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On the origin of neutrino oscillations through Lorentz violation

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Abstract. The possibility of generating neutrino masses and oscillations through Lorentz-violating models is investigated. In the first model, an interaction between a fermion doublet and a Lorentz-violating gauge field, which play the role of a regulator field and, eventually, decouples from the fermions, is considered. In this case, by solving the (non-perturbative) Schwinger-Dyson equation, we show how masses and oscillations are generated dynamically. In the second model, fermions with LV kinematics interact via a four-fermion interaction and masses are shown to be generated dynamically when using another non-perturbative method. In both models, the recovery of Lorentz invariance is discussed and it is shown that the only physical observables are the dynamical masses that lead to neutrino oscillations.

1. Introduction

The discovery of a Higgs-like scalar particle has been made recently by experiments at the Large Hadron Collider [1] and, as a consequence, the generation of masses for most of the known particles due to their coupling to the Higgs field seems now to be confirmed. An exception to this, however, comes from the fact that the origin of neutrino masses is still not well understood, although the seesaw mechanism seems the most elegant and simple for such a purpose [2]. On the other hand, seesaw mechanisms require right-handed sterile neutrinos which have not yet been discovered in Nature [3].

Thus, in this paper, we present two alternative ways, based on Lorentz-violating (LV) theories, to generate dynamically neutrino masses and, consequently, flavour oscillations. In both cases, LV higher-order space derivatives suppressed by a large mass scale M are added to the "usual" models, but the number of time derivatives is not altered, in order not to generate ghosts.

In the first model, we consider two massless bare fermions interacting with an Abelian gauge field, which has a LV propagator. The mass scale M introduced together with the LV operator allows the dynamical generation of fermion masses, as was shown in [4] with the Schwinger-Dyson approach and also leads to a finite gap equation, regulating the model. Moreover, Lorentz-violating gauge models of the form suggested in [4] can arise in the low-energy limit of some quantum gravity theories [5].

An important feature of our results is the structure of the dynamical fermion mass [4], *i.e.*

$$m_{dyn} \simeq M \, \exp(-a/e^2) \, , \tag{1}$$

where a is a positive constant and e the coupling constant. Such a dynamical mass solution, being non-analytical in e, can only be obtained by means of non-perturbative approaches, such

as the Schwinger-Dyson derivation of a gap equation used in [4] and here. In addition, it is possible to take the simultaneous limits

$$M \to \infty \quad \text{and} \quad e \to 0 \ , \tag{2}$$

in such a way that the dynamical mass (1) remains finite, corresponding to a physical fermion mass. In this limit, the gauge field decouples from the fermion fields and the former plays the role of a LV regulator field. This procedure is consistent with the string-embedding case of [5] in the limit where the density of D-particles vanishes and then Lorentz symmetry is recovered.

In the second case, we consider new LV kinematics for a four-fermion interaction model. The kinematics however are not the same as in Lifshitz-type models (see [6] for a review), and our present mechanism involves "quasi-relativistic" fermions, in the sense that, their dispersion relations differ from the relativistic case only in an intermediate energy regime governed by M, whereas are almost relativistic in both the infrared (IR) and the ultraviolet (UV) regions.

In addition to the quasi-relativistic kinematics, this model allows the dynamical generation of mass from a four-fermion interaction, however small the coupling strength is. This feature allows us to consider the limit where the couplings $g_i \to 0$ and the mass scale $M \to \infty$, *i.e.* the Lorentz symmetric limit. This is not the case for usual Lorentz symmetric four-fermion interactions, where critical couplings are naturally defined by the gap equation (see for example the Nambu-Jona-Lasinio model [7] - NJL). Nevertheless, Lifshitz four-fermion interaction models also allow dynamical mass generation for any coupling strength [8], but in such models, fermions have a dispersion relation which implies a large deviation from relativistic kinematics in the UV.

A non-trivial consequence of this model in contrast to our first model is the analytic properties of the dynamical mass, as a function of the coupling constant. Therefore, although we make use of a non-perturbative approach to calculate the dynamical mass, an expansion of the present result in the coupling constant could be obtained by one-loop calculation.

In general, neutrino oscillation models contain massive particles, but oscillations involving massless particles have been studied in [9] and also in the framework of LV models in [10]. Whilst these studies have been questioned by phenomenological constraints [11], our present models, based on higher order space derivatives, are not excluded.

Finally, we stress here an essential feature of the mechanisms described for both models. Although LV operators are suppressed by a large mass scale M and therefore the corresponding effect is negligible at the classical level, quantum corrections completely change this picture, leading to finite effects. The finite effects here are the dynamical generation of fermion masses and consequently flavour oscillations, which are present even when the LV-suppressing mass scale M is set to infinity and the couplings to zero. Note that the order of the steps followed is important: quantization is done for finite mass M and couplings e_i and only afterwards the Lorentz symmetric limit is taken.

