

# Subtleties in the *BABAR* measurement of time-reversal violation

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**Abstract.** A first measurement of time-reversal (T) asymmetries that are not also CP asymmetries has been recently achieved by the *BABAR* collaboration. In this talk, which follows the work done in Ref. [1], I discuss the subtleties of this measurement in the presence of direct CP violation, CPT violation, wrong strangeness decays and wrong sign semi-leptonic decays. In particular, I explain why, in order to identify the measured asymmetries with time-reversal violation, one needs to assume (i) the absence of wrong strangeness decays or of CPT violation in strangeness changing decays, and (ii) the absence of wrong sign decays.

## 1. Introduction

The *BABAR* collaboration has recently announced the first direct observation of time-reversal violation in the neutral  $B$  meson system [2]. The basic idea is to compare the time-dependent rates of two processes that differ by exchange of initial and final states. The measurement makes use of the EPR effect in the entangled  $B$  mesons produced in  $\Upsilon(4S)$  decays [3, 4, 5, 6, 7]. For example, one rate,  $\Gamma_{(\psi K_L)_\perp, \ell^- X}$ , involves the decay of one of the neutral  $B$ 's into a  $\psi K_L$  state, and, after time  $t$ , the decay of the other  $B$  into  $\ell^- X$ . The other rate,  $\Gamma_{(\ell^+ X)_\perp, \psi K_S}$ , involves the decay of one of the neutral  $B$ 's into  $\ell^+ X$ , and, after time  $t$ , the decay of the other  $B$  into  $\psi K_S$ . Under certain assumptions, to be spelled out below, this is a comparison between the rates of  $B_- \rightarrow \bar{B}^0$  and  $\bar{B}^0 \rightarrow B_-$ , where  $\bar{B}^0$  has a well defined flavor content ( $b\bar{d}$ ) and  $B_-$  is a CP-odd state.

Time reversal violation had been observed earlier in the neutral  $K$  meson system by CPLEAR [8]. The measurement involves the processes  $p\bar{p} \rightarrow K^- \pi^+ K^0$  and  $p\bar{p} \rightarrow K^+ \pi^- \bar{K}^0$ . Again, one aims to compare rates of processes that are related by exchange of initial and final states. One rate,  $\Gamma_{K^-, e^-}$ , involves a production of  $K^-$  and a neutral  $K$  that after time  $t$  decay into  $e^- \pi^+ \bar{\nu}$ . The other rate,  $\Gamma_{K^+, \ell^+}$ , involves the production of  $K^+$  and a neutral  $\bar{K}$  that after time  $t$  decay into  $e^+ \pi^- \nu$ . Under certain assumptions, this is a comparison between the rates of  $K^0 \rightarrow \bar{K}^0$  and  $\bar{K}^0 \rightarrow K^0$  [9]. The CPLEAR asymmetry is also a CP asymmetry, since the initial and final states are CP-conjugate. In contrast, the *BABAR* asymmetry is not a CP asymmetry.

When aiming to demonstrate that time-reversal is violated, one needs to allow for CPT violation [10, 11]. (Otherwise, T violation follows from CP violation.) For two processes to be related by time-reversal, the initial state in each of them should be the time-reversal conjugate of the final state in the other. The subtleties related to this requirement, in the context of the *BABAR* measurements, are clarified in this work. To do so, it is helpful to use only parameters



that have well-defined transformation properties under all three relevant symmetries: CP, T and CPT. In the presence of CPT violation, most of the parameters used in the literature do not have well-defined transformation under T and CPT. In particular,  $\Delta S_T^+$ , which is used to demonstrate time reversal violation by the *BABAR* collaboration, has, apart from a T-odd CPT-even part, also a T-even CPT-odd part. We introduce parameters with well-defined transformation properties under these discrete symmetries and formulate the assumptions one needs to make in order to identify the asymmetries measured by *BABAR* with time reversal violation.

## 2. Parameter definitions and transformation properties

Our definitions and notations, which use a formalism that allows for CPT violation, are described in detail in Ref. [1]. The following summarizes the necessary and novel ingredients used in our work.

