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Neutrino masses from SUSY breaking in radiative seesaw models

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Abstract. We study the possibility that neutrino masses (m_ν) are proportional to small supersymmetry (SUSY) breaking effects that take place within the sector responsible for generating m_ν at the quantum level. We specifically consider the case in which lepton number (L-number) breaking is independent of SUSY breaking ($\not{\text{SUSY}}$), so that this connection can be seen to be a consequence of the SUSY non-renormalisation theorem. We show that the simplest one-loop models realising this idea generate $LLHH$ operators suppressed by at least $\mu\, m_{\text{soft}}/M^3$ or m_{soft}^2/M^3, where m_{soft} is the scale of soft-$\not{\text{SUSY}}$ effects involving the seesaw mediators, which possess superpotential masses of order M (the L-number breaking scale). We exemplify this possibility by constructing one such model based on a one-loop type-II seesaw. This contribution summarises the work published in [1].

1. Introduction

The idea that neutrino masses (m_ν) may owe part of their smallness to small supersymmetry (SUSY) breaking effects was put forward in [2], where it relied on the observation that the R-parity conserving minimal SUSY Standard Model (MSSM) can generate non-vanishing neutrino masses if one takes into account hard SUSY breaking ($\not{\text{SUSY}}$) terms involving slepton doublets and Higgses, as for instance

$$\frac{m_{\text{soft}}}{M_X}(\tilde{L}H_u)^2 \subset \frac{1}{M_X^2}\int d^2\theta \hat{X}(\hat{L}\hat{H}_u)^2\,, \tag{1}$$

where M_X is the underlying scale mediating $\not{\text{SUSY}}$ ($\langle\hat{X}\rangle \sim F_X\theta^2$) to the visible sector, and $m_{\text{soft}} \sim F_X/M_X$ is the soft-$\not{\text{SUSY}}$ scale. Although non-standard $\not{\text{SUSY}}$ terms are typically neglected in phenomenological analysis due to their expected smallness, this is a feature that could conceivably be responsible for $m_\nu/v \ll 1$. For example, if $\not{\text{SUSY}}$ generates $(\tilde{L}H_u)^2$, then LLH_uH_u arises at the one-loop level via a gaugino-slepton loop and is suppressed by m_{soft}/M_X.

A shortcoming of the idea presented in the previous paragraph is that it does not explain why the more fundamental model is able to generate the (s)lepton number violating superfield operator (superoperator) $\hat{X}(\hat{L}\hat{H}_u)^2$ while being unable to generate SUSY conserving superoperators that, if generated, would expectedly provide dominant contributions to m_ν. For example, given $\hat{X}(\hat{L}\hat{H}_u)^2 \subset \mathcal{W}_{\text{eff}}$ one would arguably expect that a generic model would also generate

$$\frac{1}{M_X}LLH_uH_u \subset \frac{1}{M_X}\int d^2\theta(\hat{L}\hat{H}_u)^2\,. \tag{2}$$

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A possible strategy to fix this shortcoming is to consider that $\hat{X}$ carries $\hat{L}$-number, so that $\hat{X}(\hat{L}\hat{H}_u)^2$ is symmetric, and is thus allowed, while $(\hat{L}\hat{H}_u)^2$ is forbidden. This kind of strategy in which the $\not{\mathrm{SUSY}}$ sector is postulated to share a visible sector symmetry is followed in [3]. A different strategy is to postulate a visible sector symmetry that forbids chiral superoperators while allowing non-chiral. Since non-chiral superoperators contribute to the effective action as D-terms, their contribution to neutrino masses is necessarily proportional to $\not{\mathrm{SUSY}}$. For example, the authors of [4] consider an extra $U(1)$ symmetry that allows

$$\frac{m_{\rm soft}}{M_X} L H_d^\dagger N \subset \frac{1}{M_X^2}\int d^4\theta \hat{X}^\dagger \hat{L}\hat{H}_d^\dagger \hat{N} \tag{3}$$

but forbids the non-suppressed $LH_uN \subset \int d^2\theta \hat{L}\hat{H}_u\hat{N}$.

In the aforementioned works the connection between $\not{\mathrm{SUSY}}$ and m_ν results from making $\not{\mathrm{SUSY}}$ the unique provider of:

i) L-number violating couplings suitable for a radiative seesaw with just the superfield content of the MSSM [2] or for a canonical type-I seesaw [3]; or
ii) couplings for Dirac neutrino masses [3, 4].

An alternative arises when one considers models in which neutrino masses are radiatively generated. In such models, the connection between $\not{\mathrm{SUSY}}$ and m_ν is a consequence of the SUSY non-renormalisation theorem, so that one may have all the required couplings at the SUSY level while the radiative structure of a SUSY QFT precludes the generation of neutrino masses. Then, one may have L-number breaking at the superpotential level, i.e. independently of $\not{\mathrm{SUSY}}$, and neutrino masses will still be proportional to $\not{\mathrm{SUSY}}$. This dependence on $\not{\mathrm{SUSY}}$ is not entirely obvious in component fields calculations. To illustrate what we mean, consider the trilinear R-parity violation contribution to neutrino masses [5], which is generated at the one-loop level. It takes the schematic form

$$\frac{m_\nu}{v^2} \sim \frac{A - \mu\, c_\xi\, t_\beta}{M^2}\,, \tag{4}$$

where M is the loop mass scale, $\mu\, c_\xi$ is the effective μ-term and A stands for holomorphic soft-$\not{\mathrm{SUSY}}$ trilinears. From this expression, one may erroneously think that the $\propto \mu/M^2$ contribution is a SUSY conserving one, since μ is a superpotential mass term. However, a closer inspection reveals that the μ-term contribution comes from the vacuum expectation value (VEV) of an F-term, specifically $\langle F^\dagger_{H_d}\rangle = \mu\langle H_u\rangle$, and is thus intrinsically $\not{\mathrm{SUSY}}$. This is not that surprising since in the SUSY limit one has $\langle H\rangle = 0$ and therefore $\langle H_{u,d}\rangle = v_{u,d}$ are proportional to $\not{\mathrm{SUSY}}$. This suggests that $\not{\mathrm{SUSY}}$ contributions can be classified with respect to (w.r.t.) their involvement in electroweak symmetry breaking (EWSB) as follows: $\not{\mathrm{SUSY}}_{\rm EWSB}$ contributions are those which involve $\not{\mathrm{SUSY}}$ VEVs of the form

$$\langle F^\dagger\rangle = \sum_H \mu_H \langle H\rangle + \sum_H \lambda_H \langle HH'\rangle \neq 0 \;\; \text{or} \;\; \langle D\rangle = g\sum_H \langle H^\dagger \otimes_H H\rangle \neq 0\,, \tag{5}$$

where Hs are fields whose VEVs break the electroweak symmetry (EWS); while $\not{\mathrm{SUSY}}_{\rm EWS}$ contributions correspond to those in which at least one $\not{\mathrm{SUSY}}$ VEV is unrelated to EWSB.

