

Lorentz symmetry breaking: phenomenology and constraints

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Abstract.

In this talk I shall review several motivations for considering departures from exact Lorentz invariance and the different theoretical frameworks adopted to describe these departures. Among these, I shall focus on an effective field theory approach and discuss the phenomenology and constraints of Lorentz symmetry breaking in the Standard Model as well as in Gravity. In particular I will focus on current constraints on UV breaking inspired by quantum gravity scenarios and briefly discuss the open issues and future perspectives for this field of research.

1. Introduction

Understanding the nature of spacetime at the Planck scale $M_{\text{Pl}} = 1.2 \times 10^{28}$ eV and beyond is still one of the greatest challenges in modern physics. It is in fact at this scale that present theories fail to be predictive and quantum aspects of gravitation are expected to be relevant. While this endeavour has been in the past partially hampered by lack of experimental/observational tests, this situation has changed in the last fifteen years. Indeed, quantum gravity models have shown that there can be several sub-Planckian physics “relic signatures” of quantum gravitational effects able to produce potentially observable deviations from the standard predictions.

Some of these new phenomena, which comprise what is often termed “QG phenomenology”, include tests of Quantum decoherence and state collapse [1], QG imprint on initial cosmological perturbations [2], Cosmological variation of couplings [3, 4], TeV Black Holes related to extra-dimensions [5], Planck scale spacetime fuzziness [6], Generalized uncertainty principle [7, 8, 9], Violation of discrete symmetries [10] and Violation of space-time symmetries [11].

Among the above fields a preeminent role was played by physics associated to violations of space-time symmetries in the form of departures from exact local Lorentz invariance (LI). In these proceedings I will provide a succinct description of the Effective Field theory (EFT) approach to this problem and provide a concise summary of its phenomenology and associated constraints. A more extensive recent review on the same subject can be found in [12]. See also [11, 8, 13].

2. Forms of Lorentz violations

The introduction of a set of observers endowed with standard clocks and rods and the agreement on a suitable procedure of synchronisation of such clocks allows to rigorously define operatively



spacetime. Within such a setting the usual von Ignatovsky derivation (see *e.g.* [12] and references therein) of Lorentz transformations is based on four basic axioms: i) Pre-causality; ii) Spatial and temporal homogeneity; iii) Spatial isotropy; iv) Relativity principle. In view of such hypotheses, one can prove that the most general change of coordinates between two inertial reference frames reduce to the familiar Lorentz transformations with an invariant limit speed C to be determined by observations. Notice that the speed scale C is in principle arbitrary and can even be infinite — in this latter case one finds back the usual Galilean transformations.

In extending this construction to the case of a spacetime that change its nature beyond some scale it is worth revising the implications of the above mentioned assumptions [14]. Assumption i) guarantees that the temporal order of events along the history of an observer cannot be upturned by a change of reference frame. Hypothesis ii) enforces that the laws of transformation relating coordinates in two inertial frames have to be linear, i.e. independent of the location of space and time where they are performed. As for assumption iii), it mirrors the indifference of physical phenomena with respect to the orientation of reference axes. Assumption iv) embodies the notion of the equivalence of all the inertial frames. It also guarantees that the transformations gain a group structure. Finally, assumptions iii) and iv) yield the so-called Principle of Reciprocity (see *e.g.* [12] and references therein).

While assumption i) and ii) are generally assumed to hold (see however [14]) the hypothesis iii) can be relaxed, and a preferred direction can be introduced, generally reducing the numbers of generators and symmetries for the system. The resulting theory is commonly referred to as Very Special Relativity [15], and the set of transformations associated to this end up leaving invariant the four-dimensional spacetime manifold a Finslerian structure [16], i.e. explicit dependencies on the velocities are present in the symmetric metric tensor accounting for measurements of space and time intervals. In what follows we shall instead relax assumption iv) so we shall consider the case in which quantum gravitational physics could provide a preferred frame.

