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# Boson field mixing in Rindler spacetime

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**Abstract.** We study the mixing of two complex boson fields with different masses in Rindler spacetime. We find that the Bogoliubov transformation associated with field mixing combines in a non-trivial way with the thermal Bogoliubov transformation associated to the uniformly accelerated observer (Rindler observer): as a result, the spectrum of Unruh radiation gets modified.

## 1. Introduction

Mixing transformations for quantum fields [1] have been found to be non-trivial, since they are associated to inequivalent representations of the canonical commutation relations [2]. This has been shown first in the case of Dirac fermions [3] and later for other fields [4, 5, 6]. The reason for such a somewhat unexpected result resides in the fact that the mixing transformations for fields with different masses, which are just a rotation at level of fields, actually hide Bogoliubov transformations at level of creation/annihilation operators. This in turn induces a drastic change into the vacuum structure, which becomes a condensate of particle/antiparticle pairs.

These studies have been carried out only in Minkowski spacetime. It arises thus the question how the above structure appears in general frame, and in particular for the case of a uniformly accelerated observer - Rindler spacetime. In this case, it is well known that the quantization of a free field leads to Bogoliubov transformations relating the ladder operators for Rindler and Minkowski observers and giving rise to the celebrated Unruh effect, i.e. the detection of thermal radiation by a uniformly accelerated observer [7]-[12].

In this paper we consider the quantization of mixed fields in Rindler spacetime in the simplest case of two boson fields with different masses. Despite of such minimal setting, a rich mathematical structure arises, due to the combination of the two different Bogoliubov transformations involved (the one associated to mixing, the other to Rindler spacetime). The main result of our analysis is a modification of the number spectrum of particles in Unruh radiation induced by the mixing terms.

In the following we first quantize free fields in Minkowski spacetime in the hyperbolic basis, in which boost generator is diagonalized. Then we discuss the quantization for a free field in Rindler spacetime and finally we consider mixed fields for an accelerated observer. A preliminary result about the correction to Unruh effect due to mixing terms is given.

## 2. Field operator in hyperbolic representation

We consider a free complex scalar field $\phi$ with mass $m$ in a four-dimensional Minkowski spacetime

$$\phi(x) = \int d^3k \left( a_{\mathbf{k}}\, U_{\mathbf{k}}(x) + \bar{a}^\dagger_{\mathbf{k}}\, U^*_{\mathbf{k}}(x) \right), \tag{1}$$

where the operators $a_{\mathbf{k}}$ and $\bar{a}_{\mathbf{k}}$ are assumed to obey the canonical commutation relations,

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = [\bar{a}_{\mathbf{k}}, \bar{a}^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}'), \tag{2}$$

with all other commutators vanishing. These operators can be interpreted as annihilation operators of Minkowski particle and antiparticle, respectively. The Minkowski vacuum $|0_M\rangle$ is defined by

$$a_{\mathbf{k}}|0_M\rangle = \bar{a}_{\mathbf{k}}|0_M\rangle = 0, \qquad \forall \mathbf{k}. \tag{3}$$

The mode $U_{\mathbf{k}}(x)$ in Eq.(1) is a plane-wave of the type

$$U_{\mathbf{k}}(x) = \left[2\omega_k (2\pi)^3\right]^{-\frac{1}{2}} e^{i\left(\mathbf{k}\cdot\mathbf{x} - \omega_k x^0\right)}, \tag{4}$$

where

$$\mathbf{k} = (k_1, k_2, k_3), \qquad \omega_k = \sqrt{m^2 + |\mathbf{k}|^2}. \tag{5}$$

This mode is said to be of *positive frequency* with respect to the Minkowski time $x^0$, since it depends on $x^0$ as

$$U_{\mathbf{k}}(x) \propto e^{-i\omega_k x^0} \tag{6}$$

and it is a solution of the Klein-Gordon equation (we use the metric $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$)

$$\left\{ \left(\frac{\partial}{\partial x^0}\right)^2 - \sum_{j=1}^{3} \left(\frac{\partial}{\partial x^j}\right)^2 + m^2 \right\} \phi = 0. \tag{7}$$

Moreover the modes (4) are normalized with respect to the Klein-Gordon (KG) inner product

$$(\phi_1, \phi_2) = i \int d^3x \left[ \phi_2^*(x) \overleftrightarrow{\partial_0}\, \phi_1(x) \right]; \tag{8}$$

where we have implicitly assumed that the integration is performed on a hypersurface of constant $x^0$. In fact, we have

$$(U_{\mathbf{k}}, U_{\mathbf{k}'}) = \delta^3(\mathbf{k} - \mathbf{k}'), \quad (U^*_{\mathbf{k}}, U^*_{\mathbf{k}'}) = -\delta^3(\mathbf{k} - \mathbf{k}'), \quad (U_{\mathbf{k}}, U^*_{\mathbf{k}'}) = 0. \tag{9}$$

The Hamiltonian and momentum operator are easily expressed in terms of operators (2). They are, respectively

$$H = \int d^3k\, \omega_k \left( a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \bar{a}_{\mathbf{k}} \bar{a}^\dagger_{\mathbf{k}} \right), \qquad \mathbf{k} = \int d^3k\, \mathbf{k} \left( a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \bar{a}_{\mathbf{k}} \bar{a}^\dagger_{\mathbf{k}} \right). \tag{10}$$

Therefore, with the particular choice of the plane-wave basis, the field quanta carry *momentum* $\mathbf{k}$ and *energy* $\omega_k$. In this basis, however, other physically relevant operators like the angular momentum, are not diagonal. The diagonalization of the angular momentum is obtained by expanding the field in the *spherical wave* basis (see, for example, [13]).

Now we are interested in the diagonalization of the *generator of boosts* [12]. We start from the expression of the generators of Lorentz transformations

$$M^{(\alpha,\beta)} = \int d^3x \, (x^\alpha \, T^{(0,\beta)} - x^\beta \, T^{(0,\alpha)}). \tag{11}$$

The generator of boost (for example along the $x^1$ axis) is the $(1,0)$ component of the above tensor. Using the standard expression for the stress tensor $T_{\mu\nu}$

$$T_{\mu\nu} = \partial_\mu\phi^* \, \partial_\nu\phi + \partial_\nu\phi^* \, \partial_\mu\phi - g_{\mu\nu}\left(\partial_\lambda\phi^* \, \partial^\lambda\phi - m^2\phi^*\phi\right) \tag{12}$$

and substituting the Minkowski expansion (1) of the field, one obtains [14]

$$M^{(1,0)} = i\int \frac{d^3k}{2\omega_k}\left(c^\dagger_{\mathbf{k}}\,\omega_k\,\frac{\partial}{\partial k_1}c_{\mathbf{k}} + \bar{c}^\dagger_{\mathbf{k}}\,\omega_k\,\frac{\partial}{\partial k_1}\bar{c}_{\mathbf{k}}\right), \tag{13}$$

where

$$c_{\mathbf{k}} \equiv \sqrt{2\omega_k}\, a_{\mathbf{k}}. \tag{14}$$

The presence of the $k_1$ derivative shows that the generator of boost in plane-wave representation is nondiagonal. We then introduce the following operators [12]:

$$d^{(\sigma)}_{\Omega,\vec{k}} = \int_{-\infty}^{+\infty} dk_1 \, p^{(\sigma)}_\Omega(k_1)\, a_{k_1,\vec{k}}\,, \qquad \bar{d}^{(\sigma)}_{\Omega,\vec{k}} = \int_{-\infty}^{+\infty} dk_1 \, p^{(\sigma)}_\Omega(k_1)\, \bar{a}_{k_1,\vec{k}}\,, \tag{15}$$

where $\vec{k} = (k_2, k_3)$, $\Omega$ is a positive parameter and $\sigma = \pm$ (the physical significance of $\Omega$ and $\sigma$ will be clarified in Section 3). The function $p^{(\sigma)}_\Omega(k_1)$ in Eq.(15) is given by

$$p^{(\sigma)}_\Omega(k_1) = \frac{1}{\sqrt{2\pi\omega_k}}\left(\frac{\omega_k + k_1}{\omega_k - k_1}\right)^{i\sigma\Omega/2}. \tag{16}$$

