

Spectra of protons and nuclei in the energy range of $10^{10} \div 10^{20}$ eV in the framework of Galactic cosmic ray origin

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Abstract. We consider the problem of the cosmic ray spectrum formation assuming that cosmic rays are produced by Galactic sources. The anomalous diffusion equation proposed in our recent papers is used to describe cosmic ray propagation in the interstellar medium. We show that in the framework of this approach and with generation spectrum exponent $\gamma = 2.85$ it is possible to reproduce locally observed basic features of cosmic rays in the energy region of $10^{10} \div 10^{20}$ eV: difference between spectral exponents of protons and other nuclei, mass composition variation, “knee” problem, flattening of the primary spectrum at $E \geq 10^{18} \div 10^{19}$ eV. The crucial model predictions for the mass composition behaviour in the ultra-high energy region are discussed.

1. Introduction

The problem of ultra-high energy cosmic ray (UHECR) origin remains unsolved, despite significant progress made in recent years. The suppression of the cosmic ray flux at energy of $(4 \div 6) \times 10^{19}$ eV being established unambiguously can not be reliably identified as GZK-cutoff due to the apparent differences between Telescope Array/HiRes and Auger mass composition results [1,2]. As the Auger data indicated a nuclear UHECR composition becoming heavier with energy in the region of $4 \div 40$ EeV, different models assuming that the UHECR spectra cutoff is not an extragalactic GZK feature are considered [3–5].

In this paper we examine the possibility for self-consistent description of all the basic features of the observed cosmic ray spectrum in the energy range of $10^{10} \div 10^{20}$ eV within the Galactic origin scenario. We assume the existence of Galactic sources that accelerate particles up to $\sim 10^{20}$ eV and take into account highly inhomogeneous (fractal-like) distribution of matter and magnetic fields in the Galaxy that leads to extremely large free paths of particles (“Levy flights”), along with an overwhelming contribution to the cosmic ray fluxes observed above $\sim 10^{18}$ eV from particles reaching the Solar System without scattering. The crucial model predictions for the mass composition behaviour in the ultra-high energy region are presented.



2. Anomalous diffusion model

In our recent papers an anomalous diffusion (AD) model for solution of the “knee” problem in primary cosmic-rays spectrum was proposed [6–9]. The “anomaly” in this model results from the presence of extremely large free paths (Levy flights) of particles between Galactic inhomogeneities and also the long-lasting trapping of particles in very strong magnetic field regions (Levy traps). Thus the following asymptotic behaviour of the probabilities of free paths ($p(x, E)$) and time, during which particles are caught in magnetic traps ($q(\tau, E)$), takes place:

$$\int_{|x|>r} p(x, E) dx \propto A(E, \alpha) r^{-\alpha}, \quad 0 < \alpha < 2; \quad \int_t^\infty q(\tau, E) d\tau \propto B(E, \beta) t^{-\beta}, \quad 0 < \beta < 1. \quad (1)$$

The cosmic rays propagation in the fractal-like interstellar Galactic medium without energy losses and nuclear interactions can be described by the anomalous diffusion equation. The equation for the concentration of particles with an energy E , generated in a fractal medium by Galactic sources with a distribution density $S(\mathbf{r}, t, E)$ can be written as

$$\frac{\partial N}{\partial t} = -D(E, \alpha, \beta) D_{0+}^{1-\beta} (-\Delta)^{\alpha/2} N(\mathbf{r}, t, E) + S(\mathbf{r}, t, E). \quad (2)$$

Here D_{0+}^μ denotes the Riemann-Liouville fractional derivative [10] and $(-\Delta)^{\alpha/2}$ is the fractional Laplacian (‘Riesz operator’) [10]. The anomalous diffusivity $D(E, \alpha, \beta) \sim A(E, \alpha)/B(E, \beta) = D_0(\alpha, \beta) E^\delta$.