In this work, we will start by analysing the space of radiative seesaw models w.r.t. their dependence on $\not{\mathrm{SUSY}}$. In order to do this in the most efficient manner, we will use supergraph techniques[1] and incorporate the $\not{\mathrm{SUSY}}$ effects by means of considering couplings to external

[1] A detailed introduction to supergraph techniques can be found in chapter 6 of [6].

$\cancel{\text{SUSY}}$ spurions [7]. This will allow one to see the $\cancel{\text{SUSY}}_{\text{EWS}}$ contributions to neutrino masses as small $\cancel{\text{SUSY}}$ effects upon a fundamentally SUSY topology.

Although L-number violation can possibly arise from $\cancel{\text{SUSY}}$, i.e. from the VEV of an auxiliary rather than scalar field, in here we assume that they are broken separately so that the non-renormalisation theorem is the only bridge between m_ν and $\cancel{\text{SUSY}}$. We thus assume that the radiative seesaw models are realised in the superpotential at a L-number breaking scale M that is higher than the scale of soft-$\cancel{\text{SUSY}}$ effects involving the seesaw mediators. We will furthermore restrict ourselves to models with the low energy Higgs sector of the MSSM.

As we will see in section 2, contributions to neutrino mass operators whose dependence on $\cancel{\text{SUSY}}$ arises entirely by means of $\cancel{\text{SUSY}}$ sources involved in EWSB are expected to be suppressed by some power of μ/M or be of dimension higher than 5 and involve gauge couplings. Thus, in a generic radiative seesaw model one expects m_ν to have the schematic dependence

$$\frac{m_\nu}{v^2} \sim \frac{\mu}{M^2} \oplus \frac{g^2 v^2}{M^3} \oplus \frac{m_{\text{soft}}}{M^2}\,, \tag{6}$$

where m_{soft}/M^2 corresponds to conceivable $\cancel{\text{SUSY}}_{\text{EWS}}$ contributions. Exploiting the power of the SUSY non-renormalisation in the space of radiative seesaw models, we then investigate if models exist in which the pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ contribution either vanishes or is subleading w.r.t. the contribution from $\cancel{\text{SUSY}}_{\text{EWS}}$. Since in such a case

$$\frac{m_\nu^{\text{LO}}}{v^2} \sim \frac{m_{\text{soft}}}{M^2}\,, \tag{7}$$

where m_{soft} is the scale of soft-$\cancel{\text{SUSY}}$ effects involving the mediators of the radiative seesaw, models in which the leading contribution comes from soft-$\cancel{\text{SUSY}}_{\text{EWS}}$ offer the possibility of explaining the smallness of m_ν with M in the vicinity of the TeV scale provided $m_{\text{soft}}/M \ll 1$. It should be noted that the smallness of m_{soft} is not constrained by lower-limits on the mass of new particles because the fields upon which these m_{soft}-effects are felt possess superpotential masses of order M. We catalogue one-loop topologies for such models in section 3. An explicit model example is presented in section 4 and consists of a one-loop type-II seesaw whose leading pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ contribution is of dimension-7 – comprising contributions $\propto \mu/M$ and $\propto g^2$ –, whereas the leading contribution from $\cancel{\text{SUSY}}_{\text{EWS}}$ is of dimension-5 and has the dimensionful dependence $\mu\, m_{\text{soft}}/M^3$ or m_{soft}^2/M^3, the latter corresponding to pure-$\cancel{\text{SUSY}}_{\text{EWS}}$ contributions.

2. Radiative seesaws in SUSY: understanding the different $\cancel{\text{SUSY}}$ contributions

In order to understand the $\cancel{\text{SUSY}}$ structure of radiatively generated neutrino mass operators (OP_ν), it is first convenient to notice that for every $\text{OP} \in \text{OP}_\nu$ there exists a superoperator $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$ such that $\text{OP} \subset \int d^4\theta\, \widehat{\text{OP}}$. Then, since any $\text{OP} \in \text{OP}_\nu$ is of the form $\text{OP} = LL \otimes \text{Higgses}$, one arrives at the conclusion that $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$ belongs to one of two classes:

$$\begin{gathered} D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \hat{A} \quad \text{or} \quad \hat{L}\hat{L} \otimes \hat{B}\,, \\ \text{with} \quad \int d^4\theta \hat{A} \supset \text{Higgses}\,, \qquad \int d^2\bar{\theta} \hat{B} \supset \text{Higgses}\,, \end{gathered} \tag{8}$$

and where $n = 0, 1, ...$ stands for conceivable insertions of superfields that yield Higgses at $\theta = 0$ (a limit hereafter denoted by $|$). Now, if we assume that only scalar and gauge vector superfields exist, we can write

$$\begin{aligned} \hat{A} &\in \hat{a} \otimes \left\{ \hat{H}, \hat{H}^\dagger, D^2\hat{Z}, \bar{D}^2\hat{Z}^\dagger, D^2\bar{D}^2\hat{V} \right\}^n\,, \\ \hat{B} &\in \hat{b}^\dagger \otimes \left\{ \hat{H}, \hat{H}^\dagger, D^2\hat{Z}, \bar{D}^2\hat{Z}^\dagger, D^2\bar{D}^2\hat{V} \right\}^n\,, \end{aligned} \tag{9}$$

where $n = 0, 1, ...$ stands for arbitrary insertions of superfields within the given set (denoted by curly braces), though constrained by internal symmetries. $\hat{a}$ and $\hat{V}$ (mod $\hat{H}^\dagger, \hat{H}$) ($\hat{b}^\dagger$ and $\hat{Z}^\dagger$ (mod $\hat{H}^\dagger$)) are real (anti-chiral) scalar superfields whose D (F) component is a constant or a product of Higgses[2]. We show in figure 1 characteristic examples of class A and B superoperators.

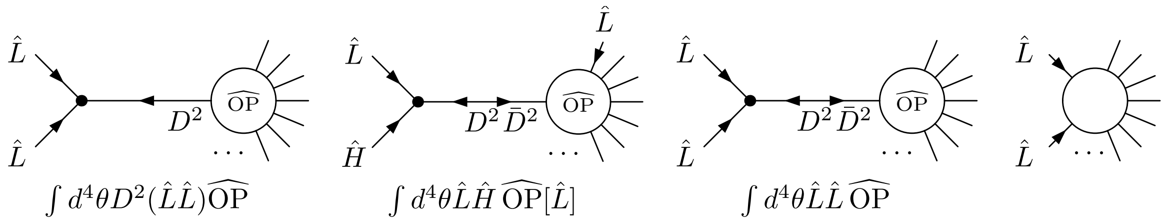

Figure 1. Characteristic examples of supergraph topologies for radiative seesaws: type-II without a chirality flip (class A), type-I and -III, type-II with a chirality flip and 1PI seesaw, respectively.