This paper is organized as follows. In the next section, the first model is considered. By making use of the Schwinger-Dyson equation for the fermion propagator the corresponding gap equations which must be satisfied by the dynamical masses are derived. Then considering the constraints arising from the gap equations, we calculate the dynamical masses for different cases. Finally, in the last subsection of section 2, the "Lorentz-Invariant limit" is discussed, in which the LV gauge field decouples from fermions, and we demonstrate that relativistic dispersion relations for fermions are indeed recovered. After that, in the section 3, the second model is presented. After discussing a few of its classical properties, we show how the fermion masses are generated by quantum corrections for any coupling strength g_i. In addition, exploring the fact that masses are generated dynamically independent of the coupling strength, we consider the possibility of taking the Lorentz symmetric limit in such a way that the dynamical masses are finite and then we show that this limit does not spoil the theory, since the quantum corrections to the

fermion self-energy and oscillation probability are kept Lorentz invariant. Finally, conclusions and outlook are presented in section 4.

2. Model I

The first LV model we consider is [12]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(1-\frac{\Delta}{M^2})F^{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - \tau A\!\!\!/)\Psi, \tag{3}$$

where $F_{\mu\nu}$ is the usual Abelian field strength for the gauge field A^μ, $\Delta = -\partial_i\partial^i$ and M is the mass scale that suppresses the LV derivative operator Δ, and can be thought of as the Plank mass, however, it will eventually be set to infinity. Ψ is a massless fermion doublet $\Psi = (\psi_1, \psi_2)$ and the flavour mixing matrix τ features the gauge couplings (e_1, e_2, ϵ) as

$$\tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} . \tag{4}$$

The Lagrangian (3), as already pointed out, can be derived from a stringy space time foam model, as shown in [5].

The gauge field bare propagator is

$$D_{\mu\nu} = -\frac{i}{1+\vec{p}^2/M^2}\left(\frac{\eta_{\mu\nu}}{\omega^2-\vec{p}^2} + \zeta\frac{p_\mu p_\nu}{(\omega^2-\vec{p}^2)^2}\right) , \tag{5}$$

where ζ is a gauge fixing parameter. Nonetheless, as we will see in the next sections, ζ does not play a role when the simultaneous limits $M \to \infty$ and e_1, e_2, $\epsilon \to 0$ that leave the dynamical masses finite are considered.

It is worth mentioning at this point that, although previous works, such as [13] and [14], have shown that the flavour mixing interaction $\overline{\Psi}\tau A\!\!\!/\Psi$ can be at the origin of a dynamically generated gauge boson mass, in the present study, this possibility is disregarded, since, as it will be demonstrated below, the flavour mixing coupling ϵ vanishes necessarily for the consistency of the model when fermion oscillations are dynamically generated.

The bare fermion propagator is $S = ip\!\!\!//p^2$, where $p_\mu = (\omega, \vec{p})$. Finally, assuming the dynamical generation of the following fermion mass matrix

$$\mathbf{M} = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix}, \quad with \quad m_\pm = \frac{m_1+m_2}{2} \pm \frac{\sqrt{(m_1-m_2)^2+4\mu^2}}{2} , \tag{6}$$

where $m_\pm$ are the mass eigenvalues, and neglecting other quantum corrections, the dressed fermion propagator G is given by

$$G = i\frac{p^2 + p\!\!\!/(m_1+m_2) + m_1m_2 - \mu^2}{(p^2-m_1^2)(p^2-m_2^2) - 2\mu^2(p^2+m_1m_2) + \mu^4}\begin{pmatrix} p\!\!\!/ - m_2 & \mu \\ \mu & p\!\!\!/ - m_2 \end{pmatrix} . \tag{7}$$

At this point, before performing any calculation, it is important to remember which conditions must be satisfied so that flavour oscillations can take place. First, we should remember that the flavour eigenstates $(\psi_{1,2})$ are connected to the mass (energy) eigenstates $(\psi_{+,-})$ by a unitary transformation, parametrized by a mixing angle θ, *i.e.*

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \tag{8}$$

and if at an initial time $t = 0$ a flavour neutrino $\psi_1(t = 0)$ is produced then, in general, the probability of obtaining the other flavour $\psi_2(t)$ at $t > 0$ is nontrivial and given by [15]

$$\mathcal{P}_{12}(t) = \sin^2(2\theta)\sin^2\left[\frac{(E_+ - E_-)\ t}{2}\right] \ , \tag{9}$$

where $E_{+,-}$ represent the energy eigenvalues. Therefore, from (9) one notes that oscillations can only occur in case there is a nontrivial mixing angle $\theta \neq 0$ and, at the same time, the energy levels are different $E_+^2 \neq E_-^2$. In particular, as will be useful later, assuming massive relativistic neutrinos for which the dispersion relation is $E = \sqrt{\vec{p}^2 + m^2}$, (9) can be rewritten as

$$\mathcal{P}_{12} \approx \sin^2(2\theta)\sin^2\left[\frac{(m_+^2 - m_-^2)\ L}{4E}\right] \ , \tag{10}$$

where $m_{+,-}$ are the mass eigenvalues and L the distance between the source and detector.

