should briefly discuss the well-known method of solution of the boundary–value problem and the widely used approximations of the differential operators by the finite differences on the grid (see, e. g., [9]).

If we change the variables $X = X(r)$, $Y = Y(\vartheta)$, $\tau = \tau(p)$ and the unknown function $U = f/f_{nm}(p) = J/J_{nm}(T)$ so that for new variables normalized to [0,1] the grids are homogeneous ($X_i = i\Delta X$, $Y_j = j\Delta Y$, $\tau_k = k\Delta\tau$), our TPE for U, which we call the normalized intensity, can be rewritten in the operator form:

$$\underbrace{\frac{\nabla\vec{V}^{sw}}{3}p\tau'\frac{\partial U}{\partial\tau}}_{A_\tau L_\tau U} = \underbrace{-\frac{X'}{r^2}\frac{\partial}{\partial X}\left(K_{rr}r^2X'\frac{\partial U}{\partial X}\right) + V_r^{sw}X'\frac{\partial U}{\partial X} + V_r^{dr}X'\frac{\partial U}{\partial X}}_{L_X U}$$

$$\underbrace{-\frac{Y'}{r^2\sin\vartheta}\frac{\partial}{\partial Y}\left(K_{\vartheta\vartheta}\sin\vartheta Y'\frac{\partial U}{\partial Y}\right) + \frac{1}{r}V_\vartheta^{dr}Y'\frac{\partial U}{\partial Y}}_{L_Y U}\underbrace{-\frac{\nabla\vec{V}^{sw}}{3}p\tau'(\ln f_{nm})'U}_{A_0 U} \tag{5}$$

and the differential operators can be approximated ($\approx$) by their difference analogues to the second order for ΔX, ΔY and to the first order for $\Delta\tau$:

$$L_X U = AX_0\left\{-\frac{\partial}{\partial X}\left[AXX\frac{\partial U}{\partial X}\right] + AXV\frac{\partial U}{\partial X}\right\} \approx \mathcal{L_X}^k\mathcal{U}_{i,j}^k \tag{6}$$

$$\mathcal{L_X}\mathcal{U}_i = \mathcal{AX}_0\left\{-\frac{\mathcal{AXX}_{i+\frac{1}{2}}(\mathcal{U}_{i+1}-\mathcal{U}_i) - \mathcal{AXX}_{i-\frac{1}{2}}(\mathcal{U}_i-\mathcal{U}_{i-1})}{\Delta^2 X} + \mathcal{AXV}\frac{(\mathcal{U}_{i+1}-\mathcal{U}_{i-1})}{2\Delta X}\right\} \tag{7}$$

$$A_\tau L_\tau U \approx \mathcal{A}_\tau\frac{(\mathcal{U}^{k+1}-\mathcal{U}^k)}{\Delta\tau}, \tag{8}$$

where $\mathcal{U}_{i,j}^k \approx U(X_i, Y_j, \tau_k)$, the coefficients $\mathcal{AX}_0$, $\mathcal{AXV}$, $\mathcal{A}_\tau$ in (7, 8) are equal to their counterparts in (5, 6) in the nodes of the grid, while $\mathcal{AXX}_{i\pm\frac{1}{2}}$ in (7) is equal to AXX in (6) at halfway between nodes X_i and $X_{i\pm1}$. The indices j, k are not repeated in (7), as well as i, j in (8). The approximation for L_Y is similar to that for L_X. Then the initial TPE can be also approximated by the finite differences on the grid

$$\begin{aligned}\mathcal{A}_\tau\frac{\mathcal{U}^{k+1}-\mathcal{U}^k}{\Delta\tau} &= \sigma_X\mathcal{L_X}^{k+1}\mathcal{U}^{k+1} + \sigma_Y\mathcal{L_Y}^{k+1}\mathcal{U}^{k+1} + (1-\sigma_X)\mathcal{L_X}^k\mathcal{U}^k + (1-\sigma_Y)\mathcal{L_Y}^k\mathcal{U}^k \\ &+ \sigma_X\mathcal{A}_0^k\mathcal{U}^k + (1-\sigma_X)\mathcal{A}_0^{k+1}\mathcal{U}^{k+1},\end{aligned} \tag{9}$$

with the weights $0 \le \sigma_{X,Y} \le 1$. For each step in τ (or momentum, or energy) using the Alternating Direction Implicit (ADI) method the system of the algebraic equations (9) with the appropriate boundary conditions can be solved. So $\mathcal{U}_{i,j}^{k+1}$ and hence the distribution function and intensity for the current energy in the whole heliosphere is calculated from the already known $\mathcal{U}_{i,j}^k$ (starting from $k = 0$, $\mathcal{U}_{i,j}^0 = 1$) and hence intensity for a little bit higher energy.