2.1. Pure-~~SUSY~~$_{EWSB}$ contributions

Pure-~~SUSY~~$_{\mathrm{EWSB}}$ contributions are generated by superoperators containing superfields whose auxiliary fields yield Higgses upon being integrated out. We have two possibilities:

$$\bar{D}^2 D^2 \hat{V}\Big| = D \supset g H^\dagger \otimes H \,, \tag{10}$$

where $\hat{V}$ is the gauge vector superfield of any symmetry under which Higgses are charged; and

$$\bar{D}^2 \hat{Z}^\dagger\Big| = F_Z^\dagger \supset \mu H \text{ or } \lambda H \otimes H' \,, \tag{11}$$

where $\hat{Z}^\dagger$ is any anti-chiral scalar superfield that has a bilinear with an Higgs or a trilinear with two Higgses. For class B superoperators (cf. Eq. (9)), $\hat{b}^\dagger$ is thus $\hat{Z}^\dagger$ or the anti-chiral projection of $\hat{V}$ ($D^2\hat{V}$), while, for class A, $\hat{a}$ is $\hat{V}$ or the real product of $\hat{b}$ ($\hat{b}^\dagger\hat{b}$).

Now, under the phenomenologically reasonable assumption of a superpotential mass term for $\hat{Z}$, the contribution of a trilinear with two Higgses adds up to an overall derivative term of the form $\square(HH')$. Moreover,

$$\langle F_Z^\dagger \rangle = \mu_Z \langle \overline{Z} \rangle + \lambda \langle HH' \rangle = 0 \,, \tag{12}$$

up to ~~SUSY~~ effects $\propto (m^2_{\mathrm{soft}})_{\overline{Z}}/|\mu_Z|^2$. Therefore, the contribution that arises from a trilinear with two Higgses is more appropriately classified as a ~~SUSY~~$_{\mathrm{EWS}}$ contribution. (For a detailed discussion see the original work [1].)

Neutrino mass operators that come from a class A or B superoperator via a gauge vector superfield must have mass dimension higher than 5, since $\hat{V}$ is an hypercharge singlet. The least is a dimension-6 operator

$$\int d^4\theta \left\{ \hat{V} D^2(\hat{L}\hat{L}) \,, D^2\hat{V}\hat{L}\hat{L} \right\} \otimes \hat{H}' \supset LLH^\dagger HH' \,, \tag{13}$$

that is conceivable if there exists a hypercharge +1 Higgs (H'). On the other hand, if the low energy Higgs sector coincides with that of the MSSM, the leading pure-~~SUSY~~$_{\mathrm{EWSB}}$ contributions that are independent of $\langle F_Z^\dagger \rangle$ correspond to the dimension-7 operators

$$LL \otimes \left\{ H_u H_u \,, H_u H_d^\dagger \,, H_d^\dagger H_d^\dagger \right\} \otimes \left\{ H_u^\dagger H_u \,, H_d^\dagger H_d \right\} \,. \tag{14}$$

[2] Here and throughout the text, "mod X" means modulo insertions of X.

Since realistic SUSY models have Higgs bilinears, be them dynamically generated or otherwise, it is conceivable that in general models there are pure-$\not{\text{SUSY}}_{\text{EWSB}}$ contributions to $LLHH$. While pure-$\not{\text{SUSY}}_{\text{EWSB}}$ contributions are indeed guaranteed to exist in any model (see section 2.3), there are models in which they appear only at an higher order of perturbation theory, while the leading order (LO) contribution is proportional to $\not{\text{SUSY}}_{\text{EWS}}$.

2.2. $\not{SUSY}_{EWS}$ contributions

$\not{\text{SUSY}}_{\text{EWS}}$ contributions are most conveniently understood by means of employing spurionic superfields: objects that are superfields at the formal level but that have constant θ-dependent values. The underlying idea is to perform perturbation theory in superspace while considering possible couplings to spurions (thus parameterising $\not{\text{SUSY}}$) in order to obtain the effective superoperators and see whether the θ-integration projects the spurions in a way that $\int d^4\theta\, \widehat{\text{OP}} \supset \text{OP} \in \text{OP}_\nu$. We can thus identify three cases:

$$\text{a)} \int d^4\theta\, \hat{X}\, \widehat{\text{OP}}\,, \qquad \text{b)} \int d^4\theta\, \hat{X}^\dagger\, \widehat{\text{OP}}\,, \qquad \text{c)} \int d^4\theta\, \hat{Y}\, \widehat{\text{OP}}\,, \tag{15}$$

modulo $D^2\hat{X}$, $\bar{D}^2\hat{X}^\dagger$ and $D^2\bar{D}^2\hat{Y}$ insertions, and where $\hat{X}$ and $\hat{Y}$ are F- and D-term $\not{\text{SUSY}}$ spurions, respectively.

In this work we are strictly interested in the case in which the $\not{\text{SUSY}}$ spurions are neutral under any symmetry of visible sector superfields. In such a framework, one expects that both $\{\hat{X}, \hat{X}^\dagger, \hat{Y}\}\widehat{\text{OP}}$ (cases a, b and c, respectively) and $\widehat{\text{OP}}$ are generated up to some order in perturbation theory. For example, if a model generates

$$\frac{1}{M^2}\int d^4\theta \left(\frac{\hat{X}^\dagger}{M_X}\right) \hat{L}\hat{L}\hat{H}_u\hat{H}_d^\dagger \supset \frac{m_{\text{soft}}}{M^2} LLH_uH_d^\dagger\,, \tag{16}$$

then it would most certainly also generate

$$\frac{1}{M^2}\int d^4\theta \hat{L}\hat{L}\hat{H}_u\hat{H}_d^\dagger \supset \frac{\mu}{M^2} LLH_uH_u\,. \tag{17}$$

We can now ask ourselves which instances of $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$ do not yield an $\text{OP} \in \text{OP}_\nu$ in the absence of $\not{\text{SUSY}}$ spurions[3]. (This is the operational definition of superoperators that only give $\text{OP} \in \text{OP}_\nu$ from $\not{\text{SUSY}}_{\text{EWS}}$.) The general answer is:

$$\begin{aligned} &1.\ \widehat{\text{OP}} = D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \Big(\text{a superoperator whose } D\text{-term is zero at } p_{\text{ext}} = 0\Big)\,; \\ &2.\ \widehat{\text{OP}} = \hat{L}\hat{L} \otimes \Big(\text{a superoperator whose } F^\dagger\text{-term is zero at } p_{\text{ext}} = 0\Big)\,. \end{aligned} \tag{18}$$

For example, consider $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$: $\hat{X}^\dagger\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ yields an $\text{OP} \in \text{OP}_\nu$, namely LLH_uH_u, whereas $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ does not. For a non-trivial example, consider $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$ (cf. Eq. (31)).