In the next section, we will check that the dynamical masses m_1, m_2, μ assumed here can indeed be generated by quantum corrections, using the Schwinger-Dyson approach.

2.1. Schwinger-Dyson Gap equations

The Schwinger-Dyson equation for the fermion propagator has the usual structure [16], not being modified by the LV term in (3). Neglecting corrections to the wave functions, the vertices and the gauge propagator, the Schwinger-Dyson equation reads for our model

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu}\ \tau\gamma^\mu\ G\ \tau\gamma^\nu \ . \tag{11}$$

Due to the presence of the LV term in the denominator of the gauge field propagator (5), the loop integral in (11) is finite.

The Schwinger-Dyson equation above leads to four gap equations, which must be satisfied by the three masses m_1, m_2, μ:

$$\begin{aligned}
\frac{m_1}{4+\zeta} &= (e_1^2 m_1 + \epsilon^2 m_2)I_1 + (\mu^2 - m_1 m_2)(e_1^2 m_2 + \epsilon^2 m_1)I_2 \\
\frac{m_2}{4+\zeta} &= (e_2^2 m_2 + \epsilon^2 m_1)I_1 + (\mu^2 - m_1 m_2)(e_2^2 m_1 + \epsilon^2 m_2)I_2 \\
\frac{\mu}{4+\zeta} &= \mu(e_1 e_2 - \epsilon^2)[I_1 - (\mu^2 - m_1 m_2)I_2] \\
0 &= \epsilon(e_1 m_1 + e_2 m_2)I_1 + \epsilon(\mu^2 - m_1 m_2)(e_1 m_2 + e_2 m_1)I_2 \ ,
\end{aligned} \tag{12}$$

where after integrating over the frequency ω and momentum $\vec{p}$, and expanding the results so that $M >> m_1, m_2, \mu$, the expressions I_1 and I_2 become

$$I_1 \simeq \frac{1}{16\pi^2}\frac{1}{A_+^2 - A_-^2}\left[A_-^2 \ln\left(\frac{A_-^2}{M^2}\right) - A_+^2 \ln\left(\frac{A_+^2}{M^2}\right)\right] \ and\ I_2 \simeq \frac{1}{16\pi^2}\frac{1}{A_+^2 - A_-^2}\ln\left(\frac{A_-^2}{A_+^2}\right) \ , \tag{13}$$

with

$$A_\pm^2 = \frac{m_1^2 + m_2^2 + 2\mu^2}{2} \pm \frac{\sqrt{(m_1^2 - m_2^2)^2 + 4\mu^2(m_1 + m_2)^2}}{2} \ . \tag{14}$$

Furthermore, the four equations in (12) are not independent and must be satisfied by only three unknowns m_1, m_2, μ. In order to find the solutions in a more efficient way, we make use

of the following constraints which can be easily derived from (12). Considering the first two equations, for $e_1^2 e_2^2 \neq \epsilon^4$, we have

$$\begin{aligned} I_1 &= \frac{1}{4+\zeta}\,\frac{e_2^2 m_1^2 - e_1^2 m_2^2}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)} \\ (\mu^2 - m_1 m_2) I_2 &= \frac{1}{4+\zeta}\frac{m_1 m_2 (e_1^2 - e_2^2) + \epsilon^2 (m_2^2 - m_1^2)}{(e_1^2 e_2^2 - \epsilon^4)(m_1^2 - m_2^2)} \ , \end{aligned} \tag{15}$$

and the third and forth equations lead to the following constraints respectively

$$\mu(m_1 + m_2)(e_2 m_1 + e_1 m_2)(e_1 - e_2) = 0 \quad \textit{and} \quad \epsilon(e_2 m_1 + e_1 m_2) = 0 \ . \tag{16}$$

According to the constraints above, we are left with few possibilities to study in the next section.

2.2. Solutions of the Gap Equations - Dynamical Fermion Masses and Mixing
In what follows, by making choices that satisfy the constraints (16), we focus on different solutions to the gap equations (12) which allow for $\mu \neq 0$ (non-vanishing mixing angle), which according to (9) is a necessary, but not sufficient condition, for flavour oscillations.

2.2.1. The case $m_1 = m_2 = 0$ and $\mu \neq 0$: In this case, the mass eigenvalues and eigenstates are, respectively,

$$m_\pm = \pm\mu \ , \quad \textit{and} \quad \psi_\pm = \frac{1}{\sqrt{2}}(\psi_2 \pm \psi_1) \tag{17}$$

such that the mixing angle (8) is $\theta = -\pi/4$, in our conventions. No oscillations (10) among the fermion flavours are allowed in such a case, since $m_+^2 = m_-^2$.