Usually after this point the process is repeated for even smaller energy and so on. However we here stop for a while to calculate and memorize (for some r_0, ϑ_0) the final difference approximation of any TPE term (diffusion, convection, drift) and of all necessary coefficients (e. g., $\mathcal{A}_\tau$, $\nabla\vec{V}^{sw}$ etc.) for the current energy.

2.3. Decomposition of the calculated J into the partial intensities

Note that TPE (5) in (r_0, ϑ_0) can be considered as the Transport Ordinary differential Equation (TOE) (10) with respect to $f(\tau)$ and in ∇–notations with the natural initial condition (11):

$$\frac{df(\vec{r}_0,\tau)}{d\tau} = \underbrace{\frac{3}{\nabla\vec{V}^{sw}}\nabla(K\nabla f)}_{T^{diff}(\vec{r}_0,\tau)} \underbrace{-\frac{3}{\nabla\vec{V}^{sw}}\vec{V}^{sw}\nabla f}_{\breve{T}^{conv}(\vec{r}_0,\tau)} \underbrace{-\frac{3}{\nabla\vec{V}^{sw}}\vec{V}^{dr}\nabla f}_{T^{drift}(\vec{r}_0,\tau)} \tag{10}$$

$$f|_{\tau=0} = f_{nm}(p_{max}), \qquad \tau = \ln\frac{p_{max}}{p} \tag{11}$$

This Cauchy problem could be solved if one knew all terms $T^{diff}, \breve{T}^{conv}, T^{drift}$ in (r_0, ϑ_0) for $\tau > 0$ $(T < T_{max})$. That is why we have already calculated and memorized the approximations of these terms while solving the algebraic equations (9) for U. Then integrating the initial value problem (10,11) we can calculate the partial "intensities" related to diffusion, convection and drift and their sum $\breve{J}_p^{dcd}$ (12).

$$\breve{J}_p^{dcd}(\vec{r}_0,T) = \underbrace{p^2\int_0^\tau T^{diff}d\tau}_{J_p^{diff}} + \underbrace{p^2\int_0^\tau \breve{T}^{conv}d\tau}_{\breve{J}_p^{conv}} + \underbrace{p^2\int_0^\tau T^{drift}d\tau}_{J_p^{drift}} \tag{12}$$

$$\breve{J}_p^{adiab} = J - \breve{J}_p^{dcd} \tag{13}$$

$$J = J_p^{diff} + \breve{J}_p^{conv} + J_p^{drift} + \breve{J}_p^{adiab} \tag{14}$$

But we can't find the partial "intensity" related to adiabatic loss, as we used part of the related term as the left–hand side of (10). However, knowing the total J, we can calculate the partial "intensity" related to adiabatic energy loss as the difference between J and $\breve{J}_p^{dcd}$. So the decomposition (14) is what we tried to get. In this paragraph we marked the partial intensities with the inverted commas to emphasizes their strangeness, as they can be negative if the process leads to a reduction in intensity. In the following we omit the inverted commas. Besides, we marked by the breve ($\breve{\ }$) the terms and partial intensities, related to the convection and adiabatic energy loss, which we consider incorrect and try to improve (see our note in 2.1 and the next subsection).