It is possible to show that the set of functions $\left\{p^{(\sigma)}_\Omega\right\}$ forms a complete orhonormal set, i.e.

$$\sum_\sigma \int_0^{+\infty} d\Omega \; p^{(\sigma)}_\Omega(k_1)\, p^{(\sigma)*}_\Omega(k_1') = \delta(k_1 - k_1'), \tag{17}$$

$$\int_{-\infty}^{+\infty} dk_1 \; p^{(\sigma)*}_\Omega(k_1)\, p^{(\sigma')}_{\Omega'}(k_1) = \delta_{\sigma\sigma'}\,\delta(\Omega - \Omega'). \tag{18}$$

We note that $d^{(\sigma)}_{\Omega,\vec{k}}$ and $\bar{d}^{(\sigma)}_{\Omega,\vec{k}}$ are linear combinations of the Minkowski annihilation operators alone. Therefore they also annihilate the Minkowski vacuum,

$$d^{(\sigma)}_{\Omega,\vec{k}}\,|0_M\rangle = \bar{d}^{(\sigma)}_{\Omega,\vec{k}}\,|0_M\rangle = 0\,, \qquad \forall\sigma,\,\Omega,\,\vec{k}\,. \tag{19}$$

Moreover, by virtue of Eqs.(17), they satisfy the canonical commutation relations,

$$[d^{(\sigma)}_{\Omega,\vec{k}}\,,\, d^{(\sigma')\dagger}_{\Omega',\vec{k}'}] = [\bar{d}^{(\sigma)}_{\Omega,\vec{k}},\, \bar{d}^{(\sigma')\dagger}_{\Omega',\vec{k}'}] = \delta_{\sigma\sigma'}\,\delta(\Omega - \Omega')\,\delta^2(\vec{k} - \vec{k}'), \tag{20}$$

with all other commutators vanishing. Because of Eqs.(19) and (20), the operators $d^{(\sigma)}_{\Omega,\vec{k}}$ and $\bar{d}^{(\sigma)}_{\Omega,\vec{k}}$ could also be called annihilation operators of a Minkowski particle.

Now, inverting Eq.(15) by use of Eq.(18), we have

$$a_{\mathbf{k}} = \sum_{\sigma} \int_0^{+\infty} d\Omega \, p_{\Omega}^{(\sigma)*}(k_1) \, d_{\Omega,\vec{k}}^{(\sigma)} \,. \tag{21}$$

Substituting Eq.(21) into Eq.(13), we find

$$M^{(1,0)} = \int_0^{+\infty} d\Omega \int d^2k \sum_{\sigma} \sigma \, \Omega \, \left( d_{\Omega,\vec{k}}^{(\sigma)\dagger} \, d_{\Omega,\vec{k}}^{(\sigma)} + \bar{d}_{\Omega,\vec{k}}^{(\sigma)\dagger} \, \bar{d}_{\Omega,\vec{k}}^{(\sigma)} \right) . \tag{22}$$

This shows that the new Minkowski particles $d_{\Omega,\vec{k}}^{(\sigma)}$ diagonalize the generator of boosts. Note that, since a boost transformation involves both time and space coordinates, the Hamiltonian operator preserves its diagonal structure provided that it is expressed in terms of the new time coordinate (as we shall see later).

Now we want to determine the expression of the modes associated with the operators $d_{\Omega,\vec{k}}^{(\sigma)}$. For this purpose, let us consider the field expansion (1). After inserting Eq.(21), we obtain

$$\phi(x) = \int_0^{+\infty} d\Omega \int d^2k \, \sum_{\sigma} \left( d_{\Omega,\vec{k}}^{(\sigma)} \, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)}(x) + \bar{d}_{\Omega,\vec{k}}^{(\sigma)\dagger} \, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)*}(x) \right), \tag{23}$$

where

$$\widetilde{U}_{\Omega,\vec{k}}^{(\sigma)}(x) = \int_{-\infty}^{+\infty} dk_1 \, p_{\Omega}^{(\sigma)*}(k_1) \, U_{\mathbf{k}}(x), \tag{24}$$

with $U_{\mathbf{k}}(x)$ defined in Eq.(4). The last integral is explicitly solved in Appendix; introducing the *hyperbolic* (or Rindler) coordinates $(\eta, \xi)$, related to the Minkowski coordinates by

$$x^0 = \xi \sinh \eta \,, \qquad x^1 = \xi \cosh \eta, \qquad -\infty < \eta, \xi < \infty, \tag{25}$$

one obtains[1]

$$\widetilde{U}_{\Omega,\vec{k}}^{(\sigma)}(x) = \frac{e^{\sigma\pi\Omega/2}}{2\sqrt{2}\pi^2} \, K_{i\sigma\Omega}(\mu_k \xi) \, e^{i\left(\vec{k}\cdot\vec{x} - \sigma\Omega\eta\right)}, \tag{26}$$

where $K_{i\sigma\Omega}(\mu_k\xi)$ is the *modified Bessel function of second kind* and

$$\mu_k = \sqrt{m^2 + k_2^2 + k_3^2}. \tag{27}$$

It can be shown that the set of modes $\left\{ \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)}, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)*} \right\}$ is complete and orthonormal with respect to the Klein-Gordon inner product, i.e.

$$\left( \widetilde{U}_{\Omega',\vec{k}'}^{(\sigma')}, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)} \right) = -\left( \widetilde{U}_{\Omega',\vec{k}'}^{(\sigma')*}, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)*} \right) = \delta_{\sigma\sigma'} \delta(\Omega - \Omega') \delta^2(\vec{k} - \vec{k}') \,, \qquad \left( \widetilde{U}_{\Omega',\vec{k}'}^{(\sigma')}, \widetilde{U}_{\Omega,\vec{k}}^{(\sigma)*} \right) = 0. \tag{28}$$

Moreover it is interesting to note that, with respect to the Minkowski coordinates, the conserved quantity associated to these new field quanta is the relativistic generalization of the so-called *center of mass* (see for example [15]).

[1] To be precise, we remark that Eq.(26) is valid only on the Rindler manifold $R_+ \cup R_-$, since the coordinate transformation (25) holds only on these regions. One could also introduce Rindler-type coordinates to cover the remaining regions of the spacetime (see, for example, [11]), but this is not necessary in our case. The correct *global* functions, namely the Gerlach's Minkowski Bessel modes, are defined in Ref.[16].

## 3. From Minkowski to Rindler

The foregoing discussion is completely within the usual framework of the *Minkowski quantization*. We now describe an alternative scheme first discussed by Fulling [9] and called the *Rindler-Fulling quantization* (note that in this Section we will closely follow Ref.[12]).
For this purpose, we make use of the coordinates (25) (observe that $\vec{x} = (x^1,\, x^2)$ is common to both sets of coordinates). The line element

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = (dx^0)^2 - (dx^1)^2 - \sum_{j=2}^{3} (dx^j)^2 \tag{29}$$

takes the form

$$ds^2 = \xi^2 d\eta^2 - d\xi^2 - \sum_{j=2}^{3} (dx^j)^2. \tag{30}$$

We remark that the Rindler coordinates cover only two portions of Minkowski spacetime, namely the "positive"(or right) Rindler wedge, if $\xi > 0$:

$$R_+ = \{x | x^1 > |x^0|\}, \tag{31}$$

and the "negative"(or left) Rindler wedge, if $\xi < 0$:

$$R_- = \{x | x^1 < -|x^0|\}. \tag{32}$$

Since the metric in the Rindler coordinates does not depend on $\eta$, the vector

$$B = \frac{\partial}{\partial \eta} \tag{33}$$

with components

$$B^\eta = 1, \quad B^\xi = B^j = 0, \qquad j = 2, 3, \tag{34}$$

is a timelike Killing vector. Using Eq.(25), it follows that

$$B = \frac{\partial x_0}{\partial \eta}\frac{\partial}{\partial x_0} + \frac{\partial x_1}{\partial \eta}\frac{\partial}{\partial x_1} = x^1 \frac{\partial}{\partial x_0} + x^0 \frac{\partial}{\partial x_1}\,, \tag{35}$$

i.e., $B$ is the *boost Killing vector*.