The Green’s function $G(\mathbf{r}, t, E; E_0)$, corresponding to (2), can be found from the equation

$$\frac{\partial G(\mathbf{r}, t, E; E_0)}{\partial t} = -D(E, \alpha, \beta) D_{0+}^{1-\beta} (-\Delta)^{\alpha/2} G(\mathbf{r}, t, E; E_0) + \delta(\mathbf{r}) \delta(t) \delta(E - E_0). \quad (3)$$

With the use of the Laplace-Fourier transformations and formulae [10]

$$\int_0^\infty e^{-\lambda t} D_{0+}^\beta G(\mathbf{r}, t, E; E_0) dt = \lambda^\beta \int_0^\infty e^{-\lambda t} G(\mathbf{r}, t, E; E_0) dt = \lambda^\beta \tilde{G}(\mathbf{r}, \lambda, E; E_0),$$

$$\int_{\mathbb{R}^3} e^{i\mathbf{k}\mathbf{r}} (-\Delta)^{\alpha/2} G(\mathbf{r}, t, E; E_0) d\mathbf{r} = |k|^\alpha \int_{\mathbb{R}^3} e^{i\mathbf{k}\mathbf{r}} G(\mathbf{r}, t, E; E_0) d\mathbf{r} = |\mathbf{k}|^\alpha \tilde{G}(\mathbf{k}, t, E; E_0)$$

we obtain

$$\tilde{G}(\mathbf{k}, \lambda, E; E_0) = \delta(E - E_0) \lambda^{\beta-1} \int_0^\infty \exp\left(-\left[\lambda^\beta + D(E, \alpha, \beta) |\mathbf{k}|^\alpha\right] y\right) dy.$$

Then the inverse transform yields

$$G(\mathbf{r}, t, E; E_0) = \delta(E - E_0) \left(D(E_0, \alpha, \beta) t^\beta\right)^{-3/\alpha} \Psi_3^{(\alpha, \beta)} \left(|\mathbf{r}| \left(D(E_0, \alpha, \beta) t^\beta\right)^{-1/\alpha}\right), \quad (4)$$

where

$$\Psi_3^{(\alpha, \beta)}(r) = \int_0^\infty g_3^{(\alpha)} \left(r \tau^{\beta/\alpha}\right) g_1^{(\beta, 1)}(\tau) \tau^{3\beta/\alpha} d\tau$$

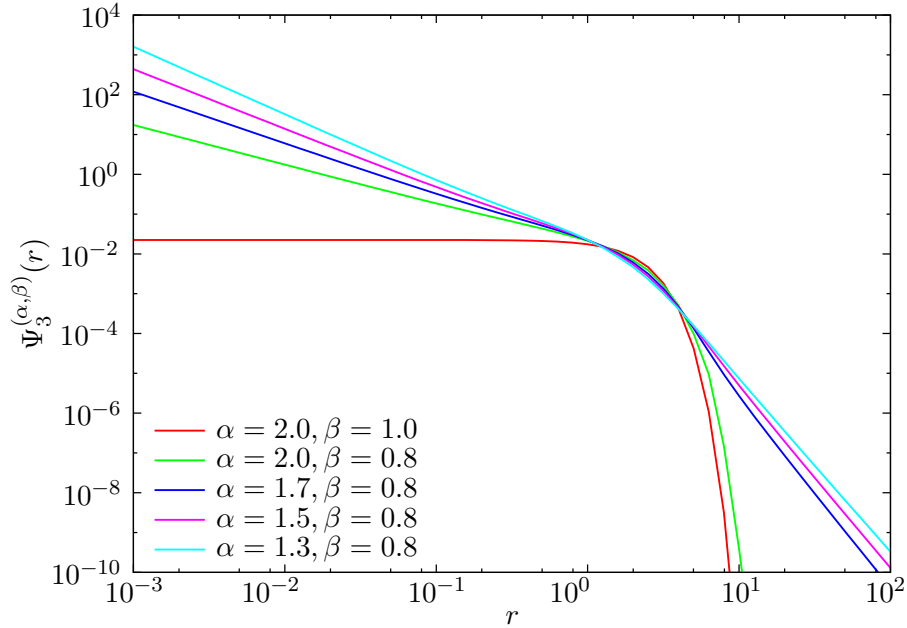


Figure 1. Fractional stable distributions densities $\Psi_3^{(\alpha, \beta)}(r)$

is the density of fractional stable distribution [11], $g_3^{(\alpha)}(r)$ ($\alpha \leq 2$) the three-dimensional spherically-symmetrical stable distribution and $g_1^{(\beta, 1)}(t)$ is the one-sided stable distribution with characteristic exponent $\beta \leq 1$ [11]. The probability densities of $\Psi_3^{(\alpha, \beta)}(r)$ are shown in figure 1.