It is easily seen that both diffusion and drift partial intensities can be further decomposed into their components. As $K_{rr} = K_\parallel \cos^2\chi + K_{\perp,r}\sin^2\chi$ (with χ being the regular HMF Parker spiral angle) and $K_{\vartheta\vartheta} = K_{\perp,\vartheta}$, the diffusion partial intensity can be decomposed into those related to the diffusion along the regular HMF in the radial direction and perpendicular to the regular HMF in the radial and latitudinal directions, $J_p^{diff} = J_{\parallel,r}^{diff} + J_{\perp,r}^{diff} + J_{\perp,\vartheta}^{diff}$.

$$J_p^{diff} \propto \int_0^\tau \frac{1}{r^2}\frac{\partial}{\partial r}\left(K_\parallel\cos^2\chi r^2\frac{\partial J}{\partial r}\right)d\tau + \int_0^\tau \frac{1}{r^2}\frac{\partial}{\partial r}\left(K_{\perp,r}\sin^2\chi r^2\frac{\partial J}{\partial r}\right)d\tau$$
$$+ \int_0^\tau \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(K_{\perp,\vartheta}\sin\vartheta\frac{\partial J}{\partial\vartheta}\right)d\tau \tag{15}$$

Similarly as $\vec{V}^{dr} = \vec{V}^{dr}_{reg} + \vec{V}^{dr}_{cs}$, the drift partial intensity can be decomposed into those related to the drift in the regular HMF in the radial and latitudinal directions and along the heliospheric current sheet in the radial direction, $J_p^{drift} = J_{reg,r}^{drift} + J_{cs,r}^{drift} + J_{reg,\vartheta}^{drift}$. The terms corresponding to these parts of the diffusion and drift partial intensities are written in (15-16).

$$J_p^{drift} \propto \int_0^\tau -V_r^{dr,reg}\frac{\partial J}{\partial r}d\tau + \int_0^\tau -V_r^{dr,cs}\frac{\partial J}{\partial r}d\tau + \int_0^\tau -\frac{1}{r}V_\vartheta^{dr,reg}\frac{\partial J}{\partial \vartheta}d\tau \tag{16}$$

2.4. On the convection and adiabatic energy loss partial intensities

The transport equation for the cosmic ray distribution function in the form (1), first suggested probably in [7], is simple to solve. However, as we mentioned in subsection 2.1, each of its second and third terms by itself is not equal to the divergence of the convection $\vec{S}^{conv}$ and adiabatic energy loss S^{adiab} fluxes, respectively. As was shown in [10] (see also the references therein and [11]) the right expression for these fluxes in terms of f are

$$\vec{S}^{conv} = 4\pi p^2 C_{CG} f \vec{V}^{sw} \tag{17}$$

$$S^{adiab} = \langle \dot{p} \rangle = \frac{1}{3} p \vec{V}^{sw} \vec{G}, \tag{18}$$

where $C_{CG} = -1/3\partial(\ln f)/\partial(\ln p)$ and $\vec{G} = \nabla f/f$ are the Compton-Getting factor and the relative gradient of the distribution function, respectively. So the TPE for f with each its term equal to the divergence of the corresponding flux is

$$-\frac{\partial f}{\partial t} = \underbrace{-\nabla(K\nabla f)}_{\text{diffusion}} + \underbrace{\nabla\left(C_{CG} f \vec{V}^{sw}\right)}_{\text{convection}} + \underbrace{\frac{1}{3p^2}\frac{\partial\left(p^3 \vec{V}^{sw}\nabla f\right)}{\partial p}}_{\text{adiabatic loss}} + \underbrace{\vec{V}^{dr}\nabla f}_{\text{drift}} = 0 \tag{19}$$

The TPE in form (19) looks quite different from (1). However as was shown in [10] and [7] these two forms are equivalent. So the right expression for the convective partial intensity would be

$$J_p^{conv} = p^2 \frac{3}{\nabla \vec{V}^{sw}} \int_0^\tau -\nabla\left(C_{CG} f \vec{V}^{sw}\right) d\tau \tag{20}$$

instead of that implied in (10, 12). As to the adiabatic loss partial intensity it still can be calculated using (13), where however J_p^{dcd} includes the proper J_p^{conv} (20).

3. The illustrations

To solve the boundary–value problem (2-4) we need the models of the coefficients of the transport equation and the GCR nonmodulated spectrum. Here we shall not consider them as we used the same coefficients and $J_{nm}(T)$ as in [1, 2, 3, 12, 13, 14], where they (and even their questionable features) were described and discussed. As to the physical task we are most interested in understanding the role of the magnetic drift and other processes associated with the dominating HMF polarity A in the long–term variations of the GCR intensity. So we choose for illustration the same task as in [12, 14] that is the calculation of the space-energy distribution of the GCR intensity during the consecutive solar minima 21/22 (between solar cycles 21 and 22) and 22/23 characterized by similar behavior of the solar activity and the opposite HMF polarity.