In the following we give the general form of type-1 and -2 superoperators. For this, let $\hat{Z}^\dagger$ and $\hat{V}$ denote any superfields whose $\hat{Z}^\dagger$ (mod $\hat{H}^\dagger$) and $\hat{V}$ (mod $\hat{H}, \hat{H}^\dagger$) parts satisfy Eqs. (11) and (10), respectively. Type-1 and -2 superoperators that only give $\text{OP} \in \text{OP}_\nu$ are then:

$$1.\text{a)}\ D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \Big\{\hat{Z}^\dagger, D^2\hat{V}\Big\} \otimes \Big\{\hat{H}^\dagger, \bar{D}^2\hat{Z}^\dagger, D^2\hat{Z}, D^2\bar{D}^2\hat{V}\Big\}^{n'}\ ;$$

$$1.\text{b)}\ D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \Big\{\hat{Z}, \bar{D}^2\hat{V}\Big\} \otimes \Big\{\hat{H}, \bar{D}^2\hat{Z}^\dagger, D^2\hat{Z}, D^2\bar{D}^2\hat{V}\Big\}^{n'}\ ;$$

$$1.\text{c)}\ D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \Big\{(\hat{H}^\dagger)^k, (\hat{H})^k\Big\} \otimes \Big\{\bar{D}^2\hat{Z}^\dagger, D^2\hat{Z}, D^2\bar{D}^2\hat{V}\Big\}^{n'}\ ;$$

[3] To simplify the discussion, from now on any $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$ is defined modulo $\not{\text{SUSY}}$ insertions.

$$2.\text{b})\ \hat{L}\hat{L} \otimes \left\{\hat{H}, D^2\hat{Z}, \bar{D}^2\hat{Z}^\dagger, D^2\bar{D}^2\hat{V}\right\}^n ; \tag{19}$$

where $n, n', k = 0, 1, \ldots$ stand for any number of insertions, though constrained by internal symmetries.

2.3. Are there models in which the pure-$\cancel{SUSY}_{EWSB}$ subset of OP_ν is empty?

First, let us analyse possible contributions of gauge vector superfields. As there are $U(1)_Y$ and $SU(2)_L$ charges flowing in internal lines of any supergraph contributing to $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$, it seems natural to think that insertions of external $\hat{V}_{U(1)_Y}$ and $\hat{V}^\alpha_{SU(2)_L}$ into internal lines can map any superoperator to one which yields a pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ $\text{OP} \in \text{OP}_\nu$. An example is

$$D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u \to D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{V}_{U(1)_Y} , \tag{20}$$

as we will see in the context of the model example presented in section 4. Although supergraphs with any given number of external $\hat{V}$s can be constructed from any underlying $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$, the so obtained $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$ may vanish as the supergraphs add up to zero. More generally, $\hat{V}$s insertions can be seen to correspond to terms in the $\hat{V}$-expansion of gauge completed superoperators. For example, $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{V}_{U(1)_Y}$ is a term in the $\hat{V}$-expansion of

$$D^2(\hat{L}\hat{L}e^{-2g'Y_L\hat{V}_{U(1)_Y}})\hat{H}_u\hat{H}_u e^{-2g'Y_{H_u}\hat{V}_{U(1)_Y}} . \tag{21}$$

Next, we will see that for models in which there exists a Higgs bilinear, there is always a pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ contribution proportional to μ. First, pick a $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$. Now, each supergraph contributing to $\widehat{\text{OP}}$ has either at least one external Higgs locally connected to loop superfields (say, class-a) or none (class-b). Without loss of generality, say that for a particular supergraph belonging to class-a the vertex is $\hat{H}\hat{X}_1\hat{X}_2$, where $\hat{X}$s are loop superfields. One can then see (cf. figure 2) that an insertion of $\hat{H}^\dagger$ ($\hat{H}$) followed by an insertion of $\hat{H}$ ($\hat{H}^\dagger$) leads to a supergraph for the superoperator

$$\hat{H}^\dagger\hat{H}\,\widehat{\text{OP}} . \tag{22}$$

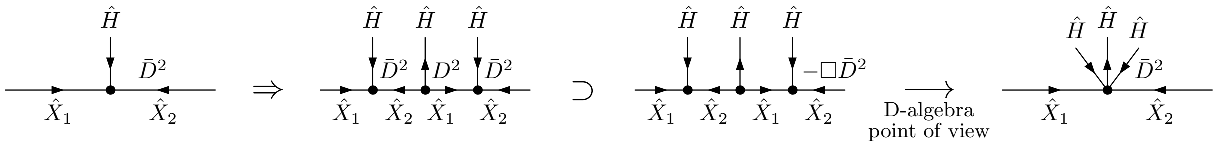

Figure 2. A $\hat{H}\hat{X}_1\hat{X}_2$ vertex (leftmost diagram) implies a non-vanishing $\hat{H}\hat{H}^\dagger\hat{H}\hat{X}_1\hat{X}_2$ interaction that is local in θ, i.e. "a vertex" from the D-algebra point of view (rightmost diagram).

By means of the procedure described in figure 3, each class-b supergraph can also be transformed into a supergraph for $\hat{H}^\dagger\hat{H}\,\widehat{\text{OP}}$.

Hence, if class-a or -b supergraphs for superoperator $\widehat{\text{OP}}$ do not add up to zero, the transformed ones do not add up to zero for $\hat{H}^\dagger\hat{H}\,\widehat{\text{OP}}$ either. Now, if there exists a Higgs bilinear, $\hat{H}^\dagger\hat{H}\,\widehat{\text{OP}}$ yields a pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ $\text{OP} \in \text{OP}_\nu$ regardless of $\widehat{\text{OP}} \in \widehat{\text{OP}}_\nu$. One expects such contributions to m_ν to be proportional to

$$\lambda^2 \left(\frac{\mu}{M_X}\right)^{2 \text{ or } 1} , \tag{23}$$

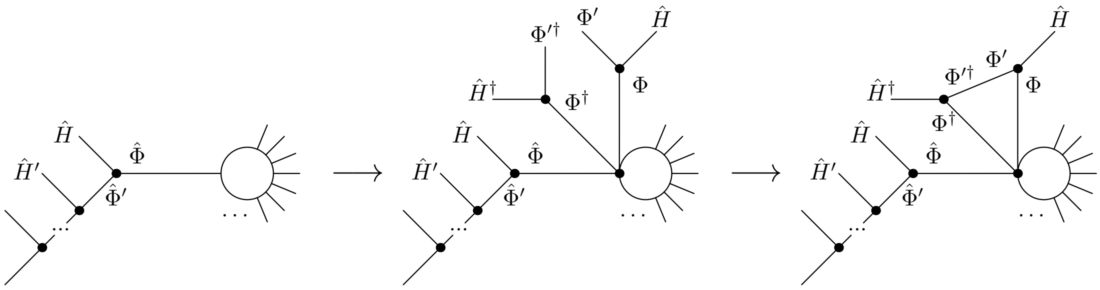

Figure 3. Schematic of a procedure to go from a class-b supergraph for $\widehat{\mathrm{OP}}$ (leftmost diagram) to a supergraph for $\hat{H}^\dagger\hat{H}\,\widehat{\mathrm{OP}}$ (rightmost diagram) by means of a double insertion in the loop line to which the 1PR leg is attached (middle diagram). The dot at which the lines of $\hat{\Phi}^\dagger$ and the two $\hat{\Phi}$s meet is a vertex in the sense of figure 2. In order to describe all conceivable assignments of chiralities to external and internal superfields, the chiralities of $\hat{H}$, $\hat{H}'$, $\hat{\Phi}$ and $\hat{\Phi}'$ are left unspecified. However, $\hat{H}$, $\hat{\Phi}$ and $\hat{\Phi}'$ have the same chirality, as is implied by the vertex. Moreover, and so that all conceivable propagators are described, we also do not specify how $\hat{\Phi}$ is connected to the loop(s) (depicted by the circle), nor how $\hat{\Phi}'$ is connected to $\hat{H}'$.

for class A or B superoperators, respectively, and where λ is the coupling strength of $\hat{H}$s to the loop(s). We will see an example of this in the model example of section 4, where an insertion of $\hat{H}^\dagger\hat{H}$ into the LO topologies leads to dimension-7 contributions proportional μ/M^4 and μ^2/M^5 (cf. Eq. (34)).