Using the Green's function (4), we can write a solution of the anomalous diffusion equation for a point pulse source $S(\mathbf{r}, t, E) = S_{\text{im}} E^{-\gamma} \exp(-E/E_0(z)) \delta(\mathbf{r}) \Theta(T - t) \Theta(t)$ with emission time T and an exponential cutoff at energy E_0

$$N(\mathbf{r}, t, E) = \frac{S_{\text{im}} E^{-\gamma}}{D(E, \alpha, \beta)^{3/\alpha}} \times \left[\int_{\max[0, t-T]}^t d\tau \tau^{-3\beta/\alpha} \Psi_3^{(\alpha, \beta)} \left(|\mathbf{r}| (D(E, \alpha, \beta) \tau^\beta)^{-1/\alpha} \right) \right] \exp \left(-\frac{E}{E_0(z)} \right).$$

3. Energy spectrum of cosmic rays

Types of sources and their distribution in space suggest their separation into three parts as follows:

$$J(\mathbf{r}, t, E) = J_G(\mathbf{r}, E) + J_L(\mathbf{r}, t, E) + J_{NS}(\mathbf{r}, E).$$

Here

- J_G is the global spectrum component determined by the multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources;
- J_L is the local component, i.e. the contribution nearby ($r < 1$ kpc) young ($t < 10^6$ yr) sources;
- J_{NS} is the flux of non-scattered particles.

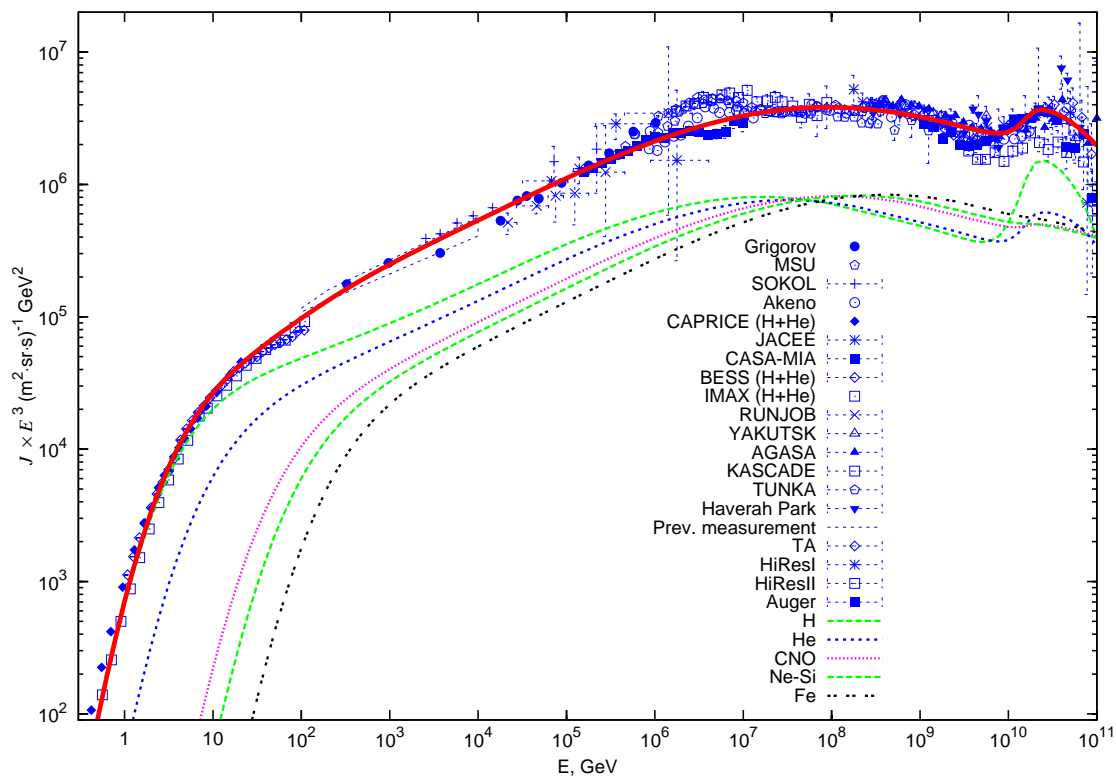


Figure 2. Comparison of our calculations of primary cosmic ray spectra with experimental data in wide energy range of $10^{10} \div 10^{20}$ eV. Curves marked 'H', 'He', 'CNO', 'Ne-Si', 'Fe' and 'Total' represent our results.