### 2.1. CPT violation

We define decay amplitudes and inverse-decay amplitudes

$$\begin{aligned} A_f &\equiv A(B^0 \rightarrow f) = \langle f|T|B^0 \rangle, & \bar{A}_f &\equiv A(\bar{B}^0 \rightarrow f) = \langle f|T|\bar{B}^0 \rangle, \\ A_f^{\text{ID}} &\equiv A(f^T \rightarrow B^0) = \langle B^0|T|f^T \rangle, & \bar{A}_f^{\text{ID}} &\equiv A(f^T \rightarrow \bar{B}^0) = \langle \bar{B}^0|T|f^T \rangle, \end{aligned} \quad (1)$$

where  $f^T$  is T-conjugate (*i.e.* reversed spins and momenta) to  $f$ . Using these amplitudes, we define complex parameters,  $\theta_f$ , representing CPT violation in the decay:

$$\theta_f = \theta_f^R + i\theta_f^I \equiv \frac{A_f^{\text{ID}}/\bar{A}_f^{\text{ID}} - \bar{A}_f/A_f}{A_f^{\text{ID}}/\bar{A}_f^{\text{ID}} + \bar{A}_f/A_f}. \quad (2)$$

For final CP eigenstates,  $\theta_f \neq 0$  breaks CPT and CP, but not T. The complex parameter  $z \equiv z^R + iz^I$  represents CP and CPT violation in mixing, while the real parameter,  $R_M$ , represents T and CP violation in mixing. For these two parameters we use the standard definitions which can be found in Ref. [1]. We further define the phase convention independent combination of amplitudes and mixing parameters,

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} \sqrt{\frac{1+\theta_f}{1-\theta_f}} = \frac{q}{p} \frac{A_f^{\text{ID}}}{\bar{A}_f^{\text{ID}}} \sqrt{\frac{1-\theta_f}{1+\theta_f}}. \quad (3)$$

In the CPT limit  $\theta_f = 0$  and the standard definition of  $\lambda_f$  is recovered. It is convenient to introduce the following functions of  $\lambda_f$ :

$$C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f \equiv \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \quad G_f \equiv \frac{2\text{Re}(\lambda_f)}{1 + |\lambda_f|^2}, \quad (4)$$

with  $C_f^2 + G_f^2 + S_f^2 = 1$ . The transformation rules for these parameters under T, CP and CPT, are summarized in Table 1. As explained above, it is very convenient for our purposes to use parameters that are either even or odd under all three transformations. Using superscript + for T-even, and – for T-odd, we define the following combinations:

$$C_f^\pm = \frac{1}{2}(C_f \mp C_{\bar{f}}), \quad S_f^\pm = \frac{1}{2}(S_f \mp S_{\bar{f}}), \quad G_f^\pm = \frac{1}{2}(G_f \pm G_{\bar{f}}), \quad \theta_f^\pm = \frac{1}{2}(\theta_f \pm \theta_{\bar{f}}). \quad (5)$$

A summary of the transformation properties of these parameters is provided in Table 2.

We emphasize that the definition of  $\lambda_f$  in Eq. (3) and, consequently, the definitions of  $S_f$ ,  $C_f$  and  $G_f$  in Eq. (4) differ from the standard definitions of these parameters in the literature.

**Table 1.** Transformation rules of the various parameters under T, CP and CPT.

Parameter	T	CP	CPT
$R_M$	$-R_M$	$-R_M$	$R_M$
$z$	$z$	$-z$	$-z$
$\lambda_f$	$1/\lambda_{\bar{f}}$	$1/\lambda_{\bar{f}}$	$\lambda_f$
$S_f$	$-S_{\bar{f}}$	$-S_{\bar{f}}$	$S_f$
$C_f$	$-C_{\bar{f}}$	$-C_{\bar{f}}$	$C_f$
$G_f$	$G_{\bar{f}}$	$G_{\bar{f}}$	$G_f$
$\theta_f$	$\theta_{\bar{f}}$	$-\theta_{\bar{f}}$	$-\theta_f$