In order to understand the physical relevance of the Rindler coordinates, we now consider a world line such that

$$\xi(\tau) = \mathrm{const} \equiv a^{-1}, \qquad \vec{x}(\tau) = \mathrm{const}, \tag{36}$$

where $\tau$ is the proper time measured along the line. Substituting Eq.(36) into the line element (30), we find that

$$\eta(\tau) = a\tau. \tag{37}$$

Therefore, for an observer moving along the line (36), the Rindler time $\eta$ is the same as the proper time $\tau$ up to the scale factor $a$. Combining this result with Eq.(35), we find that the time evolution of this observer actually an infinite succession of Minkowski boost transformations. The hypersurfaces $\eta = $ const describe events which are simultaneous from the point of view of this observer. We also note that, by our convention, $\eta(\tau)$ is a decreasing function of $\tau$ if $a < 0$, which occurs when the world line lies in the negative wedge $R_-$.

Coming back to the Minkowski coordinates, the world line (36) takes the form

$$x^0(\tau) = a^{-1} \sinh a\tau, \quad x^1(\tau) = a^{-1} \cosh a\tau, \quad \vec{x}(\tau) = \mathrm{const}. \tag{38}$$

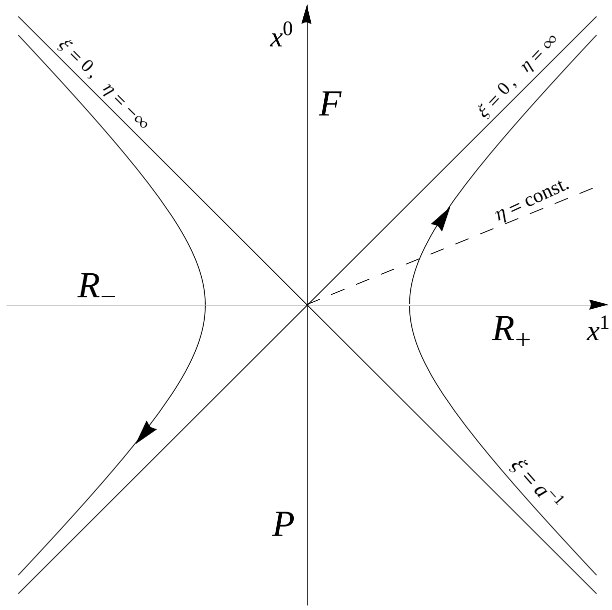


**Figure 1.** The proper coordinate system of a uniformly accelerated observer in the Minkowski spacetime.The hyperbola represents the world line of an observer with proper acceleration $a$. The asymptotes $x^0 = \pm x^1$ appear to this observer as future and past horizonts, respectively. The Rindler observer cannot receive any signals from the region $F$ and $R_-$ and cannot send signals to $P$ and $R_-$. Therefore the wedges $R_+$ and $R_-$ are causally separeted. The regions $F$ and $P$, however, are not covered by the proper coordinate system (25).

This is an hyperbola in the $(x^0, x^1)$ plane with asymptotes $x^0 = \pm x^1$ (see figure below). Varying $a$, we obviously obtain different hyperbolas with the same characteristics. We can think at the Rindler spacetime as the collection of these world lines.
It is not difficult to show that Eq.(38) represents the world line of an uniformly accelerated observer with proper acceleration $|a|$ (see, for example, [10]), and this is the reason why Rindler spacetime is generally regarded as the "natural"manifold in which to describe accelerate motion. When $a$ is positive, the accelerated observer will be referred to as *the Rindler observer.*

One may now wonder which is the most important difference between the Minkowski and the Rindler metrics. About this, we note that a Rindler observer, namely a uniformly accelerated observer in $R_+$, is *causally separeted* from $R_-$. In addition he cannot receive any signal from the future wedge ($x^0 > |x^1|$). Therefore, the null hyperplane $x^0 = |x^1|$ appears to him as a *future event horizon* (in the same way, it is easy to show that the null hyperplane $x^0 = -|x^1|$ appears to him as a *past event horizon*). Clearly for a Minkowski observer there is no horizont at all!

In what follows we will also denote the set of coordinates $(\eta, \xi, \vec{x})$ by $x$; therefore the symbol $x$ refers to a spacetime point, rather than its representative in a particular coordinate system.

After these general considerations about the Rindler metric, we work in the Rindler coordinates to solve the Klein-Gordon equation (7), which then takes the form

$$\left\{\frac{1}{\xi^2}\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\xi^2} - \frac{1}{\xi}\frac{\partial}{\partial\xi} - \sum_{j=2}^{3}\left(\frac{\partial}{\partial x^j}\right)^2 + m^2\right\}\phi = 0. \tag{39}$$

We look for solutions which are of positive frequency with respect to the Rindler time $\eta$

$$u_{\mathbf{k}}^{(\sigma)} = \frac{\theta(\sigma\xi)}{2\Omega\sqrt{2\pi}}\, h_{\mathbf{k}}^{(\sigma)(\xi)}\, e^{i\left(\vec{k}\cdot\vec{x} - \sigma\Omega\eta\right)}, \tag{40}$$

where $\sigma = +$ refers to the positive wedge $R_+$ (Eq.(31)) and $\sigma = -$ to the negative wedge $R_-$ (Eq.(32)), $\theta$ is the Heaviside step function and the subscript $\mathbf{k}$ stands for $(\Omega, \vec{k})$. According to our previous considerations, we remark that the $\theta$ function has been inserted in Eq.(40) in order to restrict these modes to only one of the two causally separeted wedges ($R_+$ for $\sigma = +$ and $R_-$ for $\sigma = -$).
Observe that the reason why we have chosen $u_{\mathbf{k}}^{(\sigma)}$ such that

$$u_{\mathbf{k}}^{(\sigma)} \propto e^{-i\sigma\Omega\eta} \tag{41}$$

(and not $u_{\mathbf{k}}^{(\sigma)} \propto e^{-i\Omega\eta}$) is that the boost Killing vector $B$, Eq.(33), is future oriented in $R_+$, while it is past oriented in $R_-$.