3. The EFT approach to Lorentz Violation

In the absence of a definitive theory of quantum gravity, several ideas have been present for discussing possible Lorentz breaking scenarios. As said, we shall focus here on the most conservative approach based on an EFT framework within which to describe and study the empirical effects of small deviations from exact LI. Such a framework goes under the name of Standard Model Extension (SME). This consists in supplementing the standard model of particle physics with all the possible Lorentz violating operators that can be written without changing the field content or violating gauge symmetry. The operators appearing in the SME can be conveniently classified according to their mass dimension and behaviour under CPT given that Lorentz violation does not imply CPT violation for local EFTs, while CPT violation does imply Lorentz violation in local EFT [17, 18].

The total SME consists of hundreds of operators, many of which are very tightly constrained, however it is usually simplified by imposing that subgroups of the Lorentz group or discrete symmetries such as CPT are preserved. As often done, we shall consider here that rotational invariance is a symmetry of nature in one particular frame which is generally identified with the rest frame of the cosmic background radiation (CBR). Given that our motion with respect to this frame is only $\beta \simeq 10^{-3}$ this does not significantly affect our results.

With the assumption of rotational invariance the number of possible operators for the SME drops significantly and it makes sense, from an EFT point of view, to classify the operators for their relative mass dimensions. In this sense one generally considers separately the case of operators of mass dimension 3,4 and that of higher order operators (mass dimension 5 and 6). The Minimal Standard Model Extension or mSME, is the subset of the SME which considers only mass dimension 3 and 4 operators i.e. only those which naively (without considering anisotropic scaling techniques [19]) power counting renormalizable. In what follows we shall discuss in detail

the resulting EFT for leptons and photons (QED) and neutrinos given that these elementary particles are the most relevant for the current phenomenological constraints.

3.1. QED sector

The high energy ($M_{\text{Pl}} \gg E \gg m$) rotational invariant dispersion relations e.g. for the mSME of QED can be expressed as (see e.g. [11] and references therein)

$$E_{\text{el}}^2 = m_e^2 + p^2 + f_e^{(1)}p + f_e^{(2)}p^2 \quad \text{electrons} \quad (1)$$

$$E_\gamma^2 = (1 + f_\gamma^{(2)})p^2 \quad \text{photons} \quad (2)$$

Noticeably in this limit the mSME reduces to the phenomenological theory for Lorentz breaking proposed by Coleman and Glashow where particles dispersion relation pertain their relativistic form but are characterised by different limit speeds [20, 21, 22].

Going further one can consider dimension 5 operators. In [23] it was found that there are essentially only three operators of dimension five, quadratic in the fields, that can be added to the QED Lagrangian preserving rotational and gauge invariance, but breaking local LI. These are all \mathcal{CPT} odd. The corresponding high energy dispersion relations respectively for photons and electrons are

$$\omega_\pm^2 = k^2 \pm \xi \frac{k^3}{M_{\text{Pl}}}, \quad (3)$$

where the + and – signs denote right and left circular polarisation, and

$$E_\pm^2 = p^2 + m^2 + \eta_\pm \frac{p^3}{M_{\text{Pl}}}, \quad (4)$$

where the + and – signs now denoting positive and negative helicity states.

It is quite important to note that for positrons, the \mathcal{CPT} odd character of the relevant operators implies that the same dispersion relation holds, with $\eta_{\bar{q}\pm} = -\eta_{q\mp}$ where \bar{q} and q denote respectively anti-fermion and fermion [24, 25].

Finally, one can consider \mathcal{CPT} even, rotationally invariant mass dimension five and six LIV terms has been computed in [26]. From these operators, the dispersion relations of fermions and photons can be easily computed, yielding

$$E^2 - p^2 - m^2 = \frac{\alpha_R^{(6)} E^3}{M_{\text{Pl}}^2} (E + sp) + \frac{\alpha_L^{(6)} E^3}{M_{\text{Pl}}^2} (E - sp) + \frac{m}{M_{\text{Pl}}} (\alpha_R^{(5)} + \alpha_L^{(5)}) p^2 + \alpha_R^{(5)} \alpha_L^{(5)} \frac{p^4}{M_{\text{Pl}}^2} \quad (5)$$

$$\omega^2 - k^2 = \beta^{(6)} \frac{k^4}{M_{\text{Pl}}^2}, \quad (6)$$

where m is the electron mass and where $s = \sigma \cdot \mathbf{p}/|\mathbf{p}|$ is the helicity of the fermions, and we have neglected terms of order m^2/M_{Pl}^2 stemming from the $\tilde{\alpha}^{(6)}$ terms as they are highly suppressed.