3. Models in which $m_\nu^{\mathbf{LO}}$ is proportional to soft-$\cancel{\mathrm{SUSY}}$ involving seesaw mediators

To construct models of this kind one may first choose a set of superoperators from those identified in Eq. (19), and then postulate the existence of superfields that generate the topologies for these superoperators. Finally, one should pick an internal symmetry group that forbids all couplings that could generate a pure-$\cancel{\mathrm{SUSY}}_{\mathrm{EWSB}}$ OP $\in$ OP$_\nu$ up to the same order of perturbation theory. We cannot think of any serious obstruction that would compromise this procedure for constructing general models of this kind. In fact, in the next section we give a proof of existence based on a one-loop type-II seesaw, also showing that this kind of models need not be complicated.

In here, we shall identify the simplest of such model-topologies. To be precise, we consider one-loop topologies that generate the lowest dimension superoperators that can only give OP $\in$ OP$_\nu$ from $\cancel{\mathrm{SUSY}}_{\mathrm{EWS}}$. From Eq. (19) one can see that the candidate superoperators are

$$\begin{aligned}
&1.\mathrm{a})\ D^2(\hat{L}\hat{L})\hat{H}_d^\dagger \otimes \left\{\hat{H}_d^\dagger, \bar{D}^2\hat{H}_d^\dagger, D^2\hat{H}_u\right\} \cup D^2(\hat{L}\hat{L}\hat{H}_u)\hat{H}_d^\dagger\,;\\
&1.\mathrm{b})\ D^2(\hat{L}\hat{L})\hat{H}_u \otimes \left\{\hat{H}_u, \bar{D}^2\hat{H}_d^\dagger, D^2\hat{H}_u\right\} \cup D^2(\hat{L}\hat{L}\hat{H}_u)\hat{H}_u\,;\\
&1.\mathrm{c})\ D^2(\hat{L}\hat{L}) \otimes \left\{D^2\hat{H}_u \otimes \left\{D^2\hat{H}_u, \bar{D}^2\hat{H}_d^\dagger\right\}, \bar{D}^2\hat{H}_d^\dagger\bar{D}^2\hat{H}_d^\dagger,\right.\\
&\qquad\qquad \left. D^2(\hat{H}_u\hat{H}_u), \bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)\right\} \cup D^2(\hat{L}\hat{L}\hat{H}_u\hat{H}_u) \cup 1.\mathrm{a} \cup 1.\mathrm{b}\,;\\
&2.\mathrm{b})\ \hat{L}\hat{L} \otimes \left\{\hat{H}_u \otimes \left\{\hat{H}_u, D^2\hat{H}_u, \bar{D}^2\hat{H}_d^\dagger\right\}, D^2\hat{H}_u \otimes \left\{D^2\hat{H}_u, \bar{D}^2\hat{H}_d^\dagger\right\},\right.\\
&\qquad\qquad \left.\bar{D}^2\hat{H}_d^\dagger\bar{D}^2\hat{H}_d^\dagger, D^2(\hat{H}_u\hat{H}_u), \bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)\right\}.
\end{aligned} \tag{24}$$

Then, from D-algebra considerations, and discarding topologies with self-energies[4], one obtains the following list of possibilities:

- $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$, $\hat{L}\hat{L}D^2(\hat{H}_u\hat{H}_u)$, $D^2(\hat{L}\hat{L})\hat{H}_d^\dagger\hat{H}_d^\dagger$ and $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$
  – type-II without a chirality flip;
- $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PR)
  – type-II with a chirality flip, type-I and -III;
- $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PI).

The corresponding supergraph topologies are depicted in figure 4.

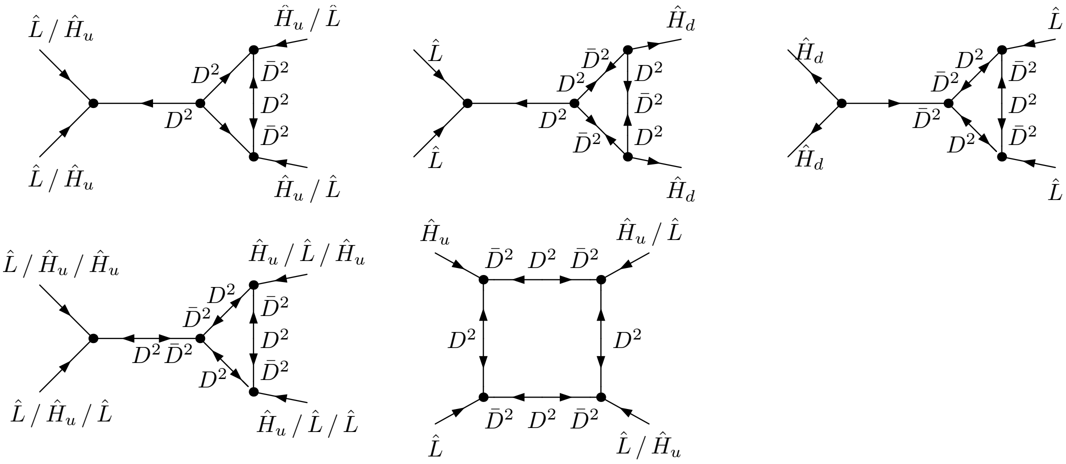

Figure 4. One-loop supergraph topologies that are identified in the text. From left to right: $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$ or $\hat{L}\hat{L}D^2(\hat{H}_u\hat{H}_u)$, $D^2(\hat{L}\hat{L})\hat{H}_d^\dagger\hat{H}_d^\dagger$, $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$, $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PR) and $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PI).

We calculated the contribution of these topologies to OP_ν up to order 3 in m_{soft}, finding three kinds of leading dimensionful suppression factors:

- $\mu\, m_{\mathrm{soft}}/M^3$ or m_{soft}^2/M^3 – for $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$ and $\hat{L}\hat{L}D^2(\hat{H}_u\hat{H}_u)$;
- $\mu\, m_{\mathrm{soft}}^2/M^4$ or m_{soft}^3/M^4 – for $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$;
- m_{soft}^2/M^3 – for $D^2(\hat{L}\hat{L})\hat{H}_d^\dagger\hat{H}_d^\dagger$ and $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (both 1PR and 1PI).