In order to determine the expression of $h_{\mathbf{k}}^{(\sigma)}$, we substitute Eq.(40) into (39); it follows that

$$\left\{\frac{d^2}{d\xi^2} + \frac{1}{\xi}\frac{d}{d\xi} + \frac{\Omega^2}{\xi^2} - \mu_k^2\right\} h_{\mathbf{k}}^{(\sigma)} = 0. \tag{42}$$

This is again solved in terms of modified Bessel functions of second kind; in particular, requiring that these functions are delta-normalized, one obtains (see [12] for details)

$$h_{\mathbf{k}}^{(\sigma)} = \sqrt{\frac{2}{\pi}} A_{\mathbf{k}}^{(\sigma)} \left(\frac{\alpha\mu_k}{2}\right)^{i\Omega} \Gamma(i\Omega)^{-1} K_{i\Omega}(\mu_k \xi), \tag{43}$$

where

$$A_{\mathbf{k}}^{(+)} = R_{\mathbf{k}}^* \left(\tfrac{\alpha\mu_k}{2}\right)^{-i\Omega} \Gamma(i\Omega)/|\Gamma(i\Omega)|, \tag{44}$$

$$A_{\mathbf{k}}^{(-)} = R_{\mathbf{k}} \left(\tfrac{\alpha\mu_k}{2}\right)^{i\Omega} \Gamma(-i\Omega)/|\Gamma(i\Omega)|, \tag{45}$$

with $R_{\mathbf{k}} = \left[(\alpha\mu_k/2)^{-i\Omega}\, \Gamma(i\Omega)/|\Gamma(i\Omega)|\right]^2$ and $\alpha$ arbitrary postive constant of dimension of lenght.
By using Eq.(43), it is possible to show that the complete set of *Rindler modes* $\{u_{\mathbf{k}}^{(\sigma)}, u_{\mathbf{k}}^{(\sigma)*}\}$ is orthonormal with respect to the KG inner product. To see this, we note that in the Rindler coordinates Eq.(8) reduces, if the integration is performed on a hypersurface of constant $\eta$, to

$$(\phi_1, \phi_2) = i \int_{-\infty}^{+\infty} \frac{d\xi}{|\xi|} \int d^2x\, \phi_2^* \overleftrightarrow{\partial}_\eta\, \phi_1. \tag{46}$$

By virtue of the $h_{\mathbf{k}}^{(\sigma)}$ normalization, this formula gives

$$(u_{\mathbf{k}}^{(\sigma)}, u_{\mathbf{k}'}^{(\sigma')}) = -(u_{\mathbf{k}}^{(\sigma)*}, u_{\mathbf{k}'}^{(\sigma')*}) = \delta_{\sigma\sigma'}\, \delta(\Omega - \Omega')\, \delta^2(\vec{k} - \vec{k}'), \qquad (u_{\mathbf{k}}^{(\sigma)}, u_{\mathbf{k}'}^{(\sigma')*}) = 0. \tag{47}$$

Now we are ready to describe the Rindler-Fulling quantization scheme. For this purpose, by exploiting the completeness of the Rindler modes, we take the following expansion for the Klein-Gordon field

$$\phi(x) = \int_0^{+\infty} d\Omega \int d^2k \sum_\sigma \left(b_{\mathbf{k}}^{(\sigma)}\, u_{\mathbf{k}}^{(\sigma)}(x) + \bar{b}_{\mathbf{k}}^{(\sigma)\dagger}\, u_{\mathbf{k}}^{(\sigma)*}(x)\right), \tag{48}$$

where the subscript $\mathbf{k}$ stands for $(\Omega, \vec{k})$ as discussed above. Observe that, although the Minkowski expansion (1) holds for all the points of spacetime, the Rindler expansion (48) is valid only in

the Rindler manifold $R_+ \cup R_-$.
The operators $b_{\mathbf{k}}^{(\sigma)}$ and $\bar{b}_{\mathbf{k}}^{(\sigma)}$ are assumed to obey the canonical commutation relations,

$$[b_{\mathbf{k}}^{(\sigma)}, b_{\mathbf{k}'}^{(\sigma')\dagger}] = [\bar{b}_{\mathbf{k}}^{(\sigma)}, \bar{b}_{\mathbf{k}'}^{(\sigma')\dagger}] = \delta_{\sigma\sigma'}\, \delta(\Omega - \Omega')\, \delta^2(\vec{k} - \vec{k}'), \tag{49}$$

with other commutators vanishing. They are called annihilation operators of Rindler-Fulling particles and antiparticles, respectively. The Rindler-Fulling vacuum $|0_R\rangle$ is defined by

$$b_{\mathbf{k}}^{(\sigma)}|0_R\rangle = \bar{b}_{\mathbf{k}}^{(\sigma)}|0_R\rangle = 0, \qquad \forall\sigma, \mathbf{k}. \tag{50}$$

To conclude this Section, we observe that, by expressing the Minkowski Hamiltonian (10) in terms of $b_{\mathbf{k}}^{(\sigma)}$ and $\bar{b}_{\mathbf{k}}^{(\sigma)}$, we obtain the Rindler-Fulling Hamiltonian $H_R$:

$$H_R = H_R^{(+)} - H_R^{(-)}, \tag{51}$$

where

$$H_R^{(\sigma)} = \int_0^{+\infty} d\Omega \int d^2k\ \Omega \left(b_{\mathbf{k}}^{(\sigma)\dagger}\, b_{\mathbf{k}}^{(\sigma)} + \bar{b}_{\mathbf{k}}^{(\sigma)}\, \bar{b}_{\mathbf{k}}^{(\sigma)\dagger}\right). \tag{52}$$

The minus sign in front of $H_R^{(-)}$ is due to the fact that $B$ is past oriented in $R_-$. We will have more to say about $H_R$ in the next Section.

## 4. Unruh effect

Let us now study the connection between the Minkowski and the Rindler-Fulling quantization schemes. For this purpose we equate the two alternative expansions (1) and (48) for the field on a spacelike hypersurface $\Sigma$ which lies in the Rindler manifold $R_\pm$ (for instance $\Sigma$ may be chosen to be the hyperplane of constant $\eta$). Forming the Klein-Gordon inner product of the field $\phi$ with the Rindler mode $u_{\mathbf{k}}^{(\sigma)}$ and using the orthonormal properties (47), we obtain the *thermal Bogoliubov transformation*

$$b_{\mathbf{k}}^{(\sigma)} = \int d^3k' \left(\alpha_{\mathbf{k}\mathbf{k}'}^{(\sigma)*}\, a_{\mathbf{k}'} + \beta_{\mathbf{k}\mathbf{k}'}^{(\sigma)*}\, \bar{a}_{\mathbf{k}'}^{\dagger}\right), \tag{53}$$

where

$$\alpha_{\mathbf{k}\mathbf{k}'}^{(\sigma)} = (u_{\mathbf{k}}^{(\sigma)}, U_{\mathbf{k}'}), \tag{54}$$

$$\beta_{\mathbf{k}\mathbf{k}'}^{(\sigma)} = (u_{\mathbf{k}}^{(\sigma)}, U_{\mathbf{k}'}^{*}) \tag{55}$$

are the so-called *thermal Bogoliubov coefficients*. Similarly forming the inner product of $\phi$ with $u_{\mathbf{k}}^{(\sigma)*}$, we obtain

$$\bar{b}_{\mathbf{k}}^{(\sigma)} = \int d^3k' \left(\alpha_{\mathbf{k}\mathbf{k}'}^{(\sigma)*}\, \bar{a}_{\mathbf{k}'} + \beta_{\mathbf{k}\mathbf{k}'}^{(\sigma)*}\, a_{\mathbf{k}'}^{\dagger}\right). \tag{56}$$

The relation (56) follows from the (53) on interchange of particles with antiparticles, as it should. The explicit calculation of the coefficients (54) is performed in [12]; it gives

$$\alpha_{\mathbf{k}\mathbf{k}'}^{(\sigma)} = \frac{1}{2\pi}\, \delta^2(\vec{k} - \vec{k}')\, e^{\pi\Omega/2}\, |\Gamma(i\Omega)| \left(\frac{\Omega}{\omega_{k'}}\right)^{1/2} \left(\frac{\omega_{k'} + k_1'}{\omega_{k'} - k_1'}\right)^{-i\sigma\Omega/2}, \tag{57}$$

$$\beta_{\mathbf{k}\mathbf{k}'}^{(\sigma)} = \frac{1}{2\pi}\, \delta^2(\vec{k} + \vec{k}')\, e^{-\pi\Omega/2}\, |\Gamma(i\Omega)| \left(\frac{\Omega}{\omega_{k'}}\right)^{1/2} \left(\frac{\omega_{k'} + k_1'}{\omega_{k'} - k_1'}\right)^{-i\sigma\Omega/2}. \tag{58}$$

We recall that the first subscript $\mathbf{k}$ of $\alpha$ and $\beta$ on the right-hand side stands for $(\Omega, \vec{k})$ while the second subscript $\mathbf{k}'$ for $(k_1', \vec{k}')$.