In [12, 14] we first chose the parameters of the models in such a way that the calculations basically described the latitude, radial and energy dependencies of the observations for $A < 0$ (1987) and $A > 0$ (1997) solar minima. Then we repeated the calculations with the same parameters but switched off the drift, that is omitted the drift terms in (9) or used $A = 0$ as

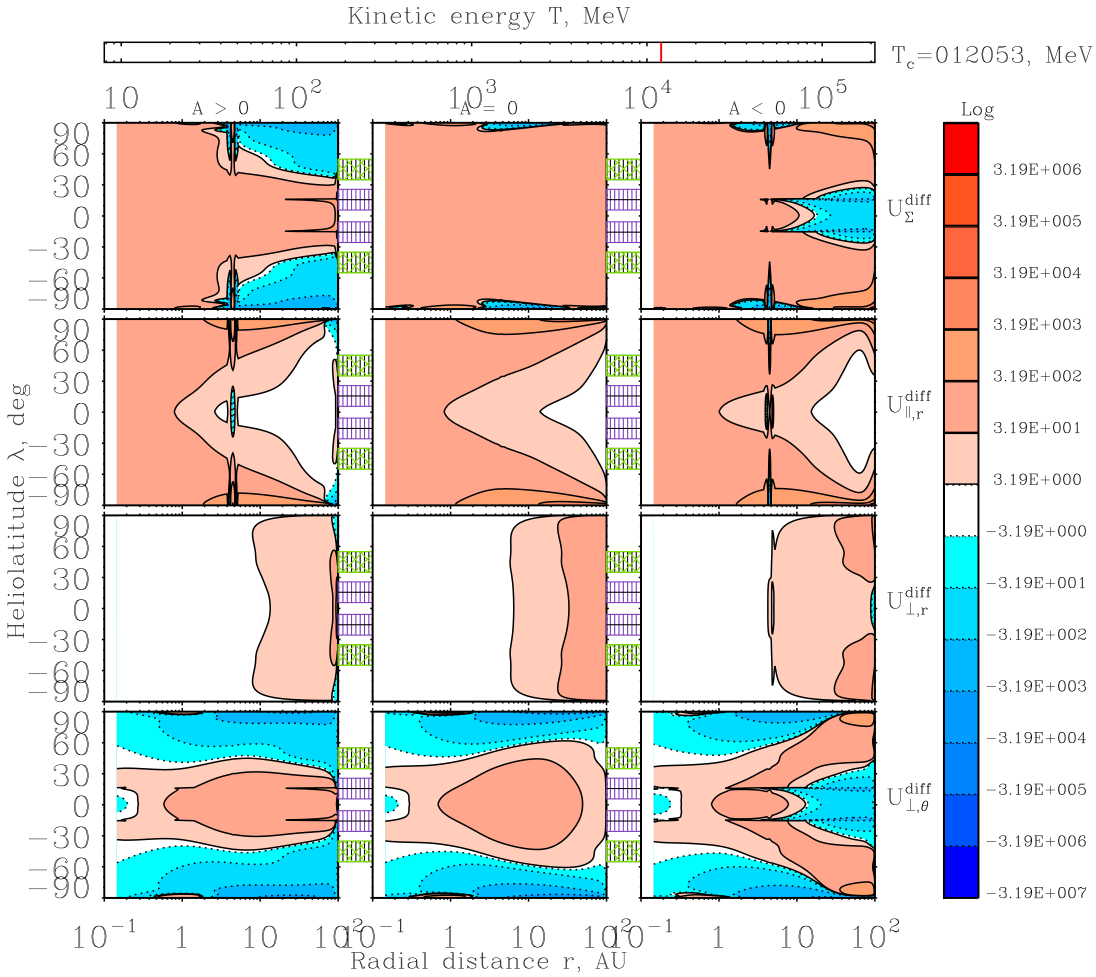

Figure 1. The spatial distribution of the normalized diffusion partial intensity and its three components for $T \approx 12$ GeV. Three columns of panels are for cases $A > 0$, $A = 0$ and $A < 0$. The upper row is for the total normalized diffusion partial intensity U_p^{diff}, while the next three rows are for $U_{\|,r}^{diff}$, $U_{\perp,r}^{diff}$, $U_{\perp,\vartheta}^{diff}$. The energy scale is the upper horizontal bar while the normalized intensity scale is in the right vertical panel. The horizontal bands between the columns mark the latitudes where the TPE coefficients (the solar wind and drift velocities and diffusion tensor) greatly change (see [3, 12, 13, 14]) for details.