The absence of a contribution linear in m_{soft} for some topologies is most easily seen to stem from the fact that one-loop topologies for $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$, as well as the one-loop 1PI parts of $D^2(\hat{L}\hat{L})\hat{H}_d^\dagger\hat{H}_d^\dagger$ and $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$, use vertices of a single chirality. Moreover, and in regard to $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$, the leading contributions from the $\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$ piece are $\mu H_u H_d^\dagger$ and $A^* H_d^\dagger H_d^\dagger$.

We also found that one-loop realisations with self-energies had leading dimensionful suppression factors that ranged from $\mu\, m_{\mathrm{soft}}/M^3$ or m_{soft}^2/M^3 to $\mu\, m_{\mathrm{soft}}^2/M^4$ or m_{soft}^3/M^4.

If we take $\mu \sim m_{\mathrm{soft}}$, we may say that in one-loop models of this kind $LLHH$ operators have a dimensionful suppression of at least m_{soft}^2/M^3. This result is naively expected for type-II seesaws without a chirality flip, since $\int d^4\theta D^2(\hat{L}\hat{L})\hat{H}\hat{H}$ has mass dimension 7. For other realisations this dependence is not trivial, since for an underlying superoperator $\hat{L}\hat{L}\hat{H}\hat{H}$ one in general expects a m_{soft}/M^2 dependence. It is worthy to mention that at higher loops there are topologies that lead to the naively expected dimensionful suppression of m_{soft}/M^2. Indeed, the one-loop exception

[4] They are analysed in appendix C of [1].

can be seen to be a consequence of the fact that the holomorphy of the superpotential restricts the non-trivial 1PI parts of one-loop topologies in a way that all couplings and masses involved must have the same chirality.

4. A model example

To construct a model example we follow the recipe outlined at the beginning of the previous section and choose a model based on an one-loop topology for the superoperator $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$ (cf. figure 4). As the most general set of scalar superfields and superpotential terms in this topology has size 7 and 5, respectively, we can choose to supplement the model with a new $U(1)$ symmetry, say $U(1)_X$, that is carried by the superfields in the loop (say $\hat{X}$s). These will be responsible for communicating L-number breaking to the actual leptons via the exchange of a type-II seesaw mediator, say $\hat{\Delta}$.

In order to make the coupling $\hat{\Delta}^\dagger\hat{H}_u\hat{H}_u$ genuinely radiative, while allowing for a mass term for $\hat{\Delta}$, we have to link it to the VEV of a superoperator of at least dimension 4 in superfields. One simple example is

$$\hat{\rho}^\dagger\hat{\Delta}^\dagger\hat{H}_u\hat{H}_u \to \langle\rho^\dagger\rangle\hat{\Delta}^\dagger\hat{H}_u\hat{H}_u + \hat{\rho}^\dagger\hat{\Delta}^\dagger\hat{H}_u\hat{H}_u \,. \tag{25}$$

To achieve this we will postulate a $U(1)$ L-number symmetry that gets broken by the VEV of the scalar component of $\hat{\rho}$. We leave unspecified the actual mechanism driving $\langle\rho\rangle \neq 0$. Now, being L-number breaking communicated by Xs, the simplest choice is then to couple $\hat{X}$s directly to $\hat{\rho}$ by making a $\hat{\rho}^\dagger$ insertion in the loop line where chirality flips. This leads to the the left-hand side diagram of figure 5. We will furthermore take $\hat{X}_1$ and $\hat{X}_2$ to possess superpotential mass terms already at the L-number symmetric phase, even though it is not required by the topology.

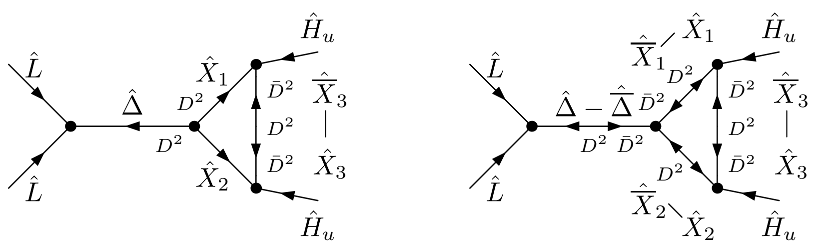

Figure 5. Leading order subset of $\widehat{\mathrm{OP}}_\nu$ in the model example.

A model consistent with this description is summarised in table 1 and its most general renormalisable superpotential is given by[5]

$$\begin{aligned}\mathcal{W} \;&:=\; \mathcal{W}_{\mathrm{MSSM}} + M_\Delta\hat{\Delta}\hat{\overline{\Delta}} + \sum_{i=1}^{2} M_{X_i}\hat{X}_i\hat{\overline{X}}_i + \lambda\hat{\rho}\hat{X}_3\hat{\overline{X}}_3 \\ &\quad + \hat{H}_u\left(\lambda_1\hat{X}_1\hat{\overline{X}}_3 + \lambda_2\hat{X}_2\hat{X}_3\right) + \hat{\Delta}\left(\lambda_L\hat{L}\hat{L} + \lambda_X\hat{X}_1\hat{X}_2\right) + \bar{\lambda}_X\hat{\overline{\Delta}}\hat{\overline{X}}_1\hat{\overline{X}}_2 \,.\end{aligned} \tag{26}$$

In the absence of the last term the model acquires the R-symmetry shown in the last column of table 1. This term allows for a chirality flipped type-II seesaw of superoperator $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$, as shown in the right-hand side supergraph of figure 5. The broken L-number phase corresponds to

$$\lambda\hat{\rho}\hat{X}_3\hat{\overline{X}}_3 \to M_{X_3}\hat{X}_3\hat{\overline{X}}_3 + \lambda\hat{\rho}\hat{X}_3\hat{\overline{X}}_3 \,, \quad M_{X_3} := \lambda\langle\rho\rangle \,. \tag{27}$$

[5] We assume that the $\hat{u}^c\hat{d}^c\hat{d}^c$ term is forbidden by, for instance, R-parity or baryon number conservation.

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Table 1. Extension of the MSSM in the model example. We omitted the conjugates of $\hat{\Delta}$ and $\hat{X}_{1,2}$. $U(1)_R$ stands for an R-symmetry that is acquired as $\bar{\lambda}_X \to 0$.