Among the gap equations (12), only the third is not trivial, and leads to

$$\frac{1}{4+\zeta} = (e_1 e_2 - \epsilon^2)(I_1 - \mu^2 I_2) \ . \tag{18}$$

Since $A_\pm^2 = \mu^2$, the expressions (13) become

$$I_1 \simeq \frac{-1}{16\pi^2}\left(1 + \ln\left(\frac{\mu^2}{M^2}\right)\right), \quad \textit{and} \quad I_2 \simeq \frac{-1}{16\pi^2}\frac{1}{\mu^2} \ , \tag{19}$$

and we finally obtain

$$\mu \simeq M \exp\left(\frac{-8\pi^2}{(4+\zeta)(e_1 e_2 - \epsilon^2)}\right) \ , \tag{20}$$

which, as expected, is non-analytic in the coupling constant and, therefore, could not be found by perturbative methods. Note that for this solution have a meaning it is necessary that $e_1 e_2 > \epsilon^2$, otherwise $\mu^2 > M^2$.

2.2.2. The case $e_2 m_1 + e_1 m_2 = 0$ and $m_1^2 \neq m_2^2$: In this situation, since the first equation (15) leads to $I_1 = 0$, the expression (13) for I_1 implies that

$$A_+^2 = A_-^2 = \exp(-1) M^2 \ , \tag{21}$$

which can not be considered physical, because the dynamical masses are then necessarily of the order M. This possibility should therefore be disregarded, specially because we will eventually take the limit $M \to \infty$.

2.2.3. The case $m_1 = -m_2 \neq 0$: It can be seen from eqs.(12) that to have $m_1 = -m_2 \equiv m$, it is necessary that $e_1 = e_2$, such that both constraints (16) are satisfied. Also, in such a case, eqs.(12) become

$$\frac{1}{4+\zeta} = (e^2 - \epsilon^2)[I_1 - (\mu^2 + m^2)I_2] \ , \tag{22}$$

with $A_\pm^2 = m^2 + \mu^2$, such that we find

$$m^2 + \mu^2 = M^2 \exp\left(\frac{-16\pi^2}{(4+\zeta)(e^2-\epsilon^2)}\right) \ , \tag{23}$$

which has a physical meaning only if $e^2 > \epsilon^2$. Moreover, this condition allows one to consider the limit $\epsilon \to 0$ without affecting the mass eigenvalues or mixing angles (see below). Therefore, as long as ϵ can be set to zero, no dynamical generation of vector boson masses is allowed, as already mentioned.

Once again, flavour oscillations do not take place because the energy eigenvalues are the same. The mass eigenvalues and mixing angle θ (8) are given by

$$m_\pm = \pm\sqrt{m^2+\mu^2} \ , \quad \tan\theta = \frac{-\mu}{m + \sqrt{m^2+\mu^2}} \ , \tag{24}$$

whereas the mass eigenstates are

$$\psi_\pm = \frac{1}{N_\pm}\left(\psi_1 + \frac{\mu}{m \pm \sqrt{m^2+\mu^2}}\psi_2\right) \quad with \quad N_\pm^2 = \frac{2m^2 + 2\mu^2 \pm 2m\sqrt{m^2+\mu^2}}{2m^2 + \mu^2 \pm 2m\sqrt{m^2+\mu^2}} \ . \tag{25}$$

2.2.4. The case $m_1 = m_2 \neq 0$, Dynamical Flavour Oscillations: We find here from eqs.(12) that necessarily $e_1 = e_2 = e$, $\epsilon = 0$ and $\mu^2 = m^2$ which implies that

$$\mu^2 = m_1 m_2 = m^2 \quad \text{and} \quad I_1 = \frac{1}{(4+\zeta)e^2} \ . \tag{26}$$

Then, noting that, in this case, $A_-^2 = 0$ and $A_+^2 = 4m^2$, the dynamical mass is

$$m = \frac{M}{2}\exp\left(-\frac{8\pi^2}{(4+\zeta)e^2}\right) \tag{27}$$

which, as expected, is not perturbative in e. In this situation, the mass matrix has identical elements and, as a consequence, different eigenvalues

$$m_+ = 2m = M\exp\left(-\frac{8\pi^2}{(4+\zeta)e^2}\right) \quad , \quad m_- = 0 \ , \tag{28}$$

and the corresponding mass eigenstates are the same as the eigenstates given by eq.(17). The mixing angle (8) is $\theta = \mp\,\pi/4$, depending on the sign of $\mu = \pm\, m$, respectively.

Therefore, from what we saw above, *i.e.* $m_+^2 \neq m_-^2$ and $\theta \neq 0$, we note that because of the constraints (16), this is the only case in the present model (3) where non trivial oscillations among fermion flavours take place. Furthermore, in this case necessarily $\epsilon \to 0$, avoiding the dynamical generation of gauge boson masses, and thus the latter play a consistent role as a regulator field.