these terms are proportional to A. And we noted that the calculated intensity almost everywhere decreased by a factor of 3-5 when compared with the intensities for both $(A > 0)$– and $(A < 0)$–periods. In [3] we showed that the same is true for all other phases of solar cycle except the years near solar maxima. So we made the conclusion that in the models used the drift contribution is large for both polarities, that is it strongly influences both 11- and 22-year cycles in the GCR intensity. It should be mentioned that the active role of drift in forming the 11-y variation of the GCR intensity followed from several model calculations [15, 16]. Note that in both of these works as well as in our calculations the first generation drift models were used. We used the simple model for the regular and current sheet drift velocities as in [17].

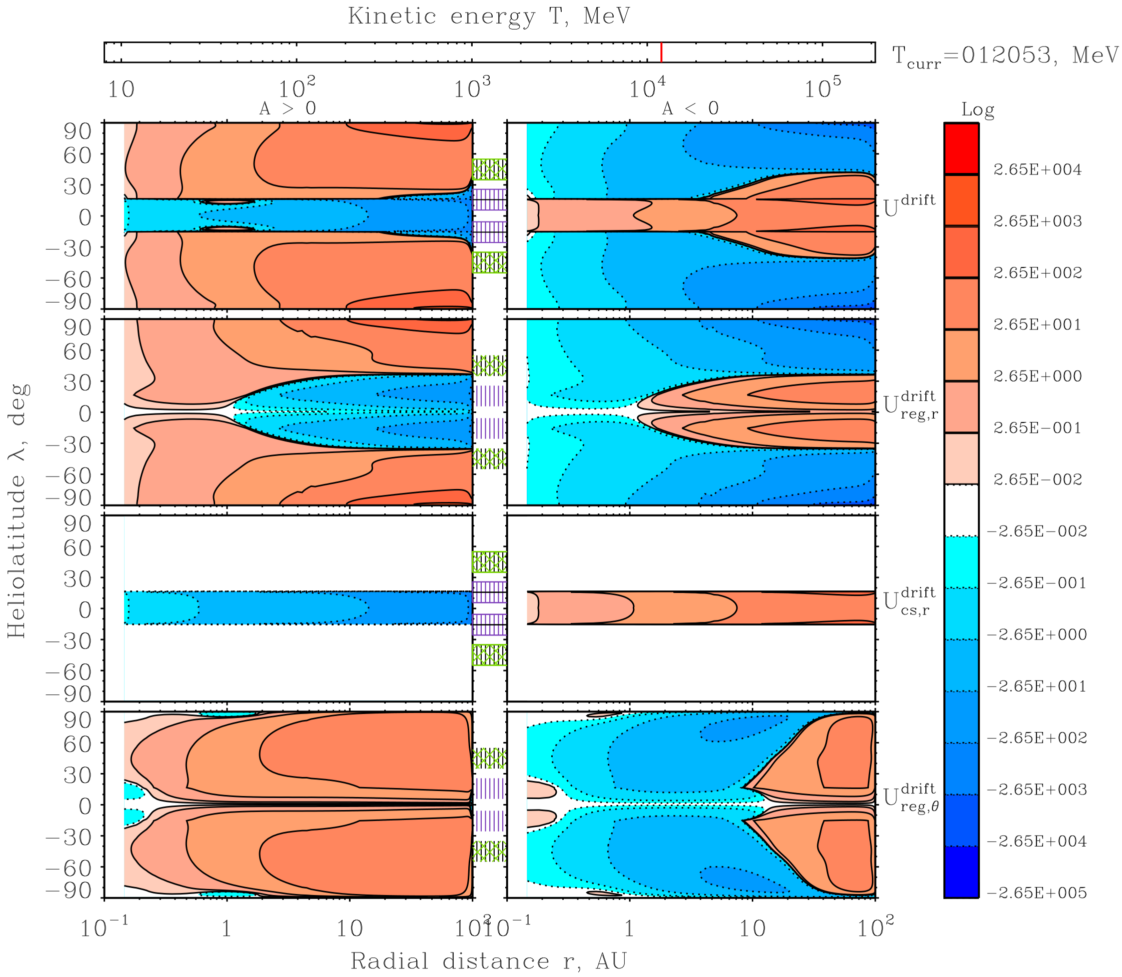

Figure 2. The same as in Fig.1 but for the drift partial intensity and without $A = 0$ column.

In this paper we try to prove that the partial intensities can be useful for revealing the mechanisms of the GCR intensity modulation and the heliospheric structure. The detailed picture of the space, energy and time distributions of different partial intensities will be discussed elsewhere. Here only two topics will be considered, namely, the spatial distributions for neutron monitor energy ($T \approx 12$ GeV) and all three cases of the HMF polarity: 1) of the diffusion and drift partial intensities and their parts and 2) of the drift partial intensity compared with the sum of all other partial intensities.

In Fig.1 the normalized diffusion partial intensity and its parts (as in (15)) are shown as functions of the radial distance and latitude for all $A > 0$, $A = 0$ and $A < 0$ cases. It should be noted that the vertical feature seen at a glance at $r = 5$ AU in the upper three rows of Fig.1 is due to somewhat poor choice of $X(r)$, when $X(r)$ is continuous but its derivative, $X'(r)$, breaks at $r = 5$ AU. So the terms including its second derivative (such as those connected with the diffusion in the radial direction) demonstrate such unusual feature. It turns out that for such energies the total diffusion partial intensity (that is the contribution of the diffusion in the calculated intensity), which in case of the absence of drift ($A = 0$) is positive almost everywhere (except small regions near the poles) is negative in the far from the Sun regions for the case $A > 0$ in the middle and high latitudes and for the case $A < 0$ in the equatorial latitudes.