	$SU(2)_L \otimes U(1)_Y$	$U(1)_X$	$U(1)_L$	$U(1)_R$
$\hat{\Delta}$	$(\mathbf{3}, 1)$	0	-2	4
$\hat{\rho}$	$(\mathbf{1}, 0)$	0	2	0
$\hat{X}_1$	$(\mathbf{2}, -1/2)$	1	1	-2
$\hat{X}_2$	$(\mathbf{2}, -1/2)$	-1	1	0
$\hat{X}_3$	$(\mathbf{1}, 0)$	1	-1	0
$\hat{\bar{X}}_3$	$(\mathbf{1}, 0)$	-1	-1	2

The set of LO superoperators that break L-number proceed from the two supergraphs of figure 5 and are

$$D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\,, \quad \hat{L}\hat{L}\hat{H}_u\hat{H}_u\,. \tag{28}$$

To see that there is indeed no other contributions up to the same order of perturbation theory, notice that a suitably defined L-number symmetry is recovered as any coupling in $\{\lambda_1, \lambda_2, \lambda_L\}$, or both λ_X and any in $\{\bar{\lambda}_X, M_\Delta, M_{X_1}, M_{X_2}\}$, goes to zero. Thus, any superoperator that breaks L-number is proportional to

$$\mathbf{a} := \lambda_1\lambda_2\boldsymbol{\lambda_L}\lambda_X^* \quad \text{or} \quad M_\Delta M_{X_1}M_{X_2}\,\mathbf{b} := \lambda_1\lambda_2\boldsymbol{\lambda_L}\bar{\lambda}_X M_\Delta M_{X_1}M_{X_2}\,. \tag{29}$$

In the $p_{\text{ext}} \to 0$ limit the LO coefficients for these operators read

$$-\left(\frac{\mathbf{a}M_{X_3}}{32\pi^2 M_\Delta^2}\right)C_0\,, \qquad \left(\frac{\mathbf{b}M_{X_3}M_{X_1}M_{X_2}}{32\pi^2 M_\Delta}\right)D_{0,0}\,, \tag{30}$$

respectively, and where C_0 and D_0 are abbreviations of scalar one-loop 3- and 4-point integrals. Then, in the SUSY limit we have

$$\begin{aligned}\int d^4\theta\, D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u \;&=\; -\Box(\tilde{L}\tilde{L})\left[\tilde{H}_u\tilde{H}_u + 2F_{H_u}H_u\right] - \Box(H_uH_u)\left[LL + 2F_L\tilde{L}\right] \\ &\quad +4\,(p_L + p_{\tilde{L}})^2\, L\tilde{H}_u\tilde{L}H_u\,,\end{aligned} \tag{31}$$

while $\int d^4\theta\hat{L}\hat{L}\hat{H}_u\hat{H}_u = 0$. Hence, there is no pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ contribution to neutrino masses at LO. To arrive at this same conclusion by a different route, consider the following. First, we note that only the first supergraph has a non-vanishing 1PI part:

$$\int d^4\theta\, \hat{\Delta}^\dagger\hat{H}_u\hat{H}_u = 2\tilde{\Delta}^{\dagger\dot{\alpha}}(p_{\tilde{H}_u} + p_{H_u})_{\beta\dot{\alpha}}\tilde{H}_u^\beta H_u + F_\Delta^\dagger\left(\tilde{H}_u\tilde{H}_u + 2F_{H_u}H_u\right) - \Delta^\dagger\Box(H_uH_u)\,. \tag{32}$$

Then, by adding these operators to the classical Lagrangian, one concludes that $\overline{\Delta}$ acquires a VEV, since $\langle F_{H_u}\rangle = \mu^*\langle H_d^\dagger\rangle \neq 0$ gives a (tadpole) contribution to $F_\Delta^\dagger \supset M_\Delta\overline{\Delta}$. However, such a VEV has no effect upon m_ν in the absence of $\overline{\Delta} - \Delta$ mixing. This VEV is no longer inconsequential when $\cancel{\text{SUSY}}_{\text{EWS}}$ contributions are considered because the B-term $B_\Delta \Delta\overline{\Delta}$ provides the mixing, so that $\langle\overline{\Delta}\rangle \to \langle\Delta\rangle$.

Due to its instructive value, we will now analyse in terms of component fields the reason behind the absence of pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ contributions to $LLHH$. In order for the left-hand side supergraph of figure 5 to contribute to $LLHH$, the three-scalar coupling $\Delta^\dagger H_u H_u$ would have to be generated. At LO there are in fact three topologies contributing to this coupling as we show in figure 6. However, in the $p_{\text{ext}} \to 0$ limit, the spinor loop topology cancels exactly the scalar loop topologies. This result can also be understood to be a consequence of the fact that one cannot draw diagrams for $\Delta^\dagger H_u H_u$ that are both holomorphy compliant and have at least an external $F^\dagger - F$ pair. In the case of the second supergraph, one can see that it would require the generation of the coupling $F_{\overline{\Delta}} H_u H_u$, whereas the holomorphy of the superpotential makes it impossible to to draw any component field diagram for such a coupling.

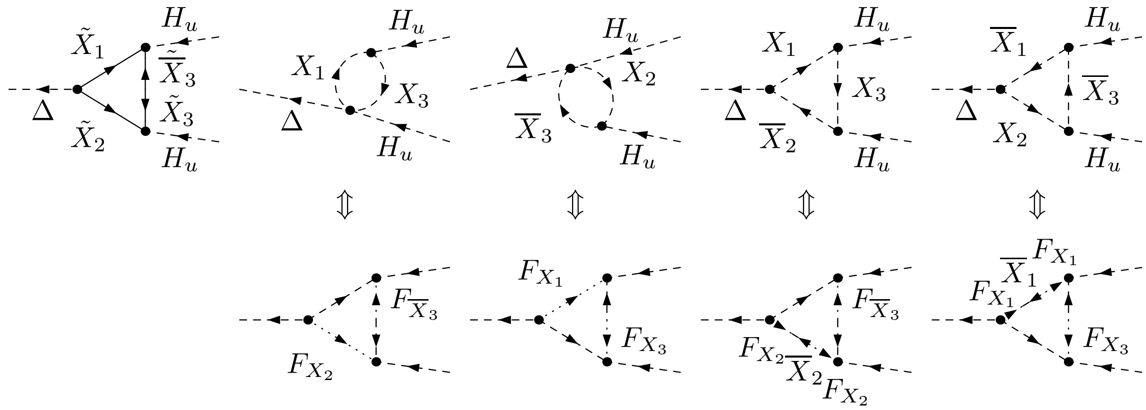

Figure 6. Leading order diagrams for $\Delta^\dagger H_u H_u$ in the model example.

The pure-$\cancel{\text{SUSY}}_{\text{EWSB}}$ subset of OP_ν can be most easily identified by reminding the analysis carried out in section 2.3. One may then see that it comprises the superoperators

$$D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{V}_{U(1)_Y}\,,\quad D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{V}_{SU(2)_L}\,,\quad D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{H}_u^\dagger\hat{H}_u\,,\quad \hat{L}\hat{L}\hat{H}_u\hat{H}_u\hat{H}_u^\dagger\hat{H}_u\,, \tag{33}$$

which generate the dimension-7 operators

$$\begin{aligned}
-\frac{1}{64\pi^2 M_\Delta^2 M_X}\Bigg(\mathbf{a}\Bigg[&\frac{g^2}{2c_w^2}\,(LH_u)\,(LH_u)\,H_u^\dagger H_u + \left(\frac{g^2 c_{2w}}{2c_w^2} + \frac{|\mu|^2(|\lambda_1|^2+|\lambda_2|^2)}{6M_X^2}\right)(LH_u)\,(LH_u)\,H_d^\dagger H_d \\
&+\left(g^2 - \frac{|\mu|^2(|\lambda_1|^2+|\lambda_2|^2)}{3M_X^2}\right)(LH_u)\,(H_uH_d)\,H_d^\dagger L\Bigg] \\
&+\frac{\mathbf{b}M_\Delta\mu\,(|\lambda_1|^2+|\lambda_2|^2)}{6M_X^2}\,(LH_u)\,(LH_u)\,(H_uH_d)\Bigg)\,,
\end{aligned} \tag{34}$$

where we have taken the simplifying limit $M_{X_{1,2,3}} = M_X$.