Substituting Eqs.(57) and (58) into the transformation (53), we can rewrite it into a very transparent form,

$$b^{(\sigma)}_{\Omega,\vec{k}} = \sqrt{1+N(\Omega)}\, d^{(\sigma)}_{\Omega,\vec{k}} + \sqrt{N(\Omega)}\, \bar{d}^{(-\sigma)\dagger}_{\Omega,-\vec{k}}\,, \tag{59}$$

where the operator $d^{(\sigma)}_{\Omega,\vec{k}}$ has been defined in Eq.(15) and

$$N(\Omega) = \left(e^{2\pi\Omega} - 1\right)^{-1}. \tag{60}$$

We remark that the same result, up to an irrelevant[2] global phase factor, can be obtained by equating expansions (23) and (48) and forming the inner product of both sides with the Rindler mode $u^{(\sigma)}_{\mathbf{k}}$. Moreover, by adopting the notation

$$\mathbf{k} = (\Omega, \vec{k})\,, \qquad \bar{\mathbf{k}} = (\Omega, -\vec{k})\,, \tag{61}$$

it follows from Eq.(59) that

$$\begin{aligned} b^{(\sigma)\dagger}_{\mathbf{k}}\, b^{(\sigma)}_{\mathbf{k}} + \bar{b}^{(\sigma)}_{\bar{\mathbf{k}}}\, \bar{b}^{(\sigma)\dagger}_{\bar{\mathbf{k}}} &= d^{(\sigma)\dagger}_{\mathbf{k}}\, d^{(\sigma)}_{\mathbf{k}} + \bar{d}^{(\sigma)}_{\bar{\mathbf{k}}}\, \bar{d}^{(\sigma)\dagger}_{\bar{\mathbf{k}}} + N(\Omega)\sum_{\sigma}(d^{(\sigma)\dagger}_{\mathbf{k}}\, d^{(\sigma)}_{\mathbf{k}} + \bar{d}^{(\sigma)}_{\bar{\mathbf{k}}}\, \bar{d}^{(\sigma)\dagger}_{\bar{\mathbf{k}}}) \\ &+ \sqrt{N(\Omega)(1+N(\Omega))}\sum_{\sigma}(\bar{d}^{(-\sigma)}_{\mathbf{k}}\, d^{(\sigma)}_{\bar{\mathbf{k}}} + d^{(\sigma)\dagger}_{\bar{\mathbf{k}}}\, \bar{d}^{(-\sigma)\dagger}_{\mathbf{k}}). \end{aligned} \tag{62}$$

Therefore

$$\sum_{\sigma}\sigma\,(b^{(\sigma)\dagger}_{\mathbf{k}}\, b^{(\sigma)}_{\mathbf{k}} + \bar{b}^{(\sigma)}_{\bar{\mathbf{k}}}\, \bar{b}^{(\sigma)\dagger}_{\bar{\mathbf{k}}}) = \sum_{\sigma}\sigma\,(d^{(\sigma)\dagger}_{\mathbf{k}}\, d^{(\sigma)}_{\mathbf{k}} + \bar{d}^{(\sigma)^\dagger}_{\bar{\mathbf{k}}}\, \bar{d}^{(\sigma)}_{\bar{\mathbf{k}}}). \tag{63}$$

This shows that

$$H_R = M^{(1,0)}, \tag{64}$$

i.e., the Rindler Hamiltonian $H_R$ also generates boost. This also could have been predicted from Eqs.(33) and (35). To be precise, the identity (64) is valid on the Rindler manifold $R_+ \cup R_-$; therefore $H_R$ is the restriction of $M^{(1,0)}$ to $R_+ \cup R_-$.

Now, by using the Bogoliubov transformation (59) and recalling that $d^{(\sigma)}_{\mathbf{k}}$ annihilates the Minkowski vacuum, we can calculate the expected number of the Rindler-Fulling particles in this state, i.e.[3]

$$\langle 0_M|\, b^{(\sigma)\dagger}_{\mathbf{k}}\, b^{(\sigma)}_{\mathbf{k}'}\, |0_M\rangle = N(\Omega)\, \delta(\Omega - \Omega')\, \delta(\vec{k} - \vec{k}')\,. \tag{65}$$

This equation remains valid if $b^{(\sigma)}_{\mathbf{k}}$ is replaced $\bar{b}^{(\sigma)}_{\mathbf{k}}$. We thus deduce a striking property: the expected number spectrum of the Rindler-Fulling particles in the Minkowski vacuum is given by the *Bose distribution function*. By using Eq.(60) and recalling that the proper energy of the Rindler particles seen by an observer with acceleration $a$ is $a\Omega$ (since his proper time $\tau$ is related to the Rindler time $\eta$ by Eq.(37)), we find that

$$T = \frac{a}{2\pi}\left(= \frac{\hbar a}{2\pi c k_B}\right). \tag{66}$$

[2] The phase factor appears to be irrelevant since we are actually interested in the calculation of the expectation value $\langle 0_M|b^{(\sigma)\dagger}_{\mathbf{k}}\, b^{(\sigma)}_{\mathbf{k}'}|0_M\rangle$.

[3] The divergence in Eq.(65) is due to the fact that the creation operators $a^\dagger_{\mathbf{k}}$, $b^{(\sigma)\dagger}_{\mathbf{k}}$, $d^{(\sigma)}_{(\Omega,\vec{k})}$ do not produce normalizable states when operated on the appropriate vacua. This can be cured by forming suitable wave packets and working with properly normalizable states (see [7]).

In summary, for a uniformly accelerated observer, the Minkowski vacuum is seen to be equivalent to a thermal bath with temperature $T$ proportional to the magnitude of the acceleration. This temperature is called *Davies-Unruh temperature*.

## 5. Mixing of boson fields in Minkowski and Rindler spacetime

Flavor mixing in QFT has been widely analyzed in the last two decades both for fermion [3] and boson [4] cases. Here we start with the study of the simplest possible case, i.e. the mixing of two boson fields "seen" by a Minkowski observer. For this purpose, we define the mixing relations as

$$\phi_A(x) = \phi_1(x)\,\cos\theta + \phi_2(x)\,\sin\theta\,, \tag{67}$$

$$\phi_B(x) = -\phi_1(x)\,\sin\theta + \phi_2(x)\,\cos\theta\,, \tag{68}$$

where generically we denote the mixed fields with suffices $A$ and $B$ and $\theta$ is the mixing angle. $\phi_i(x)$, $i = 1, 2$, are free complex fields with masses $m_i$. Their conjugate momenta are

$$\pi_i(x) = \partial_0\phi_i^\dagger \tag{69}$$

and the commutation relations are the usual ones:

$$[\phi_i(x), \pi_j(x')]_{x^0=x'^0} = [\phi_i^\dagger(x), \pi_j^\dagger(x')]_{x^0=x'^0} = i\delta^3(\mathbf{x}-\mathbf{x}')\delta_{ij}, \qquad i,j = 1,2, \tag{70}$$

with the other equal-time commutators vanishing. The Fourier expansions of $\phi_i(x)$ are defined in Eq.(1), while the expansions of momenta can be obtained by Eq.(69). The (Minkowski) vacuum state is now generalized as

$$|0_M\rangle \equiv |0_M\rangle_1 \otimes |0_M\rangle_2, \tag{71}$$

where $|0_M\rangle_i$ is the vacuum state for field with mass $m_i$.