**Table 2.** Eigenvalues of the various parameters under T, CP and CPT.

Parameter	T	CP	CPT
$R_M, S_f^-, C_f^-, G_f^-$	-	-	+
$z, \theta_f^+$	+	-	-
$\theta_f^-$	-	+	-
$S_f^+, C_f^+, G_f^+$	+	+	+
$S_{fCP}, C_{fCP}$	-	-	+
$\theta_{fCP}$	+	-	-
$G_{fCP}$	+	+	+

Our definition lends itself straightforwardly to the theoretical analysis that we are doing. The standard definition lends itself straightforwardly to the description of the experimentally measured rates. In practice, inverse decays are not accessible to the experiments. In particular, experiments are not sensitive to  $\lambda_f$ , as defined in Eq. (3), but to the related observable  $\lambda_f^e$ , defined via

$$\lambda_f^e \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} = \lambda_f (1 - \theta_f) , \quad (6)$$

where the second equation holds to first order in  $\theta_f$ . Accordingly, the experiments are sensitive to the following parameters:

$$\begin{aligned} C_f^e &= C_f + (1 - C_f^2) \theta_f^R , \\ S_f^e &= S_f (1 - C_f \theta_f^R) - G_f \theta_f^I , \\ G_f^e &= G_f (1 - C_f \theta_f^R) + S_f \theta_f^I . \end{aligned} \quad (7)$$

The distinction between the “theoretical” and “experimental” parameters is crucial for understanding the subtleties in the interpretation of the *BABAR* measurements. Of course, in the absence of CPT violation, the two sets of definitions coincide.

## 2.2. Wrong-strangeness decays

Among the final CP eigenstates, we focus on decays into the final  $\psi K_{S,L}$  states (neglecting effects of  $\epsilon_K$ ). We distinguish between the right strangeness decays and the wrong strangeness

decays, and define

$$\begin{aligned}\hat{C}_{\psi K} &\equiv \frac{1}{2}(C_{\psi K_S} + C_{\psi K_L}), & \Delta C_{\psi K} &\equiv \frac{1}{2}(C_{\psi K_S} - C_{\psi K_L}), \\ \hat{S}_{\psi K} &\equiv \frac{1}{2}(S_{\psi K_S} - S_{\psi K_L}), & \Delta S_{\psi K} &\equiv \frac{1}{2}(S_{\psi K_S} + S_{\psi K_L}), \\ \hat{G}_{\psi K} &\equiv \frac{1}{2}(G_{\psi K_S} - G_{\psi K_L}), & \Delta G_{\psi K} &\equiv \frac{1}{2}(G_{\psi K_S} + G_{\psi K_L}), \\ \hat{\theta}_{\psi K} &\equiv \frac{1}{2}(\theta_{\psi K_L} - \theta_{\psi K_S}), & \Delta \theta_{\psi K} &\equiv \frac{1}{2}(\theta_{\psi K_L} + \theta_{\psi K_S}).\end{aligned}\quad (8)$$

In the limit of no wrong strangeness decay,  $\lambda_{\psi K_S} = -\lambda_{\psi K_L}$  [12] (Ref. [12] assumes CPT conservation, but this is not a necessary assumption) and, consequently,  $\Delta C_{\psi K}, \Delta G_{\psi K}, \Delta S_{\psi K}$  and  $\Delta \theta_{\psi K}$  vanish.

### 2.3. Wrong-sign decays

Among the flavor specific final states, we focus on decays into final  $\ell^\pm X$  states. Here we distinguish between the right sign decays and the wrong sign decays, and define  $C_\ell^\pm$ ,  $S_\ell^\pm$  and  $G_\ell^\pm$  according to Eq. (5), with  $f = \ell^\pm$ , and a super-index + (−) denoting a T conserving (violating) combination. Taking the wrong sign decays to be much smaller in magnitude than the right sign decays, we have  $|\lambda_{\ell^+}| \ll 1$  and  $|\lambda_{\ell^-}^{-1}| \ll 1$ . We will work to first order in  $|\lambda_{\ell^+}|$  and in  $|\lambda_{\ell^-}^{-1}|$ , which means that we set  $C_\ell^+ = 1$  and  $C_\ell^- = 0$ . On the other hand, the four other relevant parameters are linear in  $|\lambda_{\ell^+}|$  and in  $|\lambda_{\ell^-}^{-1}|$ :