We can further simplify eq. (5) by noting that high-energy ($E \sim p$) fermions states are almost exactly chiral, and by grouping terms with same powers of the momentum. In this case we get

$$E^2 = p^2 + m^2 + \frac{m}{M_{\text{Pl}}} \eta^{(2)} p^2 + \eta_\pm^{(4)} \frac{p^4}{M_{\text{Pl}}^2} \quad (7)$$

where $R = +$, $L = -$ and we have labelled $\eta^{(n)}$ is the dispersion coefficient of the LIV p^n term in the dispersion relation for the fermion. We choose $\eta^{(n)}$ as the fermion coefficient symbol as this

nomenclature is common in the literature. Similarly, we shall use $\xi^{(n)}$ for the generic dispersion coefficient for a photon (so in (6) we shall take $\beta^{(6)} = \xi^{(4)}$).

In (7) the quadratic modification generated by the dimension five operator is suppressed by a factor of order m/M_{Pl} and hence it can be safely neglected, provided that $E > \sqrt{mM_{\text{Pl}}}$. By \mathcal{CPT} , the dispersion relation of the anti-fermion is given by (5), with the replacements $s \rightarrow -s$ and $p \rightarrow -p$. If q, \bar{q} denote a charge fermion and anti-fermion, then the relevant anti-fermion coefficient $\eta_{\bar{q}}^{(6)}$ is such that $\eta_{\bar{q}_{\pm}}^{(6)} = \eta_{q_{\mp}}^{(6)}$, where \bar{q}_{\pm} indicates an anti-fermion of positive/negative helicity (and similarly for the q_{\pm}). This different behaviour between even and odd powers of “ n ” type dispersion relations leads to quite distinct phenomenologies.

3.2. Neutrino sector

Neutrino physics can be affected in several ways by Lorentz breaking operators. The most common channels for casting constraints are observations concerning the speed of neutrinos with respect to light, modified oscillations between neutrinos flavors and threshold reactions.

The generic neutrino LIV operators, at any mass dimension, have been categorized in [27]. Also in this case, a significant reduction in the number of terms can be achieved by requiring that the LIV operators are rotationally symmetric. Let us then focus on the Lagrangian for neutrinos with LIV operators of mass dimension up to six involving a vector field u^a coupled to a Dirac neutrino ψ ¹ in a mass eigenstate with mass m that are quadratic in matter fields and hence modify the free field equations. We shall further assume that the Lorentz violation is diagonal in the mass basis. This might be justified by the idea that any theory of quantum gravity inducing such LIV must reduce to general relativity in the infrared, hence any Lorentz violation induced by quantum gravity would be primarily controlled by the charges that couple to gravity. Of course, this does not mean that the coefficients for each mass eigenstate are the same, as RG effects would not allow them to be the same at any energy.

With these assumptions, yield a high energy neutrino dispersion relation of the form [12]

$$E_i^2 = p^2 + N_i^2 \quad (8)$$

$$N_i^2 = m_i^2 + 2(a_i + b_i)p - (c_i + d_i)p^2 + 2(\zeta_{i,1} - \zeta_{i,2})\frac{p^3}{M_{\text{Pl}}} + 2\frac{\alpha_{i,L}^{(6)}p^4}{M_{\text{Pl}}^2}$$

As before, the term proportional to m/M_{Pl} has been ignored. The net result of the LIV terms is to modify the dispersion relation of each mass eigenstate according to

$$E^2 - p^2 - (m_i)^2 = \sum_{n=1}^4 \xi_i^{(n)} \frac{|p|^n}{M_{\text{Pl}}^{n-2}}, \quad (9)$$

where $\xi_i^{(n)}$ is a coefficient that depends on the relevant terms in the Lagrangian (8), so that constraints on the $\xi_i^{(n)}$ can always be translated in constraints on the coefficients of Eq. (8). The corresponding anti-particle dispersion relation is easily derived by considering the behaviour of each term under CPT and given by

$$E^2 - p^2 - (m_i)^2 = \sum_{n=1}^4 (-1)^n \xi_i^{(n)} \frac{|p|^n}{M_{\text{Pl}}^{n-2}}. \quad (10)$$

One can then leave the index n as a free phenomenological parameter and consider the cases $n = 2, 3, 4$ separately (the case $n = 1$ would produce huge effects at low energy and is strongly constrained).