The flux of non-scattered particles J_{NS} is determined by the injected flux

$$S_0 E^{-\gamma} \exp(-E/E_0(z))$$

and the Levy flight probability $p(E, > r)$. Taking into account that for a particle with energy E the probability $p(E, > r) \sim A(E, \alpha) r^{-\alpha}$, $A(E, \alpha) \sim E^{\delta_L}$, we have

$$J_{NS}(r, E) = S_{NS} E^{-\gamma+\delta_L} \exp(-E/E_0(z)) r^{-\alpha}.$$

Since $J_{NS} \sim r^{-\alpha}$, $\alpha > 1$, we assume that this component is also formed by nearby ($r < 1$ kpc) sources, defining the spectrum in ultrahigh energy region, and provides the observed flattening of the spectrum at $E \geq 10^{18}$ eV.

Finally, for the global stationary component formed by old distant sources and the local component from the nearest sources the expression obtained by the solution of anomalous

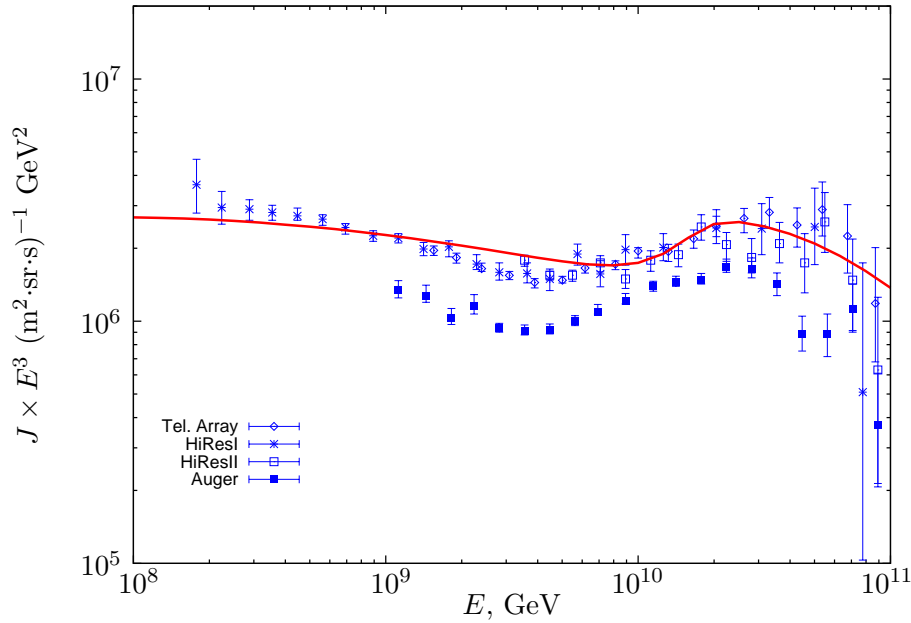


Figure 3. Comparison of our calculations of primary cosmic ray spectra with experimental data of Telescope Array/HiRes [12] and Auger [13] at ultrahigh energies.

diffusion equation is as follows:

$$J(\mathbf{r}, t, E) = \frac{v}{4\pi} \left[S_s E^{-\gamma-\delta/\beta} + \frac{S_{im} E^{-\gamma}}{D(E, \alpha, \beta)^{3/\alpha}} \times \right. \\ \times \sum_{\substack{r_j < 1 \text{ kpc} \\ t_j < 10^6 \text{ yr}}} \int_{\max[0, t_j - T]}^{t_j} d\tau \tau^{-3\beta/\alpha} \Psi_3^{(\alpha, \beta)} \left(|\mathbf{r}_j| (D(E, \alpha, \beta) \tau^\beta)^{-1/\alpha} \right) + \\ \left. + S_{NS} \sum_{r_j < 1 \text{ kpc}} E^{-\gamma+\delta_L} |\mathbf{r}_j|^{-\alpha} \right] \exp \left(-\frac{E}{E_0(z)} \right). \quad (5)$$

In (5) the first term is the contribution from the multiple old distant sources, the second term is the contribution from nearby young sources and the third term is the flux of non-scattered particles. To calculate the energy spectrum from nearby young sources, simulation of the Poisson ensemble of sources was carried out [14]. The Poisson distribution parameter (average number of sources in the local region) was chosen ~ 10 . This estimation corresponds to number of the well-known nearest supernova remnants and pulsars with $t < 10^6$ yr. Coordinates and times of birth of the sources were generated randomly and uniformly in the space region $r < 10^3$ pc and in the time interval $10^4 \leq t < 10^6$ yr.