Exploiting the completeness of the sets $\{U_{\mathbf{k},1}\,, U^*_{\mathbf{k},1}\}$ and $\{U_{\mathbf{k},2}\,, U^*_{\mathbf{k},2}\}$, we can now adopt for the mixed fields the following expansions

$$\phi_A(x) = \int d^3k \left(a_{\mathbf{k},A} U_{\mathbf{k},1}(x) + \bar{a}^\dagger_{\mathbf{k},A} U^*_{\mathbf{k},1}(x)\right), \tag{72}$$

$$\phi_B(x) = \int d^3k \left(a_{\mathbf{k},B} U_{\mathbf{k},2}(x) + \bar{a}^\dagger_{\mathbf{k},B} U^*_{\mathbf{k},2}(x)\right). \tag{73}$$

Using the orthonormality condition (9) of the Minkowski modes, it is easy to verify that the flavor operator $a_{\mathbf{k},A}$ is given by

$$a_{\mathbf{k},A} = (\phi_A, U_{\mathbf{k},1})\,. \tag{74}$$

In order to explicitly evaluate this inner product, we substitute Eq.(67) for the mixed field $\phi_A$. We then obtain[4]

$$a_{\mathbf{k},A} = \cos\theta\, a_{\mathbf{k},1} + \sin\theta\,\left(\rho^{\mathbf{k}\,*}_{12}\, a_{\mathbf{k},2} \,+\, \lambda^{\mathbf{k}}_{12}\,\bar{a}^\dagger_{-\mathbf{k},2}\right), \tag{75}$$

where the *mixing Bogoliubov coefficients* $\tilde{\rho}^{\mathbf{k}}_{12}$ and $\tilde{\lambda}^{\mathbf{k}}_{12}$ are given by

$$\rho^{\mathbf{k}}_{12} = |\rho^{\mathbf{k}}_{12}|\, e^{i\left(\omega_{k,2}-\omega_{k,1}\right)x^0}, \qquad \lambda^{\mathbf{k}}_{12} = |\lambda^{\mathbf{k}}_{12}|\, e^{i\left(\omega_{k,1}+\omega_{k,2}\right)x^0}, \tag{76}$$

[4] To simplify the notation, in what follows we will omit the time dependence.

with

$$|\rho_{12}^{\mathbf{k}}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{k,1}}{\omega_{k,2}}}+\sqrt{\frac{\omega_{k,2}}{\omega_{k,1}}}\right), \qquad |\lambda_{12}^{\mathbf{k}}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{k,1}}{\omega_{k,2}}}-\sqrt{\frac{\omega_{k,2}}{\omega_{k,1}}}\right). \tag{77}$$

It can be easily shown that

$$|\rho_{12}^{\mathbf{k}}|^2 - |\lambda_{12}^{\mathbf{k}}|^2 = 1. \tag{78}$$

In a similar way, we can derive the following expressions for the remaining flavor operators

$$\bar{a}_{\mathbf{k},A} = -(\phi_A, U^*_{\mathbf{k},1})^\dagger = \cos\theta\,\bar{a}_{\mathbf{k},1} + \sin\theta\left(\rho_{12}^{\mathbf{k}*}\,\bar{a}_{\mathbf{k},2} + \lambda_{12}^{\mathbf{k}}\,a^\dagger_{-\mathbf{k},2}\right), \tag{79}$$

$$a_{\mathbf{k},B} = (\phi_B, U_{\mathbf{k},2}) = \cos\theta\,a_{\mathbf{k},2} - \sin\theta\left(\rho_{12}^{\mathbf{k}}\,a_{\mathbf{k},1} - \lambda_{12}^{\mathbf{k}}\,\bar{a}^\dagger_{-\mathbf{k},1}\right), \tag{80}$$

$$\bar{a}_{\mathbf{k},B} = -(\phi_B, U^*_{\mathbf{k},2})^\dagger = \cos\theta\,\bar{a}_{\mathbf{k},2} - \sin\theta\left(\rho_{12}^{\mathbf{k}}\,\bar{a}_{\mathbf{k},1} - \lambda_{12}^{\mathbf{k}}\,a^\dagger_{-\mathbf{k},1}\right). \tag{81}$$

The relation (78) guarantees that the operators $a_{\mathbf{k},A}$, $\bar{a}_{\mathbf{k},A}$, $a_{\mathbf{k},B}$, $\bar{a}_{\mathbf{k},B}$ also (independently) satisfy the canonical commutation relations at equal times. In Ref.[4] the above flavor ladder operators have been derived by means of the algebraic generator of the mixing relations Eqs.(67),(68).

Let us now extend our considerations to the hyperbolic representation. For this purpose, we expand the fields $\phi_i$ as in Eq.(23). Once again it is possible to exploit the completeness of the sets $\left\{\widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),i}, \widetilde{U}^{(\sigma)*}_{(\Omega,\vec{k}),i}\right\}$ and adopt for $\phi_A$ and $\phi_B$ the following expansions

$$\phi_A(x) = \int_0^{+\infty} d\Omega \int d^2\vec{k}\sum_\sigma\left(d^{(\sigma)}_{(\Omega,\vec{k}),A}\,\widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),1}(x) + \bar{d}^{(\sigma)\dagger}_{(\Omega,\vec{k}),A}\,\widetilde{U}^{(\sigma)*}_{(\Omega,\vec{k}),1}(x)\right), \tag{82}$$

$$\phi_B(x) = \int_0^{+\infty} d\Omega \int d^2\vec{k}\sum_\sigma\left(d^{(\sigma)}_{(\Omega,\vec{k}),B}\widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),2}(x) + \bar{d}^{(\sigma)\dagger}_{(\Omega,\vec{k}),B}\widetilde{U}^{(\sigma)*}_{(\Omega,\vec{k}),2}(x)\right). \tag{83}$$

In order to determine the "new" flavor operators, we perform the same calculation as above. We find that

$$\begin{aligned} d^{(\sigma)}_{(\Omega,\vec{k}),A} &= \left(\phi_A,\, \widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),1}\right) \\ &= \cos\theta\, d^{(\sigma)}_{(\Omega,\vec{k}),1} + \sin\theta\int_0^{+\infty} d\Omega'\sum_{\sigma'}\left(d^{(\sigma')}_{(\Omega',\vec{k}),2}\,\mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} + \bar{d}^{(\sigma')\dagger}_{(\Omega',-\vec{k}),2}\,\mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}\right), \end{aligned} \tag{84}$$

$$\begin{aligned} \bar{d}^{(\sigma)}_{(\Omega,\vec{k}),A} &= -\left(\phi_A,\, \widetilde{U}^{(\sigma)*}_{(\Omega,\vec{k}),1}\right)^\dagger \\ &= \cos\theta\, \bar{d}^{(\sigma)}_{(\Omega,\vec{k}),1} + \sin\theta\int_0^{+\infty} d\Omega'\sum_{\sigma'}\left(\bar{d}^{(\sigma')}_{(\Omega',\vec{k}),2}\,\mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} + d^{(\sigma')\dagger}_{(\Omega',-\vec{k}),2}\,\mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}\right), \end{aligned} \tag{85}$$

and similar expressions for $d^{(\sigma)}_{(\Omega,\vec{k}),B}$ and $\bar{d}^{(\sigma)}_{(\Omega,\vec{k}),B}$. The Bogoliubov coefficients $\mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}$ and $\mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}$ are given by

$$\left(\widetilde{U}^{(\sigma')}_{(\Omega',\vec{k}'),2}, \widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),1}\right) = \mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}\;\delta^2(\vec{k}-\vec{k}')\,, \qquad \left(\widetilde{U}^{(\sigma')*}_{(\Omega',\vec{k}'),2}, \widetilde{U}^{(\sigma)}_{(\Omega,\vec{k}),1}\right) = \mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}}\;\delta^2(\vec{k}+\vec{k}'), \tag{86}$$