¹ No Majorana, LIV operators can be constructed in the rotational invariant case [27] albeit it is not clear if they could be dynamically generated e.g. by the same mechanism providing mass to such neutrinos.

Note also that many existing neutrino oscillation experiments measure the transition probability between neutrino flavours P_{IJ} (I and J here labelling neutrino flavours) which is affected by the modified dispersion relation (8).

$$P_{IJ} = \delta_{IJ} - \sum_{i,j>i} 4F_{IJij} \sin^2 \left(\frac{\delta N_{ij}^2 L}{4E} \right) + 2G_{IJij} \sin^2 \left(\frac{\delta N_{ij}^2 L}{2E} \right), \quad (11)$$

with $\delta N_{ij}^2 = N_i^2 - N_j^2$ and F_{IJij} and G_{IJij} are functions of the mixing matrixes. Many of these experiments also quote results on a deviation of the neutrino speed from that of light, i.e.

$$\left(\frac{\Delta c}{c} \right)_{ij}^{LIV} = E^{-2} (\delta N_{ij}^2 - \delta m_{ij}^2) \quad (12)$$

which can be easily translated into constraints on the coefficients of Eq. (8).

4. Phenomenology

The development of a systematic EFT-based approach as discussed above represented a milestone in searches for deviations from exact Lorentz invariance. The phenomenological toolkit that was developed also thanks to this systematic approach is now quite rich and can be split in two big subsets: terrestrial experiments and astrophysical observations.

4.1. Terrestrial experiments

It is an observational fact that Nature as we probe it well below the Planck scale it is Lorentz invariant to a very high degree. It is hence logic that in seeking for low energy deviations from this symmetry, as those systematically described by the minimal Standard Model extension (mSME), one has to resort to very high precision, and hence Earth based, experiments (some of them testing rotational breaking operators as well). An incomplete list includes (for more details see e.g. [11, 28, 29, 12]).

Clock comparison Experiments Two co-local atomic transition frequencies can be considered as two clocks. As the clocks move in space, they pick out different components of the Lorentz violating tensors in the mSME. This in turn yields a sidereal drift between the two clocks which could be constrained by measuring the difference between the frequencies over long periods. This technique allows to cast very high precision limits on some parameters in the mSME (generally for protons and neutrons).

Cavity Experiments The technique adopted in these experiments casts constraints on the variation of the cavity resonance frequency (with respect to a stationary frequency standard) as its orientation changes in space. While this is intrinsically similar to clock comparison experiments, these kind of experiments allows to cast constraints also on the electromagnetic sector of the mSME (as one of the “clocks” in this case involves photons).

Neutral mesons Experiments The mass difference of neutral mesons is one of the most accurately quantities in the SM. The SME operators do affect such a quantity in a Lorentz breaking way, hence one generically expects an orientation dependent change which can be constrained by looking for sidereal variations and other orientation effects. Also lifetime directional dependence has been investigated (see e.g. [30]).

Penning traps In a Penning trap a combination of static magnetic and electric fields confines a charged particle for long times. One can then problem possible deviation from exact Lorentz invariance by monitoring the particle cyclotron motion in the magnetic field and its Larmor precession due to the spin. In fact the relevant frequencies for both these motions are affected by some mSME operators and Penning traps can be set up so to make them very sensitive to differences in these frequencies.

Spin polarized torsion balances Spin-torsion balances are a very effective tools for constraining the electron sector of the mSME. An example of such balances consists in an octagonal pattern of magnets which is constructed so to have an overall spin polarization in the octagons plane. Four of these octagons are suspended from a torsion fibre in a vacuum chamber so to give an estimated net spin polarization equivalent to $\approx 10^{23}$ aligned electron spins. In order to detect Lorentz breaking effects one has again to look for orientation dependent phenomena.