The parameters of the model are $\gamma = 2.85$, $\delta = 0.27$ as it was established earlier [6], $\delta_L = \delta/2$. The cut-off energy $E_0(z) = 4 \times 10^{10} z$ GeV was fixed on the basis of comparisons with experimental data. To assess parameters $\alpha = 1.4$, $\beta = 0.8$, which are specific for our model, the results of studies of particles diffusion in cosmic and laboratory plasma have been used [15, 16]. Finally, for the anomalous diffusivity we get the value of $D_0 \approx 10^{-4} \text{ pc}^{1.4} / \text{yr}^{0.8}$ [9]. The emission time $T = 10^4$ yr.

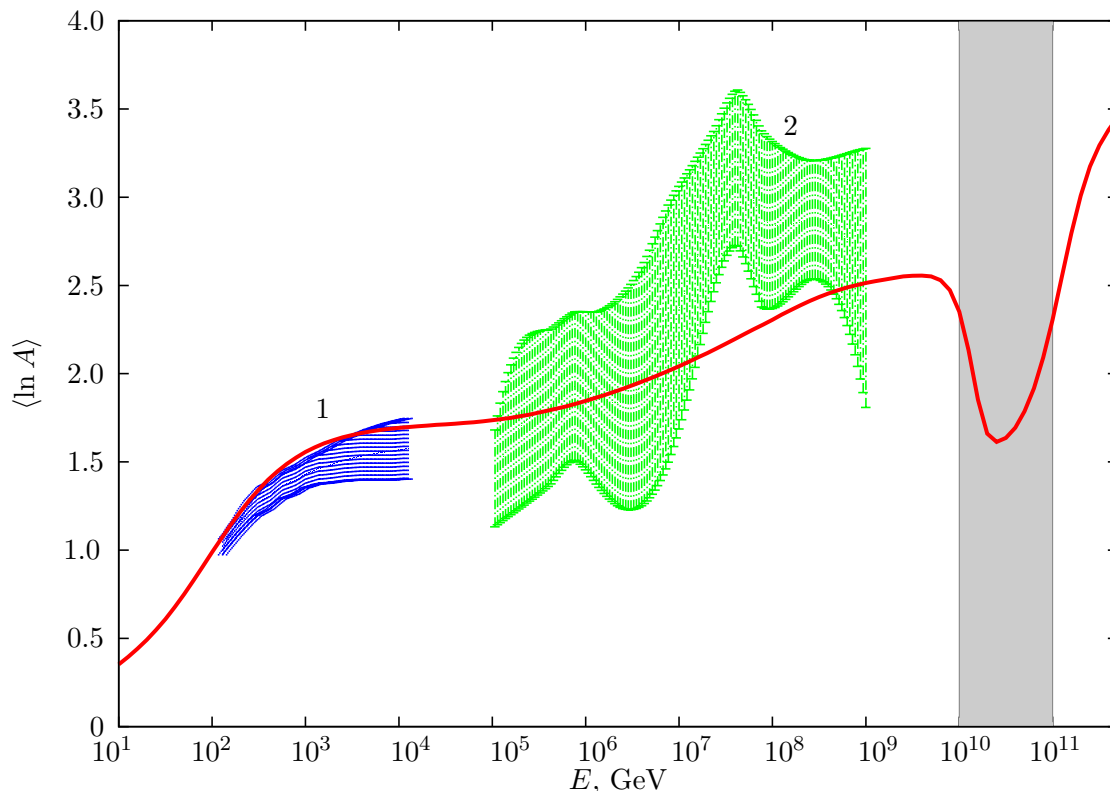


Figure 4. Mean logarithmic cosmic ray mass vs. primary particle energy. Dashed areas — experimental data [19], [20].

Our results of calculations of energy spectra of cosmic rays are shown in figures 2, 3. Note, that the “fine structure” of spectrum around the knee can be described satisfactorily in the framework of the model with two types of sources (see [17]).

An average mass number variation with energy according to our model is shown in figure 4. It was established that in the region of $(2 \div 5) \times 10^{19}$ eV the fraction of protons is $\sim 40\%$, while the fraction of iron is $\sim 15\%$. A key feature of the model is the essential dip in mean logarithm of the mass number in the region of $(4 \div 25) \times 10^{18}$ eV (from 2.6 to 1.6). The weighting of the mass composition is predicted at $E > 2.5 \times 10^{19}$ eV and $\langle \ln A \rangle$ reaches ~ 3 at $E = 2 \cdot 10^{20}$ eV. The presence of such a dip according to the experimental data could be construed in favour of our model. Note, that recent results of Auger Collaboration exhibit similar feature in $\langle \ln A \rangle$ variation with energy, although at an order of magnitude lower energies [18].