2.3. Lorentz symmetric limit

In this section, we investigate what happens when Lorentz invariance is then recovered after quantisation and dynamical mass generation. In order to recover Lorentz invariance, we take the simultaneous limits

$$M \to \infty \quad \text{and} \quad e_1,\ e_2,\ \epsilon \to 0\ , \tag{29}$$

in such a way that the dynamical masses are finite. This procedure does not depend on the gauge parameter ζ and the resulting fermion mass is set to any desired value.

Apart from setting the dynamical mass generation, we want to know whether the theory is indeed well-behaved in the limit (29) so that unwanted LV effects do not appear. To check this, we calculate the one-loop corrections to the fermion dispersion relation in the limit (29). We focus here for concreteness on the solution presented in subsection 2.2.4, with $\mu = +m$, but clearly the same conclusion holds for all the other solutions studied above. Because one of the eigenmasses vanishes leading to one-loop IR divergence, we consider $m_1 = m_2 = m$ and $m - \mu = m\delta$ with $\delta << 1$, however, δ will be eventually taken to zero.

To lowest order in momentum, we find then

$$\Sigma = \begin{pmatrix} Z^0_{diag} & Z^0_{off} \\ Z^0_{off} & Z^0_{diag} \end{pmatrix} \omega\gamma^0 - \begin{pmatrix} Z^1_{diag} & Z^1_{off} \\ Z^1_{off} & Z^1_{diag} \end{pmatrix} \vec{p}\cdot\vec{\gamma} - \mathbf{M}\ , \tag{30}$$

where $(\omega, \vec{p})$ is the external 4-momentum and

$$\begin{aligned} Z^0_{diag} &= \frac{e^2}{8\pi^2}\left(\frac{1}{4} - \frac{1}{2}\ln 2 + \frac{1}{2}\ln\delta + \ln\left(\frac{m}{M}\right)\right) \\ Z^1_{diag} &= \frac{e^2}{8\pi^2}\left(-\frac{1}{12} - \frac{1}{2}\ln 2 + \frac{1}{2}\ln\delta + \ln\left(\frac{m}{M}\right)\right) \\ Z^0_{off} &= Z^1_{off} = \frac{e^2}{16\pi^2}(\ln 2 - \ln\delta)\ . \end{aligned} \tag{31}$$

As expected, due to the Lorentz-symmetry violation nature of our model, $Z^0_{diag} \neq Z^1_{diag}$, but since

$$e^2 \ln\left(\frac{m}{M}\right) = -2\pi^2\ , \tag{32}$$

the limit (29) leads to

$$\Sigma \ \to \ -\frac{1}{4}(\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})\mathbf{1} - \mathbf{M}_R\ , \tag{33}$$

where $\mathbf{M}_R$ is the "renormalized" mass matrix. Therefore in the limit (29) the dispersion relations are relativistic and do not depend on δ, such that the limit $\delta \to 0$ will not introduce any IR divergence. Thus, these corrections can be absorbed in a fermion field redefinition, so that we are left with two free relativistic fermion flavours oscillating.

3. Model II

In this section, in order to study the dynamical generation of mass and oscillations for fermions, we present a different model consisting in fermion fields with LV kinematics interacting via four-fermion interactions. First, we assume a massive theory with one flavour only and discuss some of its classical aspects and then, we consider the massless two-flavour case and the issue of dynamical mass/oscillation generation.

3.1. One-flavour case - Massive model and classical properties
The general Lagrangian for the one-flavour case is [18]

$$\mathcal{L}_1 = \bar{\psi}\left[i\partial_0\gamma^0\left(1-\frac{a}{M^2}\Delta\right) - i\vec{\partial}\cdot\vec{\gamma}\left(1-i\frac{b}{M}\vec{\partial}\cdot\vec{\gamma}-\frac{c}{M^2}\Delta\right) - m\right]\psi + \frac{g^2}{M^2}(\overline{\psi}\psi)^2, \tag{34}$$

where g^2 is a dimensionless coupling and the mass scale M is used both to control the LV scale and the strength of the four-fermion interaction.

The choice of such a Lagrangian is motivated by a gravitational microscopic model, based on the low-energy limit of a string theory on a three brane universe, embedded, from an effective three-brane observer view point, in a bulk space-time punctured with point-like defects (*D-particles*) [17], for more details see [18]. On the other hand, this model is also compatible with the the fermionic sector of the Standard Model Extension [19] when one chooses the coefficients accordingly.