Nuclear Spin Experiments The absence of a preferred direction is also checked with great precision using nuclear spin, which translates into more stringent limits on LIV operators for the light quarks and photons (the photon LIV operator will also contribute because of the electromagnetic interactions inside the nucleon). These methods are currently able to constraints the coefficients of quarks and photons at $O(10^{-7} \div 10^{-8})$.

4.2. Astrophysical observations

Testing the higher mass dimensions operators (5 or 6) of the SME is obviously a task that requires to probe much higher energies than those achieved in terrestrial experiments. Energies up to 10^{20} eV are achieved in high energy astrophysics and as such this field has played an eminent role in QG phenomenology. One comment is in order before we further describe these tests and it concerns our parametrisation. We have introduced dimensionless coefficients $\eta^{(n)}$ and expressed the higher order terms in the dispersion relations via suitable ratios of the particle momentum to the Planck scale. Of course nothing guarantees that models of quantum/emergent spacetime will predict the Lorentz breaking scale to be coincident with the Planck scale. So, missing a derivation of our dispersion relation from a specific QG model, this has to be considered just a convenient parametrisation.

A brief list of the most commonly used observational constraints in high energy astrophysics are (for more details see again [11, 12])

Vacuum Birefringence The fact that in the “LIV extended QED” with dimension 5, CPT odd operators, opposite “helicities” have slightly different group velocities, implies that the polarisation vector of a linearly polarised plane wave with energy k rotates, during the wave propagation over a distance d . The angle of rotation will be different for different photon energies hence this effect could potentially disrupt the amount of polarisation present in a some polarised light travelling over long distances. This methods has been applied to several astrophysical sources like GRBs and Pulsar Wind Nebula (closer but more energetic). Note also that this method cannot be applied to the case of dimension 6, CPT even operators as in this case the two felicities of the photon have the same LIV coefficient, $\xi^{(4)}$.

Photon time of flight A photon dispersion relation in the form of (3) implies that photons of different colours (wave vectors k_1 and k_2) travel at slightly different speeds. We shall assume here that no birefringent effects are present, so that $\xi_+^{(n)} = \xi_-^{(n)}$ (for the case of dimension 5 CPT odd operators one can consider one helicity at a time or analyse the effect of a realistic photon beam in a more detailed way, see e.g. [31]). When propagating on a cosmological distance d , the effect of energy dependence of the photon group velocity will generically produces an energy, and travelled distance, dependent time delay which can be constrained using current astrophysical observations mainly from gamma ray bursts (GRBs) and active galactic nuclei (i.e. high redshift, high energy sources).

Threshold reactions LIV corrections are quite important in threshold processes because the LIV term (which as a first approximation can be considered as an additional mass term) only needs to be comparable to the (invariant) mass of the particles produced in the final state for strongly affecting this kind of reactions. A quite rich phenomenology of threshold reactions is introduced by LIV in EFT and threshold theorems can be generalized in this

context [32, 33]. The main conclusions of the investigation into threshold reactions are that [34, 12]

- Threshold configurations still corresponds to head-on incoming particles and parallel outgoing ones
- The threshold energy of existing threshold reactions can shift, and upper thresholds (i.e. maximal incoming momenta at which the reaction can happen in any configuration) can appear
- Pair production can occur with unequal outgoing momenta
- New, normally forbidden, reactions can be viable

All of these physical situations can be exploited to cast constraints as we shall see in the next section.

Synchrotron radiation Synchrotron emission is strongly affected by LIV [35, 36, 37, 38, 39, 40, 41] and hence effective constraints can be obtained by confronting observed and expected synchrotron spectra from energetic astrophysical sources (e.g. the Crab Nebula). For Planck scale LIV and observed energies, strong constraints can be casted for mass dimension four and five LIV operators. Dimension six operators can be effectively constrained within theoretical frameworks for which the LIV scale is supposedly lower than the Planck scale (see e.g. [41]).

5. Constraints

We shall now briefly discuss the (mainly astrophysical) constraints on Lorentz breaking mass dimension 5 and 6 for different sectors of the SME.

5.1. QED with rotational invariant CPT odd dimension 5 operators

There is a very rich literature devoted to constraining the Myers-Pospelov version of the SME. We shall not attempt here a detailed discussion but just briefly summarise the available constraints.