$$S_\ell^\pm \simeq \text{Im}(\lambda_{\ell^+} \pm \lambda_{\ell^-}^{-1}), \quad G_\ell^\pm \simeq \text{Re}(\lambda_{\ell^+} \pm \lambda_{\ell^-}^{-1}). \quad (9)$$

Equipped with these definitions, we are now ready to analyze the asymmetries measured by the *BABAR* collaboration.

### 3. The *BABAR* T-asymmetry

Consider a pair of  $B$ -mesons produced in  $\Upsilon(4S)$  decay, where one of the  $B$ -mesons decays at time  $t_1$  to a final state  $f_1$ , and the other  $B$ -meson decays at a later time,  $t_2 = t_1 + t$ , to a final state  $f_2$ . The time dependent rate for this process (to zeroth order in  $y$ ) is given by

$$\Gamma_{(f_1)_\perp, f_2} = N_{(1)_\perp, 2} e^{-\Gamma(t_1+t_2)} \times [\kappa_{(1)_\perp, 2} + C_{(1)_\perp, 2} \cos(x\Gamma t) + S_{(1)_\perp, 2} \sin(x\Gamma t)] . \quad (10)$$

where the coefficients  $N_{(1)_\perp, 2}$ ,  $\kappa_{(1)_\perp, 2}$ ,  $C_{(1)_\perp, 2}$  and  $S_{(1)_\perp, 2}$  are defined in Ref. [1]. We work to linear order in the following small parameters:

$$R_M, z^R, z^I, \theta_f^R, \theta_f^I, \hat{C}_{\psi K}, \Delta C_{\psi K}, \Delta S_{\psi K}, \Delta G_{\psi K}, S_\ell^\pm, G_\ell^\pm. \quad (11)$$

The analysis performed by *BABAR*, as described in Ref. [2], is as follows. The time dependent decay rates are measured and fitted to time-dependence of the form (10), approximating (as we do)  $y = 0$ . The quantities  $\Delta S_T^+$  and  $\Delta C_T^+$  correspond to the following combinations:

$$\begin{aligned}\Delta S_T^+ &= \frac{S_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{S_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}} \equiv S_{\ell^-, K_L}^- - S_{\ell^+, K_S}^+, \\ \Delta C_T^+ &= \frac{C_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{C_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}} \equiv C_{\ell^-, K_L}^- - C_{\ell^+, K_S}^+.\end{aligned}\quad (12)$$

We note that the normalization of Eqs. (12) removes the dependence on the total production rates and effects such as direct CP violation in leptonic decays.

We obtain the following expressions for these observables:

$$\begin{aligned}\Delta S_T^+ &= -2 \left[ (\hat{S}_{\psi K} - \hat{G}_{\psi K} \Delta \theta_{\psi K}^I) + \hat{S}_{\psi K} \hat{G}_{\psi K} (G_\ell^- - z^R) \right], \\ \Delta C_T^+ &= 2 \left[ (\hat{C}_{\psi K} + \Delta \theta_{\psi K}^R) + \hat{S}_{\psi K} (S_\ell^- - z^I) \right].\end{aligned}\quad (13)$$

If we switch off all the T-odd parameters, we are left with the following T conserving (TC) contributions:

$$\begin{aligned}\Delta S_T^+(\text{T-odd parameters} = 0) &= 2\hat{G}_{\psi K} \Delta \theta_{\psi K}^I, \\ \Delta C_T^+(\text{T-odd parameters} = 0) &= 2\Delta \theta_{\psi K}^R.\end{aligned}\quad (14)$$

These contributions are CPT violating, and they vanish in the limit of no wrong strangeness decays. Yet, since  $\Delta \theta_{\psi K}$  involves inverse decays, we are not aware of any way to experimentally bound it, and to exclude the possibility that it generates the measured value of  $\Delta S_T^+$ . We would like to emphasize, however, the following three points.