In what follows, we will assume that $m << M$. The dispersion relation for the Lagrangian (34) is

$$\omega^2 = m^2\left(\frac{1+bp^2/(Mm)}{1+ap^2/M^2}\right)^2 + p^2\left(\frac{1+cp^2/M^2}{1+ap^2/M^2}\right)^2 \ . \tag{35}$$

For all the values of a, b, c, the Lorentz symmetric limit is recovered when $M \to \infty$, at fixed p and m. In particular, we are interested in the case where $a \neq 0$ *and* $c \neq 0$. In this situation, in the IR and UV regions the dispersion relation is relativistic, however, in the intermediate regime $p \sim M$ it is not.
If we impose ω to be an increasing function of p, the different constants in the model (34) must satisfy

$$2b^2 + 4c \geq a + 2ab\, m/M \ , \tag{36}$$

and without loss of generality, since our aim here is to give emphasis on the mechanism of dynamical mass generation, we shall choose $a = c = 1$. Moreover, as shown in [18], for the case $b = 0$ the existence of a non-vanishing dynamical mass requires g to be larger than a critical coupling. However, in our case, we will eventually take $g^2 \to 0$ for the Lorentz-symmetric limit, therefore, we must have $b \neq 0$.

The bare propagator S for the model (34) is

$$S = i\ \frac{(\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+p^2/M^2) + m + bp^2/M}{(\omega^2-p^2)(1+p^2/M^2)^2 - (m+bp^2/M)^2} \ . \tag{37}$$

Then when considering $b \neq 0$, the trace of S does not vanish even in the massless case $m = 0$, which will be important for the analytical properties of the dynamical mass, as explained bellow. Thus, from now on we assume that $a = b = c = 1$.

3.2. Two-flavour case: dynamical flavour oscillations
In this section, we take the model given in (34) with $a = b = c = 1$ and consider its massless two-flavour case extension:

$$\mathcal{L}_2 = \bar{\Psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1-\frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right]\Psi + \frac{1}{M^2}(\overline{\Psi}\tau\Psi)^2, \tag{38}$$

with

$$\Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \ , \quad \text{and} \quad \tau = \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2\end{pmatrix} \ . \tag{39}$$

Now, in order to study the possibility of generating fermion masses dynamically, the usual approach which consists in introducing a Yukawa coupling of fermions to an auxiliary field ϕ, integrate over fermions, and look for a non-trivial minimum for the effective potential $V(\phi)$, which leads to a mass term in the original Yukawa interaction, is considered. This approach neglects fluctuations of the auxiliary field about its vev, but these can indeed be omitted when $\tau \to 0$, once we consider the Lorentz symmetric limit (see below).

The Lagrangian containing the auxiliary field is

$$\mathcal{L}_2' = \bar{\Psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right]\Psi - \frac{M^2}{4}\phi^2 - \phi\overline{\Psi}\tau\Psi \ , \tag{40}$$

for which the integration over ϕ leads back to the original model (38).

Note that the auxiliary field does not propagate at the tree level and its large mass suppresses fluctuations of ϕ about its vev ϕ_0, such that $\tau\phi \simeq \tau\phi_0$ can be identified with the fermion mass matrix. Because of this, it is enough to consider a homogeneous configuration for ϕ to calculate the effective potential and, consequently, its minimum. Nevertheless, it can be shown that ϕ can be interpreted as a scalar collective excitation of the original fermionic fundamental degrees of freedom and its kinetic term is generated by integrating out fermions, in case ϕ depends on spacetime coordinates, for further details see [18].

In order to integrate (40) over fermions, we should first calculate the eigenvalues, in flavour space, of the operator

$$\mathcal{O} = \begin{pmatrix} (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+\frac{p^2}{M^2}) - \frac{p^2}{M} - g_1\phi & -g_3\phi \\ -g_3\phi & (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+\frac{p^2}{M^2}) - \frac{p^2}{M} - g_2\phi \end{pmatrix} \ , \tag{41}$$

which are

$$m_\pm = (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1 + p^2/M^2) - p^2/M - h_\pm\phi \ , \tag{42}$$

where the eigenvalues $h_\pm$ of the coupling matrix τ are given by

$$h_\pm = \frac{1}{2}(g_1 + g_2) \pm \frac{1}{2}\sqrt{(g_1 - g_2)^2 + 4g_3^2} \ . \tag{43}$$

The effective potential for the auxiliary field is therefore

$$V_2 = \frac{M^2}{4}\phi^2 + i\ tr\int \frac{d^4p}{(2\pi)^4}(\ln m_+ + \ln m_-) \ . \tag{44}$$

Thus, the minimum of the potential is calculated by assuming that $(dV_2/d\phi)_{\phi_0} = 0$, leading to

$$\frac{M^2}{2}\phi_0 = \sum_{s=+,-}\frac{h_s}{\pi^3}\int p^2dp\int d\omega\left[\frac{(h_s\phi_0 + p^2/M)}{(\omega^2+p^2)(1+p^2/M^2)^2 + (h_s\phi_0 + p^2/M)^2}\right] \ , \tag{45}$$

and the integration over frequencies leads to gap equation below, regularized by the mass scale M,

$$\kappa\frac{\pi^2}{2} = \sum_{s=+,-} h_s\int_0^1 \frac{x^2dx\ (h_s\kappa + x^2)}{(1+x^2)\sqrt{x^2(1+x^2)^2 + (h_s\kappa + x^2)^2}} \ , \tag{46}$$

where $x = p/M$ and $\kappa = \phi_0/M$. We solve this integral and expand in κ to find that

$$\kappa = \frac{\alpha(h_+ + h_-)}{1 - 2(h_+^2 + h_-^2)/(5\pi^2)} + \mathcal{O}(\kappa^2) \quad with \quad \alpha = \frac{\ln(1+2/\sqrt{5}) - \arctan(1/2)}{\pi^2} \simeq\ 0.018 \ . \tag{47}$$