with

$$\mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} = \int_{-\infty}^{+\infty}\frac{dk_1}{4\pi}\left[\Big(\frac{1}{\omega_{k,1}}+\frac{1}{\omega_{k,2}}\Big)\Big(\frac{\omega_{k,1}+k_1}{\omega_{k,1}-k_1}\Big)^{i\sigma\Omega/2}\Big(\frac{\omega_{k,2}+k_1}{\omega_{k,2}-k_1}\Big)^{-i\sigma'\Omega'/2}e^{i(\omega_{k,1}-\omega_{k,2})x^0}\right], \quad (87)$$

$$\mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} = \int_{-\infty}^{+\infty}\frac{dk_1}{4\pi}\left[\Big(\frac{1}{\omega_{k,2}}-\frac{1}{\omega_{k,1}}\Big)\Big(\frac{\omega_{k,1}+k_1}{\omega_{k,1}-k_1}\Big)^{i\sigma\Omega/2}\Big(\frac{\omega_{k,2}+k_1}{\omega_{k,2}-k_1}\Big)^{-i\sigma'\Omega'/2}e^{i(\omega_{k,1}+\omega_{k,2})x^0}\right]. \quad (88)$$

The operators $d^{(\sigma)}_{(\Omega,\vec{k}),A}$, $\bar{d}^{(\sigma)}_{(\Omega,\vec{k}),A}$, $d^{(\sigma)}_{(\Omega,\vec{k}),B}$ and $\bar{d}^{(\sigma)}_{(\Omega,\vec{k}),B}$ also (independently) satisfy the canonical commutation relations at equal times.

Now we analyze the mixing effect as seen by a Rindler observer. For this purpose, we expand the mixed fields $\phi_A$ and $\phi_B$ in terms of the Rindler modes (40), obtaining

$$\phi_A(x) = \int_0^{+\infty}d\Omega\int d^2\vec{k}\sum_\sigma\Big(b^{(\sigma)}_{(\Omega,\vec{k}),A}\,u^{(\sigma)}_{(\Omega,\vec{k}),1}(x)+\bar{b}^{(\sigma)\dagger}_{(\Omega,\vec{k}),A}\,u^{(\sigma)*}_{(\Omega,\vec{k}),1}(x)\Big), \quad (89)$$

$$\phi_B(x) = \int_0^{+\infty}d\Omega\int d^2\vec{k}\sum_\sigma\Big(b^{(\sigma)}_{(\Omega,\vec{k}),B}u^{(\sigma)}_{(\Omega,\vec{k}),2}(x)+\bar{b}^{(\sigma)\dagger}_{(\Omega,\vec{k}),B}u^{(\sigma)*}_{(\Omega,\vec{k}),2}(x)\Big). \quad (90)$$

Since we are interested in the connection between the $b^{(\sigma)}_{(\Omega,\vec{k}),A}$ and $d^{(\sigma)}_{(\Omega,\vec{k}),A}$ operators, we equate expansions (82) and (89). Forming the inner product of both sides with $u^{(\sigma)}_{(\Omega,\vec{k}),1}$ and using the orthonormality of the Rindler modes, it follows that

$$b^{(\sigma)}_{(\Omega,\vec{k}),A} = \int_0^{+\infty}d\Omega'\int d^2\vec{k}'\sum_{\sigma'}\Big(d^{(\sigma')}_{(\Omega',\vec{k}'),A}\,\widetilde{\alpha}^{(\sigma,\sigma')}_{\mathbf{k}\mathbf{k}'}+\bar{d}^{(-\sigma')\dagger}_{(\Omega',-\vec{k}'),A}\,\widetilde{\beta}^{(\sigma,\sigma')}_{\mathbf{k}\bar{\mathbf{k}}}\Big), \quad (91)$$

with

$$\widetilde{\alpha}^{(\sigma,\sigma')}_{\mathbf{k}\mathbf{k}'} = \Big(\widetilde{U}^{(\sigma')}_{(\Omega',\vec{k}'),1},u^{(\sigma)}_{(\Omega,\vec{k}),1}\Big), \quad (92)$$

$$\widetilde{\beta}^{(\sigma,\sigma')}_{\mathbf{k}\bar{\mathbf{k}}} = \Big(\widetilde{U}^{(-\sigma')*}_{(\Omega',-\vec{k}'),1},u^{(\sigma)}_{(\Omega,\vec{k}),1}\Big), \quad (93)$$

where the first subscript $\mathbf{k}$ of $\widetilde{\alpha}^{(\sigma,\sigma')}_{\mathbf{k}\mathbf{k}'}$ and $\widetilde{\beta}^{(\sigma,\sigma')}_{\mathbf{k}\bar{\mathbf{k}}}$ stands for $(\Omega,\vec{k})$ while the second subscript $\tilde{\mathbf{k}}'$ for $(\Omega',-\vec{k}')$. A similar calculation can be performed for $\bar{b}^{(\sigma)}_{(\Omega,\vec{k}),A}$.

By noting that the coefficients $\widetilde{\alpha}^{(\sigma,\sigma')}_{\mathbf{k}\mathbf{k}'}$ and $\widetilde{\beta}^{(\sigma,\sigma')}_{\mathbf{k}\bar{\mathbf{k}}}$ do not depend on $\theta$ and that the equation (91), for $\theta=0$, must reduce to the transformation (59) (since $b^{(\sigma)}_{(\Omega,\vec{k}),A}\big|_{\theta=0}=b^{(\sigma)}_{(\Omega,\vec{k}),1}$, $d^{(\sigma)}_{(\Omega,\vec{k}),A}\big|_{\theta=0}=d^{(\sigma)}_{(\Omega,\vec{k}),1}$ and $d^{(-\sigma)}_{(\Omega,-\vec{k}),A}\big|_{\theta=0}=d^{(-\sigma)}_{(\Omega,-\vec{k}),1}$), one could demonstrate that (up to an irrelevant phase factor)

$$b^{(\sigma)}_{(\Omega,\vec{k}),A}=\sqrt{1+N(\Omega)}\,d^{(\sigma)}_{(\Omega,\vec{k}),A}+\sqrt{N(\Omega)}\,\bar{d}^{(-\sigma)\dagger}_{(\Omega,-\vec{k}),A}\,. \quad (94)$$

The corresponding relation between the $b^{(\sigma)}_{(\Omega,\vec{k}),B}$ and $d^{(\sigma)}_{(\Omega,\vec{k}),B}$ operators can be derived by equating expansions (83) and (90) and performing the same calculation as above.

Finally, let us calculate the expectation value of the number operator $b^{(\sigma)\dagger}_{(\Omega,\vec{k}),A}\,b^{(\sigma)}_{(\Omega,\vec{k}),A}$ with respect to the Minkowski vacuum (71) (a similar calculation can be performed for $b^{(\sigma)\dagger}_{(\Omega,\vec{k}),B}\,b^{(\sigma)}_{(\Omega,\vec{k}),B}$). To do this, we need the explicit expressions for $d^{(\sigma)}_{(\Omega,\vec{k}),A}$ (Eq.84) and $\bar{d}^{(\sigma)}_{(\Omega,\vec{k}),A}$

(Eq.(85)). Observe that the resolution of integrals (87) and (88) is not trivial. It seems reasonable to suppose that

$$\mathcal{A}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} = \tilde{\rho}^{(\sigma)\,*}_{(\Omega,\vec{k})}\,\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega'), \tag{95}$$

$$\mathcal{B}^{(\sigma,\sigma')}_{(\Omega,\Omega'),\,\vec{k}} = \tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})}\,\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega'). \tag{96}$$

The coefficients $\tilde{\rho}^{(\sigma)\,*}_{(\Omega,\vec{k})}$ and $\tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})}$ have still to be determined. They should depend on $\vec{k}$ through the sum and the difference of $\mu_{k,1}$ and $\mu_{k,2}$, respectively, with $\mu_{k,i}$ defined in Eq.(27). For $m_1 \to m_2$, we must have $\tilde{\rho}^{(\sigma)\,*}_{(\Omega,\vec{k})} \to 1$ and $\tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})} \to 0$ (see Eq.(86)). This also happens in the ultrarelativistic limit, when $|\vec{k}| = \sqrt{k_2^2+k_3^2} \gg m_i$ implying $\mu_{k,1} \approx \mu_{k,2}$.