- The appearance of the T conserving, CPT violating effects should come as no surprise. As explained in the discussion of Eq. (7), experiments can only probe  $S_{\psi K}^e$  and  $C_{\psi K}^e$ , which include these terms.
- While we are not aware of any way to constrain  $\Delta \theta_{\psi K}$  from tree level processes, it is quite possible that it affects measurable effects, such as CPT violation in mixing, via loop effects. In the absence of a rigorous framework that incorporates CPT violation, it is impossible to calculate such effects.
- It would of course be extremely exciting if the *BABAR* measurement is affected by CPT violating effects.

An additional interesting feature of Eqs. (13) is the appearance of terms that are quadratic in T-odd parameters,

$$\begin{aligned}\Delta S_T^+(\text{quadratic in T-odd parameters}) &= -2\hat{G}_{\psi K} \hat{S}_{\psi K} G_\ell^-, \\ \Delta C_T^+(\text{quadratic in T-odd parameters}) &= 2\hat{S}_{\psi K} S_\ell^-\end{aligned}\quad (15)$$

While these terms would vanish if we take all T-odd parameters to zero, they are still T-conserving. Note that since we expand to linear order in all T-odd parameters, except for  $\hat{S}_{\psi K}$ , there are additional T conserving,  $\hat{S}_{\psi K}$ -independent, contributions that are quadratic in T-odd parameters that are not presented in Eqs. (15). Since  $\hat{G}_{\psi K}^2 + \hat{S}_{\psi K}^2 \leq 1$ , the maximal absolute value of the term on the right hand side of Eq. (15) for  $\Delta S_T^+$  is 1,  $|2\hat{G}_{\psi K} \hat{S}_{\psi K} G_\ell^-| \leq 1$ . Thus, if experiments establish  $|\Delta S_T^+| > 1$ , such a result cannot be explained by this term alone.

We are now also able to formulate the conditions under which the *BABAR* measurement would demonstrate T violation unambiguously:

$$\Delta \theta_{\psi K} = G_\ell^- = S_\ell^- = 0. \quad (16)$$

In words, the necessary conditions are the following:

- The absence of wrong strangeness decays and of wrong strangeness inverse decays or, if such wrong strangeness processes occur, the absence of CPT violation in strangeness-changing decays.

- The absence of wrong sign decays or, if wrong sign decays occur, the absence of direct CP violation in semileptonic decays.

We further note that not only the T-asymmetries get T-even contributions in the presence of the aforementioned phenomena, but also the CP asymmetries get CP-even contributions, and the CPT asymmetries get CPT-even contributions. All of these effects vanish if there are neither wrong strangeness nor wrong sign (inverse) decays.

A-priori, one would expect that direct CP violation in right-strangeness decays is enough to allow for  $A_T \neq 0$  even in the T-symmetry limit. In  $\Gamma_{(\psi K_L)_{\perp}, \ell^- X}$ , the initial  $B$ -meson state is orthogonal to the one that decays to  $\psi K_L$ . In  $\Gamma_{(\ell^+ X)_{\perp}, \psi K_S}$ , the final state is the one that decays into  $\psi K_S$ . Are these two states identical? They would be if the state that does not decay to  $\psi K_L$ ,  $|B_{(\rightarrow \psi K_L)_{\perp}}\rangle$ , and the state that does not decay into  $\psi K_S$ ,  $|B_{(\rightarrow \psi K_S)_{\perp}}\rangle$ , were orthogonal to each other. In fact, this is the case if there is no direct CP violation in the  $B \rightarrow \psi K$  decays. This is presumably the reason why the theoretical paper [5] and the experimental paper [2] explicitly state that they neglect direct CP violation.