Synchrotron radiation constraints from the Crab Nebula: Presently a very important object in this sense is the Crab nebula (CN). This pulsar wind nebula had its origin by a supernova explosion observed in 1054 A.D. Its distance from Earth is approximately 1.9 kpc and it is one of the best studied sources of diffuse radio, optical and X-ray radiation. The Nebula emits an extremely broad-band spectrum (21 decades in frequency, see [40] for a comprehensive list of relevant observations) that is produced by two major radiation mechanisms. The emission from radio to low energy γ -rays ($E < 1$ GeV) is thought to be synchrotron radiation from relativistic electrons, whereas inverse Compton (IC) scattering by these electrons is the favoured explanation for the higher energy γ -rays. From a theoretical point of view, the current understanding of the whole environment is based on the so called Kennel–Coroniti [42], which accounts quite accurately for the general features observed in the CN spectrum.

Developing on the initial idea discussed in [35], a full reconstruction of the synchrotron emission processes in the CN within the SME with dimension 5 CPT operators (see Eqs. (3) and (4)) has been performed in [40]. This procedure requires fixing most of the model parameters using radio to soft X-rays observations (which are basically unaffected by LIV), a careful step by step revisitation of the basic assumptions made in the Kennel–Coroniti model and also to take into account the contribution of novel LIV effects such as vacuum Čerenkov and helicity decay. Finally, a χ^2 analysis can be performed to quantify the agreement between models and data [40]. From this analysis, one can conclude that the LIV parameters for the leptons are both constrained, at 95% CL, to be $|\eta_{\pm}| < 10^{-5}$.

Birefringence constraints: The best constraints on the photon sector are instead obtained by using birefringence effects associated with the CPT odd nature of the relevant LIV operators. Strong constraints came again from the CN [43], from which a value $|\xi^{(3)}| \lesssim 6 \times 10^{-10}$ at 95% Confidence Level (CL) was obtained by considering the observed polarization of hard-X rays [44] (see also [45]). Polarized light from GRBs has also been detected and given their cosmological distribution they could be ideal sources for improving the above mentioned constraints from birefringence. Attempts in this sense were done in the past [24, 46] but the observed of polarisation was later deemed controversial. Furthermore, GRBs for which the polarization is detected and the spectral redshift is precisely determined are scarce. In [47] this problem was circumvented by using indirect methods (the same used to use GRBs as standard candles) for the estimate of the redshift. This leads to a possibly less robust but striking constraint $|\xi^{(3)}| \lesssim 2.4 \times 10^{-15}$. A similar constraint $|\xi^{(3)}| \lesssim 10^{-15}$ was obtained in [48] by making use of two GRBs observed by the Japanese satellite IKAROS. Also in this case the redshift was determined using indirect methods such as the correlation between peak energy and luminosity of the GRB prompt emission. Remarkably, the above mentioned constraints were recently improved by using the INTEGRAL/IBIS observation of the GRB 061122, for which a redshift $z \approx 1.33$ was derived thanks to the determination of the GRB's host galaxy. In this case a constraint $|\xi^{(3)}| \lesssim 3.4 \times 10^{-16}$ was derived [49] (see also [50] for a similar previous constraint).

5.2. QED with rotational invariant CPT even dimension 5 and 6 operators

As we have shown before, this kind of operators lead to modified dispersion relations characterised by quartic terms in the particle momenta suppressed by the squared Planck mass. Furthermore the CPT even nature of the operators prevent birefringent features in the photon sector. This feature basically implies the impossibility to use any of the above discussed observation for casting effective constraints (e.g. the synchrotron emission from the CN would not provide a constraint better than 10^6 on the electron parameter given the $(E/M_{\text{Pl}})^2$ suppression). However, there are available observations exploring much higher energies, i.e. those related to the so called Ultra High Energy Cosmic Rays (UHECR).