Finally, taking into account that $h_\pm << 1$, we obtain the minimum of the potential

$$\kappa \simeq \alpha(g_1 + g_2) \ . \tag{48}$$

An interesting point to note is that the dynamical mass (48) is analytic in the coupling constants g_i, unlike the situation of Lifshitz 4-fermion interaction [8, 20] which presents the typical non-analytic form (1). We note however that the expression (48) consists of a resummation in powers of g_i and goes beyond a one-loop calculation. Nevertheless, the approximate result (48) can be obtained from the usual one-loop correction to the fermion mass. This feature is specific to the LV propagator (37), whose trace does not vanish, even in the massless case. We are therefore in the unusual situation where a fermion mass generated dynamically can be derived using a perturbative expansion, whereas a mass of the form (1) can be obtained from a non-perturbative approach only.

From the results above, the mass matrix $\mathcal{M} = \kappa M \tau$ generated dynamically is

$$\mathcal{M} = \alpha(g_1 + g_2) M \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix} \ , \tag{49}$$

such that the mass eigenvalues $m_\pm = \kappa M h_\pm$ and the mixing angle θ are given by

$$\begin{aligned} m_\pm &= \frac{\alpha}{2} M \left[(g_1 + g_2)^2 \pm \sqrt{(g_1^2 - g_2^2)^2 + 4 g_3^2 (g_1 + g_2)^2} \right] \\ \tan\theta &= \frac{g_1 - g_2}{2 g_3} + \sqrt{1 + \left(\frac{g_1 - g_2}{2 g_3} \right)^2} \ . \end{aligned} \tag{50}$$

From this we can express the dimensionless couplings g_i in terms of the masses and mixing angle

$$\begin{aligned} g_1 &= \frac{\mu_+ + \mu_- + (\mu_+ - \mu_-)\cos(2\theta)}{2\sqrt{\alpha(\mu_+ + \mu_-)}} \ , \quad g_2 = \frac{\mu_+ + \mu_- - (\mu_+ - \mu_-)\cos(2\theta)}{2\sqrt{\alpha(\mu_+ + \mu_-)}} \ , \\ g_3 &= \frac{\mu_- - \mu_+}{2\sqrt{\alpha(\mu_+ + \mu_-)}} \sin(2\theta) \qquad where \qquad \mu_\pm = \frac{m_\pm}{M} \ . \end{aligned} \tag{51}$$

This means that the couplings g_i have the following form

$$g_i = \frac{a_i}{\sqrt{M}} \ , \ i = 1, 2, 3 \ , \tag{52}$$

where the constants a_i are completely fixed by the experimental values for $m_\pm$ and θ. The expression (52) explicitly shows dependence of the couplings g_i on the scale M, *i.e.* $g_i \propto M^{-1/2}$ which means that when the Lorentz symmetric limit $M \to \infty$ is taken, $g_i \to 0$ and, therefore, we are left with two relativistic free fermions, for which flavour oscillations have been generated dynamically. In this way, any set of values for $m_\pm$ and θ can be described by the Lorentz-symmetric limit of the present model.

Furthermore, although there is no certainty about the nature of neutrinos, *i.e.* whether they are Dirac or Majorana fermions, it is likely that they are Majorana. Thus, in [18], the authors have studied two simple and consistent ways of extending the present model to the case of Majorana neutrinos, either by considering only left-handed fields or adding right-handed sterile fields. In the first extension, it has been shown that neutrino masses and oscillations can be generated dynamically without the involvement of right-handed. In the second case, a seesaw-type solution has been found, showing that the mechanism presented in this section can generate heavy masses for sterile neutrinos which suppress the masses of the light active ones.

3.3. Lorentz symmetric limit and the oscillation probability

An important point, which differs the present model from other models containing four-fermion interactions, such as the NJL model, is that, in our case, dynamical mass generation occurs for any coupling strength, and no critical coupling exists, bellow which this non-perturbative process does not occur. This feature allows us to take the Lorentz symmetric limit of the model, $M \to \infty$ which from (52) implies that $g_i \to 0$, in such a way that the dynamical mass (1) remains finite. In this limit, where the product Mg^2 goes to a finite value, we are left with free relativistic massive fermions, for which the mass has been generated by quantum corrections.