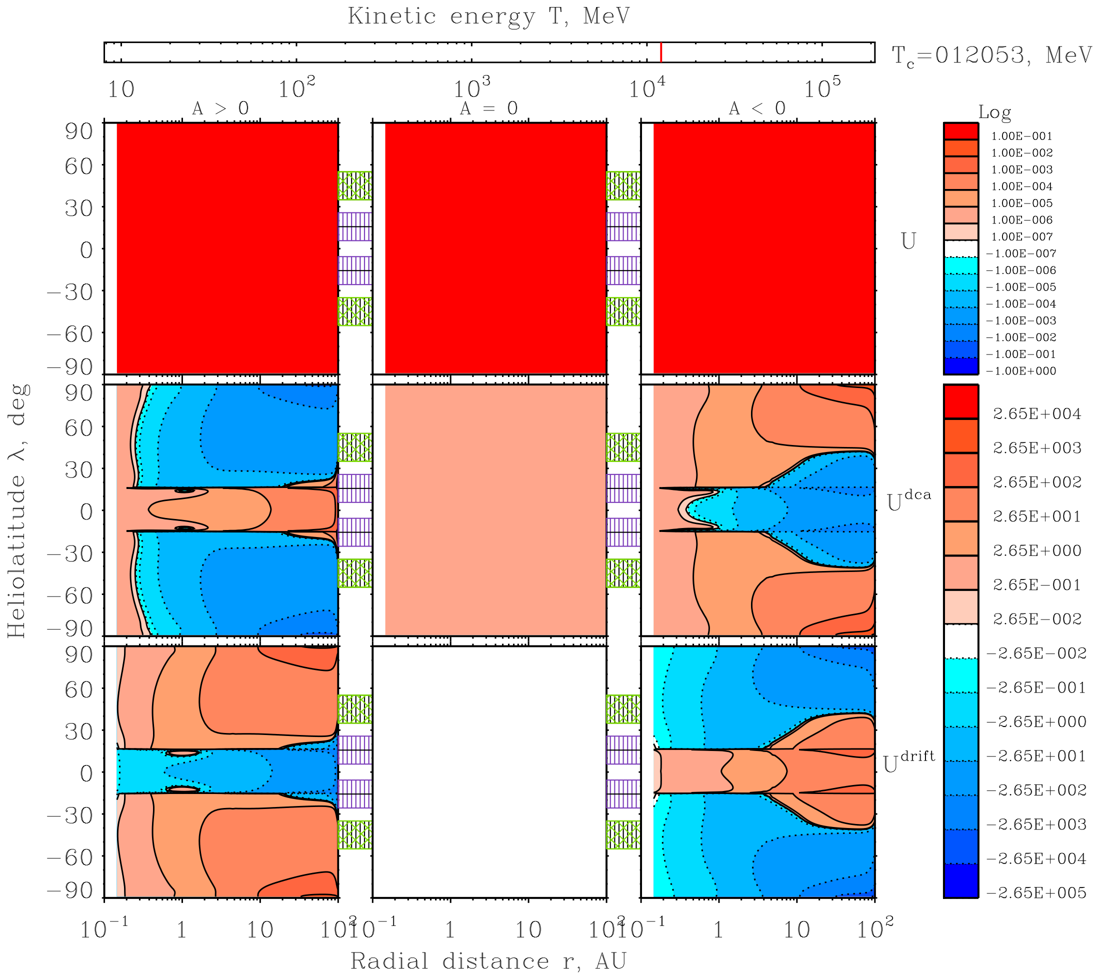

Figure 3. The balance between drift and sum of other partial intensities. Three columns of panels are for cases $A > 0$, $A = 0$ and $A < 0$. The upper row is for the total calculated intensity U, while the next two rows are for U_p^{dca} and U_p^{drift}. The energy scale is the upper horizontal bar while the intensity scales (different for U and U_p^{dca}, U_p^{drift}) are in the right vertical panels.

Besides, it turns out that the negative contribution of the diffusion for such energies for all cases (positive, zero and negative HMF polarities) is due entirely to the diffusion perpendicular to the regular HMF in the latitudinal direction. Of course, it is also easily seen that this picture reveals very detailed and unexpected heliospheric structure with respect to the diffusion of the GCRs.

Similar analysis can be also fulfilled for the drift partial intensity (see Fig.2). It also reveals the details of the drift contribution in the total calculated GCR intensity and the heliospheric structure with respect to the drift of the GCRs.

As to the main task — understanding the role of drift — in Fig.3 we separate the calculated normalized intensity U (positive (red) everywhere) into two parts much greater in absolute values and of different signs in different regions depending on the HMF polarity: the sum of the partial intensities due to the overall contribution of diffusion, convection and adiabatic cooling (U_p^{dca}) and the drift partial intensity (U_p^{drift}). It can be easily seen that there are heliospheric regions where the contribution of drift dominates over that of the combined action of the diffusion, convection and adiabatic cooling — (positive (red) drift partial intensity) at the middle and high latitudes for $A > 0$ and at equatorial latitudes for $A < 0$ — while outside these regions the combined action of the diffusion, convection and adiabatic cooling dominates.

4. Conclusions

The decomposition of the calculated GCR intensity into the partial intensities related to the main physical processes can be useful in understanding the physics of the GCR modulation (e. g., the relative significance of different processes in the long-term variations of the GCR intensity) and in the theoretical study of the heliospheric structure.

There are some unresolved problems and difficulties:

- What is the detailed physical meaning of the suggested mathematical procedure (e. g., why do the diffusion, convection and drift partial intensities depend on $\nabla\vec{V}^{sw}$)?
- How to find the partial intensities in case of any GCR acceleration (e. g., at the termination shock)?
- Is it possible to find the partial intensities in case of any other method of solving the boundary–value problem (e. g., by the stochastic modeling the GCR trajectories,[18])?

It is important that to calculate the partial intensities the errors of the grid approximation and of the solution of the boundary–value problem should be rather small (as we are interested in spatial derivatives of the intensity). These problems are even more important if we want to consider the gradients of these partial intensities.

Acknowledgments

We thank for help the Russian Foundation for Basic Research (grants 12-02-00215, 13-02-00585) and the Program of the Presidium of the Russian Academy of Sciences "Fundamental Properties of Matter and Astrophysics".

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