Now we turn to the LO subset of OP_ν, which is made of dimension-5 operators that come from $\cancel{\text{SUSY}}_{\text{EWS}}$. In figure 7 we illustrate some of the soft-$\cancel{\text{SUSY}}$ insertions behind such operators. Complete expressions for them up to order 3 in m_{soft} are given in appendix F of [1]. In here

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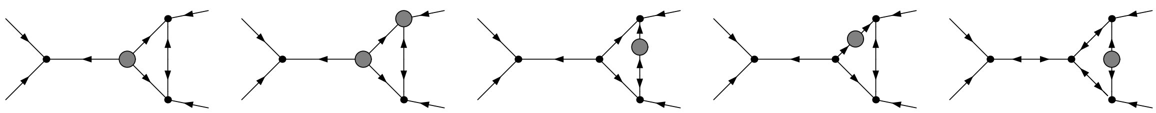

Figure 7. Example of soft-$\not{\text{SUSY}}$ insertions (grey blobs) that generate the LO subset of OP_ν in the model example. For a detailed list up to order 3 in $m_{\rm soft}$ see [1].

we take the simplifying limit $M_{X_{1,2,3}} = M_X$, $(m^2_{\rm soft})_{X_{1,2,3}} = (m^2_{\rm soft})_{\overline{X}_{1,2,3}} = m^2_{\rm soft}$, $A_{1,2} = A$ and $B_{X_{1,2,3}} = B_X$, which gives

$$\begin{aligned}
&\frac{1}{64\pi^2 M_\Delta^2}\left(\mathbf{a}\left[\frac{2m^2_{\rm soft}}{M_X}+\frac{2A}{M_X}\left(A_X^*-\frac{B_\Delta}{M_\Delta}\right)-\frac{A_X^*B_X}{M_X^2}\right]+\mathbf{b}M_\Delta\frac{B_X}{M_X^2}\right)LLH_uH_u\\
&-\frac{\mathbf{a}}{32\pi^2M_\Delta^2}\left(\frac{\mu^*}{M_X}\right)\left[A_X^*\left(1-\frac{m^2_{\rm soft}}{M_X^2}-\frac{(m^2_{\rm soft})_\Delta}{M_\Delta^2}\right)-\frac{B_\Delta}{M_\Delta}\right]LLH_uH_d^\dagger\\
&-\frac{\mathbf{a}}{192\pi^2M_\Delta^2}\left(\frac{\mu^*}{M_X}\right)^2\frac{A_X^*B_X}{M_X^2}LLH_d^\dagger H_d^\dagger\,.
\end{aligned}\tag{35}$$

Let us briefly discuss the origin of some of the terms. First, the contributions of $m^2_{\rm soft}$ (i.e. non-holomorphic mass) terms. These induce mass differences between scalar and spinor components of chiral scalar superfields while preserving the chirality flow of the line into which they are inserted. Therefore, they invalidate the conclusions related to figure 6 by introducing a mismatch in the cancellation between spinor and scalar loops, thus leading to a non-vanishing contribution to m_ν. Unlike $m^2_{\rm soft}$, B-term insertions reverse the chirality flow, thus enabling the construction of diagrams for $F_{\overline{\Delta}}H_uH_u$. This is indeed the reason behind the B_X-term contribution proportional to $\mathbf{b}$. Finally, the B_Δ contributions. As commented above, EWSB induces a VEV for $\overline{\Delta}$ which, through B_Δ, induces a VEV for Δ, thus leading to $LLH_uH_d^\dagger \subset LL\langle\Delta\rangle \subset \int d^2\theta\,\mathcal{W}$. To obtain the B_Δ coefficient of LLH_uH_u one must take into consideration the shift in $\langle\overline{\Delta}\rangle$ induced by $A_1 + A_2$.

We summarise in table 2 the contributions to m_ν that we have been discussing, and where we take the simplifying limit of $\lambda_X = \bar{\lambda}_X = \lambda_{1,2} = \lambda$, $M_{X_i} = M_\Delta = M$ and a common scale m for all the soft-$\not{\text{SUSY}}$ effects involving $\hat{\Delta}$s or $\hat{X}$s.

Table 2. Contributions to m_ν (see text) up to an overall factor of $\frac{\boldsymbol{\lambda_L}\lambda^3}{16\pi^2}\frac{v^2 s_\beta}{M}$.

Type		Intrinsic factor
One-loop dim-5		$\frac{\mu\, m}{M^2}c_\beta - \frac{5}{2}\left(\frac{m}{M}\right)^2 s_\beta$
	$\hat{V}$	$-\frac{g^2}{4c_w^2}\left(\frac{v}{M}\right)^2\left(c_\beta^2 - s_\beta^2\right)s_\beta$
One-loop dim-7	$\hat{H}^\dagger\hat{H}$ Class A	$\frac{\lvert\lambda\rvert^2}{2}\left(\frac{v\mu}{M^2}\right)^2 c_\beta^2 s_\beta$
	$\hat{H}^\dagger\hat{H}$ Class B	$-\frac{\lvert\lambda\rvert^2}{6}\left(\frac{v}{M}\right)^2\frac{\mu}{M}c_\beta s_\beta^2$

5. Conclusions

In this work we studied the connection between m_ν and $\cancel{\text{SUSY}}$ that arises when neutrino masses are radiatively generated. We started by distinguishing the different $\cancel{\text{SUSY}}$ contributions w.r.t. their involvement in EWSB. This allowed us to identify a subset of superoperators that could only contribute to m_ν through $\cancel{\text{SUSY}}$ effects unrelated to EWSB and which involved the mediators of the radiative seesaw. An interesting aspects of these effects is that they can be quite small without being in conflict with experimental data. One may then conjecture that they are (at least partially) responsible for the smallness of neutrino masses, thus allowing for a seesaw scale within foreseeable experimental reach.

We analysed the simplest (one-loop) topologies that may underlie models with this interesting feature and found that they all generated $LLHH$ operators with a leading dimensionful dependence that ranged from $\mu\, m_{\text{soft}}/M^3$ or m^2_{soft}/M^3 to $\mu\, m^2_{\text{soft}}/M^4$ or m^3_{soft}/M^4.

As an application of this more abstract analysis, we constructed a model example based on a one-loop type-II seesaw, which also served as a proof of existence of such interesting models. A phenomenological analysis of this model will be presented elsewhere.

Acknowledgments

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