Because the model (34) violates Lorentz symmetry, it is expected that space and time derivatives are dressed differently by quantum corrections. But since a consistent Lorentz symmetric limit in our case implies $g^2 \to 0$, loop corrections to the kinetic terms in the model (34) eventually vanish in this limit and the theory is therefore well-behaved.

To illustrate this point, we calculate the superficial degree of divergence D of an L-loop graph $\Sigma^{(L)}$ contributing to the self energy. Each loop integral measure carries the mass dimension 4 and each propagator (37) has mass dimension -1. The corrections p^2/M^2 are at most equal to 1, since integrals are regularized by M: they do not play a role for the superficial degree of divergence. Hence $D = 4L - I$, where I is the number of internal propagators. Momentum conservation leads to $L = I - V + 1$, where V is the number of vertices. Also, since each vertex has 4 legs, and each internal propagator relates two vertices, we have $4V = E + 2I$, where E is the number of external propagators. As a consequence, we have, as for the usual NJL model,

$$D = 2L + 2 - \frac{E}{2} \ , \quad \text{and} \quad V = L - 1 + \frac{E}{2} \ . \tag{53}$$

In our case though, each vertex brings a factor g^2/M^2, hence for the self energy ($E = 2$) we have

$$\Sigma^{(L)} \propto \left(\frac{g^2}{M^2}\right)^V M^D = Mg^{2L} \ . \tag{54}$$

Taking into account (52) when $M \to \infty$, we finally obtain

$$\Sigma^{(L)} \propto \frac{m_{dyn}^L}{M^{L-1}} \ . \tag{55}$$

The first non-trivial loop corrections to the kinetic terms occur at two loops, since the one-loop self energy is independent of the external momentum. As a consequence we are interested in $L \geq 2$, and for a *fixed* dynamical mass m_{dyn}, the property (55) therefore shows that the loop correction $\Sigma^{(L)}$ goes to 0 when $M \to \infty$: quantum corrections to the kinetic terms vanish in the Lorentz symmetric limit.

Finally, we now show that the oscillation probability for the present model is the same as the oscillation probability for relativistic neutrinos. As already seen, the general expression for the oscillation probability when considering two flavours is (9). In our case, using the LV dispersion relation (35) with $a = b = c = 1$ and considering, as usual, $m_\pm^2/p^2 \ll 1$ and $m_\pm/M << 1$, we find

$$(E_+ - E_-)t = \frac{(m_+^2 - m_-^2)L}{2E} + (m_+ - m_-)\frac{EL}{M} + \mathcal{O}(m_\pm^2/M^2) \ . \tag{56}$$

Therefore, the corresponding oscillation probability can be written as

$$\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(m_+^2 - m_-^2)L}{4E} + (m_+ - m_-)\frac{EL}{2M} + ...\right] \ , \tag{57}$$

where the first term gives the usual expression and the second term represents a correction due to the LV nature of the present model. This second term however goes to zero when $M \to \infty$, and (57) reduces to the usual relativistic oscillation probability in Lorentz-invariant vacuum, as expected.

4. Conclusions

In this article we have shown how neutrino masses and oscillations can be generated dynamically when considering two distinct Lorentz-violating models.

For the first model studied in section 2, we have considered the coupling of flavoured fermion fields to LV vector gauge bosons, with Lorentz invariance being violated in the gauge sector due to the presence of higher order space derivatives suppressed by a mass scale M. In order to study the possibility of dynamical mass generation we considered the non-perturbative Schwinger-Dyson approach. Moreover, we have shown that vector boson mass generation can be avoided by appropriate arrangement of the couplings, so that the LV vector bosons are viewed as regulator fields, with the only remnant of the LV the dynamical fermion mass. Unfortunately, the dynamical equations allow neutrino oscillations in only one case when one of the fermion mass eigenstates is massless, while the other is massive and the mixing angle is necessarily maximal $\theta = \pm\pi/4$.

Whereas in section 3, we have studied a four-fermion interaction model containing neutral fermions, representing neutrinos, with Lorentz-violating kinematics. As in the first model, Lorentz violation is achieved by higher order space derivatives suppressed by a mass scale M. Using the non-perturbative method of the effective potential, we have shown that neutrino masses are generated dynamically and remain finite even when the Lorentz symmetric limit is consistently taken. In contrast to the first model, this one can describe any set of phenomenological values for the neutrino masses and mixing angles.

The originality of our models therefore consists in generating masses and flavour oscillations from quantum corrections, which imply finite effects in the IR, even after removing the original LV regulator. Such procedures are not based on tree level processes and allow to recover the Lorentz-symmetric limit after quantisation.

An extension of the present works consists in investigating more closely the origin of LV operators, such as those present in the models discussed here, in the neutrino sector. For this, we should consider microscopic LV gravitational models coupled to fermions in which the 4-dimensional diffeomorphism symmetry is broken, as in Horava-Lifshitz Gravity for example [21].

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