4. Conclusions

Assuming the Galactic origin of cosmic rays we propose a mechanism for the formation of ultra-high energy protons and nuclei spectra observed in the Solar System, that reproduces all the basic features of cosmic ray spectrum observed experimentally.

It is shown that taking into account both the diffusion contribution from nearby young and old distant sources and also the contribution of non-scattered radiation, due to the presence of large free paths (Levy flights) caused by anomalous diffusion, a description of the experimental data in the energy range $10^{10} \div 10^{20}$ eV can be obtained if we assume that the particle injection

spectrum in the Galactic sources is

$$S = S_0 E^{-2.85} \exp(-E/E_0(z)), \quad E_0(z) = 4 \times 10^{10} z \text{ GeV}.$$

The following basic model predictions are expressed:

- the steepening of all particles spectrum in the range $E > 5 \times 10^{19}$ eV is due to the proton flux cut-off caused by energy limitation of the Galactic particle accelerators;
- at the energies $2 \div 5 \times 10^{19}$ eV the fraction of protons is ~ 40 %, and the iron fraction is about 15 % (normal composition);
- in the range $4 \div 25 \times 10^{18}$ eV mean logarithmic cosmic ray mass $\langle \ln A \rangle$ decreases rapidly from 2.6 to $\langle \ln A \rangle_{\min} \sim 1.6$;
- considerable weighting of the mass composition is observed at $E > 2.5 \cdot 10^{19}$ eV; for $E = 2 \cdot 10^{20}$ eV mean logarithmic cosmic ray mass is $\langle \ln A \rangle \sim 3$.

Acknowledgement

This work was supported by the Russian Foundation for Basic Research grant No. 14-02-31524. The authors are greatly indebted to anonymous referee for valuable comments and suggestions that helped improve the manuscript.

References

- [1] Abbasi R U et al 2008 *Phys. Rev. Lett.* **100** 101101
- [2] Abraham J et al 2008 *Phys. Rev. Lett.* **101** 061101
- [3] Aloisio R, Berezhinsky V, Gazizov A 2011 *Astropart. Phys.* **34** 620
- [4] Calvez A, Kusenko A, Nagataki S 2010 *Phys. Rev. Lett.* **105** 091101
- [5] Fang K, Kotera K, Olinto A V 2013 *JCAP* **03** 010.
- [6] Lagutin A A, Nikulin Yu A, Uchaikin V V 2001 *Nucl. Phys. B* **97** 267
- [7] Lagutin A A, Uchaikin V V 2003 *NIM B* **B201** 212
- [8] Erlykin A D, Lagutin A A, Wolfendale A W 2003 *Astropart. Phys.* **19** 351
- [9] Lagutin A A and Tyumentsev A G 2004 *Izv. Altai. Gos. Univ.* **35** 4 (in Russian)
- [10] Samko S G, Kilbas A A and Marichev O I 1993 *Fractional integrals and derivatives — Theory and Applications* (New York: Gordon and Breach) 976 p.
- [11] Uchaikin V V and Zolotarev V M 1999 *Chance and Stability* (Netherlands, Utrecht: VSP) 594 p.
- [12] Abu-Zayyad T, Aida R, Allen M (Telescope Array Experiment) 2013 *Astrophys. Journ. Lett.* **768:L1** 1
- [13] Abraham J, Abreu P, Aglietta M (Pierre Auger Collaboration) 2010 *Phys. Lett. B* **685** 239
- [14] Lagutin A A, Volkov N V and Tyumentsev A G 2013 *Bulletin of the Russian Academy of Sciences: Physics* **77** 1312
- [15] Greco A, Taktakishvili A L, Zimbardo G and et al 2003 *J. Geophys. Res.* **108** 1
- [16] Perri S and Zimbardo G 2008 *J. Geophys. Res.* **113**
- [17] Lagutin A A, Tyumentsev A G, Yushkov A V 2008 *Nucl. Phys. B (Proc. Suppl.)* **175-176** 555
- [18] Aab A, Abreu P, Aglietta M (Pierre Auger Collaboration) 2014 *Phys. Rev. D* **90** 122005
- [19] Shibata T 1999 *Nucl. Phys. B* **75A** 22
- [20] Hörandel J R 2003 *Astropart. Phys.* **19** 193