With the ansatz (95) and (96), it is possible to rewrite Eq.(94) as

$$\begin{aligned} b^{(\sigma)}_{(\Omega,\vec{k}),A} &= \sqrt{1+N(\Omega)}\,\Big[\cos\theta\, d^{(\sigma)}_{(\Omega,\vec{k}),1} + \sin\theta\,\tilde{\rho}^{(\sigma)\,*}_{(\Omega,\vec{k})}\, d^{(\sigma)}_{(\Omega,\vec{k}),2} + \sin\theta\,\tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})}\,\bar{d}^{(\sigma)\dagger}_{(\Omega,-\vec{k}),2}\Big] \\ &+ \sqrt{N(\Omega)}\,\Big[\cos\theta\,\bar{d}^{(-\sigma)\dagger}_{(\Omega,-\vec{k}),1} + \sin\theta\,\tilde{\rho}^{(-\sigma)}_{(\Omega,\vec{k})}\,\bar{d}^{(-\sigma)\dagger}_{(\Omega,-\vec{k}),2} + \sin\theta\,\tilde{\lambda}^{(-\sigma)\,*}_{(\Omega,\vec{k})}\, d^{(-\sigma)}_{(\Omega,\vec{k}),2}\Big], \end{aligned}$$

where we have used Eqs.(84) and (85).

Therefore, the expected number spectrum of the "mixed" Rindler-Fulling particles in the vacuum (71) takes the form

$$\begin{aligned} \mathcal{N}^{(\sigma)}_{(\Omega,\vec{k}),A} &\equiv \langle 0_M|\, b^{(\sigma)\dagger}_{(\Omega,\vec{k}),A}\, b^{(\sigma)}_{(\Omega',\vec{k}'),A}\,|0_M\rangle \qquad (97) \\ &= \left[(1+N(\Omega))\sin^2\theta\,\left|\tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})}\right|^2 + N(\Omega)\left(\cos^2\theta + \sin^2\theta\,\left|\tilde{\rho}^{(-\sigma)}_{(\Omega,\vec{k})}\right|^2\right)\right]\delta(\Omega-\Omega')\delta^2(\vec{k}-\vec{k}'). \end{aligned}$$

Note that, for $\theta \to 0$, Eq.(97) correctly reduces to Eq.(65), as one would expect in absence of mixing. Analogous considerations hold in the limit $m_1 \to m_2$ and in the ultrarelativistic limit, since we have $\tilde{\rho}^{(\sigma)\,*}_{(\Omega,\vec{k})} \to 1$ and $\tilde{\lambda}^{(\sigma)}_{(\Omega,\vec{k})} \to 0$.

## 6. Conclusions

We have considered the problem of quantization of two mixed charged boson fields with different masses for a uniformly accelerated observer (Rindler observer). We found that the Bogoliubov tranformations related to the field mixing on one side and to the Rindler spacetime structure on the other side, combine in a non-trivial way, affecting observable quantities such as the Unruh radiation.

Some results presented in this paper are preliminary: due to technical difficulties, it was not possible to obtain quantitative estimation of the obtained results. More study is necessary and work is in progress along this line [18].

The analysis carried out in this paper, once properly extended to the fermionic case, may serve as basis for studying neutrino oscillations in curved spacetime: this is a problem of interest since, although the gravitational interaction is relatively weak compared to electromagnetic and weak interactions, there could exist cosmological and astrophysical scenarios where gravitational fields are rather strong, affecting non trivially the propagation of particles. Previous studies on the influence of the gravitational field on neutrino mixing and oscillations can be found in Refs.[19].

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**Appendix: Explicit calculation of wave function $\widetilde{U}$**

We calculate the integral for the wave function $\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x)$ defined in Eq.(24)

$$\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x) = \int_{-\infty}^{+\infty} dk_1 \, p^{(\sigma)*}_{\Omega}(k_1) \, U_{\mathbf{k}}(x). \tag{98}$$

Substituting Eqs.(4) and (16) into Eq.(98) and using the hyperbolic coordinates (25), we obtain

$$\begin{aligned}\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x) &= \int_{-\infty}^{+\infty} dk_1 \frac{1}{\sqrt{(2\pi\omega_k)}} \left(\frac{\omega_k + k_1}{\omega_k - k_1}\right)^{-i\sigma\Omega/2} \frac{e^{i\left(k_1 x^1 + \vec{k}\cdot\vec{x} - \omega_k x^0\right)}}{\sqrt{2\omega_k (2\pi)^3}} \\ &= \frac{1}{(2\pi)^2\sqrt{2}} \int_{-\infty}^{+\infty} \frac{dk_1}{\omega_k} \left(\frac{\omega_k + k_1}{\omega_k - k_1}\right)^{-i\sigma\Omega/2} e^{i\,\xi(k_1\cosh\eta - \omega_k \sinh\eta)} e^{i\vec{k}\cdot\vec{x}}.\end{aligned} \tag{99}$$

The last integral can be solved by performing the following change of variables

$$k_1 = \mu_k \sinh t \implies dk_1 = \mu_k \cosh t \, dt. \tag{100}$$

By virtue of this transformation, we have

$$\omega_k = \mu_k \cosh t, \tag{101}$$

so the Eq.(99) becomes

$$\begin{aligned}\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x) &= \frac{1}{(2\pi)^2\sqrt{2}} \int_{-\infty}^{+\infty} dt \left(\frac{\cosh t + \sinh t}{\cosh t - \sinh t}\right)^{-i\sigma\Omega/2} e^{i\mu_k\xi(\sinh t\cosh\eta - \cosh t \sinh\eta)} e^{i\vec{k}\cdot\vec{x}} \\ &= \frac{1}{(2\pi)^2\sqrt{2}} \int_{-\infty}^{+\infty} dt \, e^{-i\sigma\Omega t} \, e^{i\mu_k\xi\sinh(t-\eta)} e^{i\vec{k}\cdot\vec{x}}.\end{aligned} \tag{102}$$

With the further substitution

$$\eta - t = t', \tag{103}$$

it follows that

$$\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x) = \frac{1}{(2\pi)^2\sqrt{2}} \int_{-\infty}^{+\infty} dt' \, e^{i\sigma\Omega t'} e^{-i\mu_k\xi\sinh t'} e^{i\left(\vec{k}\cdot\vec{x} - \sigma\Omega\eta\right)}. \tag{104}$$

By using the following integral representation of the modified Bessel function of the second kind

$$K_\alpha(x) = \frac{1}{2} e^{-i\alpha\pi/2} \int_{-\infty}^{+\infty} dt \, e^{-ix\sinh t - \alpha t}, \tag{105}$$

we finally obtain

$$\widetilde{U}^{(\sigma)}_{\Omega,\vec{k}}(x) = \frac{e^{\sigma\pi\Omega/2}}{2\sqrt{2}\pi^2} K_{i\sigma\Omega}(\mu_k\xi) \, e^{i\left(\vec{k}\cdot\vec{x} - \sigma\Omega\eta\right)}, \tag{106}$$

where we used the property $K_\alpha(x) = K_{-\alpha}(x)$.

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