However, the correct question is not whether the state that does not decay to  $\psi K_L$  is the same as the state that decays to  $\psi K_S$ . Instead, the question is whether it is the same as the state generated in the inverse decay of  $\psi K_S$ . We find that if wrong strangeness decays can be neglected, then the two processes are related by exchange of the initial and final CP-tagged states, as required for time-reversal conjugate processes.

One can raise an analogous question for the lepton-tagged states. The question to be asked is then whether the state that does not decay to  $\ell^+ X$  is orthogonal to the state that is not generated in the inverse decay of  $\ell^- X$ . We find that if wrong sign decays can be neglected, then the two processes are related by exchange of the initial and final lepton-tagged states, as required for time-reversal conjugate processes.

We conclude that if wrong strangeness and wrong sign decays can be neglected, then the two processes measured by *BABAR* represent two T-conjugate processes, and then there should be no T conserving contributions to  $\Delta S_T^+$  and  $\Delta S_T^-$ , consistent with Eqs. (14) and (15). In particular, one need not assume the absence of direct CP violation.

#### 4. Isolating parameters of interest

Combinations of observables measured by *BABAR* allow us to constrain various parameters of interest. From the results of [2] we get

$$\hat{S}_{\psi K} - \hat{G}_{\psi K} \Delta \theta_{\psi K}^I = 0.69 \pm 0.04. \quad (17)$$

and the following bounds can be deduced

$$\begin{aligned} |\hat{C}_{\psi K} + \Delta \theta_{\psi K}^R| &< 0.07, \\ |G_{\psi K_{S,L}} S_{\psi K_{S,L}} (G_{\ell}^- - z^R)| &< 0.10, \\ |S_{\psi K_{S,L}} (S_{\ell}^- - z^I)| &< 0.06, \end{aligned} \quad (18)$$

at  $2\sigma$  level. In case we assume no CPT violation naive combination of the above will lead to

$$|S_{\ell}^-| < 0.10, \quad |G_{\ell}^-| < 0.21, \quad (19)$$

at  $2\sigma$  level.

## 5. Conclusions

The *BABAR* collaboration has measured time-reversal asymmetries in  $B$  decays. Two main ingredients — the EPR effect between the two  $B$ -mesons produced in  $\Upsilon(4S)$  decays and the availability of both lepton-tagging and CP-tagging — allow the experimenters to approximately realize the main principle of time-reversal conjugate processes: exchanging initial and final states.

A precise exchange is impossible. The final state is identified by the  $B$ -meson decay, and the T-conjugate process requires, therefore, that a  $B$ -meson is produced in the corresponding inverse decay. Instead, the experimenters measure a process where the initial  $B$ -meson is identified by the decay of the other, entangled  $B$ -meson. We found however that the initial  $B$ -meson prepared by lepton tagging, and the one that would be produced in the appropriate inverse decay are not identical only if there are wrong-sign decays. The initial  $B$ -meson prepared by CP tagging, and the one that would be produced in the appropriate inverse decay, are not identical only if there are wrong-strangeness contributions and, furthermore, if there is direct CP violation or the presence of CPT violation in decays.

The effect of CPT violation in decay has gained very little attention in the literature. One reason is that it can only be probed by measuring both decay rates and inverse decay rates, but the latter are practically inaccessible to experiments. For precisely this reason, there are no bounds on these effects. In principle, they could play a significant role in the asymmetries measured by *BABAR*, in spite of the fact that they are T conserving.

Both wrong-sign and wrong-strangeness effects are expected to be very small. If so, then the asymmetry measured by *BABAR* is indeed a time-reversal asymmetry to a very good approximation. The existing experimental upper bounds on these effects are rather weak. The same set of measurements used for the time-reversal asymmetries can be used (in different combinations) to constrain also the wrong-sign and wrong-strangeness contributions.

While in this work we concentrated on a very specific measurements in  $B$  decays, our results are more general. They apply straightforwardly, with minor changes, to other meson systems. The main ideas also apply to neutrino oscillation experiments. Observation of  $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\nu_\beta \rightarrow \nu_\alpha)$  has been advocated as a way to establish T-violation. Such a result can arise, however, also from non-standard interactions in the production or the detection processes [13, 14, 15].

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