Astrophysical constraints from GZK reaction secondaries: The GZK cut off [51, 52], is a suppression of the high-energy tail of the UHECR spectrum arising from interactions with CMB photons: $p\gamma \rightarrow \Delta^+ \rightarrow p\pi^0(n\pi^+)$. When Lorentz invariance holds, this process has a threshold energy $E_{\text{th}} \simeq 5 \times 10^{19} (\omega_0/1.3 \text{ meV})^{-1} \text{ eV}$ (where ω_0 is the target photon energy). Experimentally, the presence of a suppression of the UHECR flux was claimed by the HIRES collaboration [53, 54] (but was initially claimed to be absent by the AGASA collaboration [55]). Although the cut off could be also due to the finite acceleration power of the UHECR sources, the fact that it occurs at the expected energy favours the GZK explanation. The results presented in [56] seemed to further strengthen this hypothesis (but see further discussion in the conclusions).

Significant limits on $\xi = \xi^{(4)}$ and $\eta = \eta^{(4)}$ for the electron/positron can be derived by considering UHE photons generated as secondary products of the GZK reaction [57, 58]. These UHE photons originate because the GZK process leads to the production of neutral pions that subsequently decay into photon pairs. These photons are mainly absorbed by pair production onto the CMB and radio background. Thus, the fraction of UHE photons in UHECRs is theoretically predicted to be less than 1% at 10^{19} eV [59]. Several experiments imposed limits on the presence of photons in the UHECR spectrum. In particular, the photon fraction is less than 2.0%, 5.1%, 31% and 36% (95% C.L) at $E = 10, 20, 40, 100 \text{ EeV}$ respectively [60, 61] (at lower energies even stricter limits 0.4%, 0.5%, 1.0%, 2.6% and 8.9% (95% C.L) above $E = 1, 2, 3, 5$ and 10 EeV respectively were recently found [62]).

The key idea for casting constraints here, relies on the strong dependence of the photon pair production on LIV modifications. In particular, the (lower) threshold energy can be shifted

and in general an upper threshold can be introduced [34]. If the upper threshold energy is lower than 10^{20} eV, then GZK secondary UHE photons are no longer attenuated by the CMB and can reach the Earth. Hence, they would constitute a significant fraction of the total UHECR flux and thereby they would violate the above mentioned experimental bounds [57, 58, 63].

To complete the analysis one can also notice that the γ -decay process can also imply a significant constraint. Indeed, if some UHE photon ($E_\gamma \simeq 10^{19}$ eV) is detected by experiments (and the Pierre Auger Observatory, PAO, will be able to do so in few years [60]), then γ -decay must be forbidden above 10^{19} eV [58] (note that for $n = 4$ the two photons velocities travel at the same speed, so there is in this case a single population).

The combination of above mentioned ‘‘upper threshold constraint’’ and the expected constraint from the absence of gamma decay would bound the QED LIV parameters at order $n = 4$ to the roughly rectangular region (see Figure 2, of [31] for more details) $-10^{-7} \lesssim \xi^{(4)} \lesssim 10^{-8}$ and $-10^{-7} \lesssim \eta^{(4)} \lesssim 10^{-6}$. Note that the absence of gamma decay can improve on the determination of $\eta = \eta^{(3)}$ as provided by the synchrotron constraint, leading to a double sided bound $|\eta^{(3)}| \lesssim 10^{-16}$ (see Figure 3 [58]). All of this of course relies on the actual observation of the GZK cutoff which has instead been questioned recently, see Section 7 for a dedicated discussion.

5.3. SME: Hadronic sector

The very same GZK photo-pion production is strongly affected by LIV and the constraints implied by the detection of this effect have been extensively considered in the literature [64, 65, 34, 26, 66, 67]. Nonetheless, a detailed LIV study of the GZK feature is hard to perform, because of the many astrophysical uncertainties related to the modelling of the propagation and the interactions of UHECRs. In fact, the Lorentz breaking operators could lead to relevant modifications of the GZK mean free path. Consequently, the propagated UHECR spectrum can display new features, like bumps at specific energies, suppression at low energy, recovery at energies above the cutoff. These are all features which cannot be easily conciliated with the observed spectrum, even taking into account experimental uncertainties. Furthermore, the emission of Cherenkov γ -rays and pions in vacuum would lead to sharp suppression of the spectrum above the relevant threshold energy. After a detailed statistical analysis of the agreement between the observed UHECR spectrum and the theoretically predicted one in the presence of LIV and assuming pure proton composition, the final constraints implied by UHECR physics are (at 99% CL) [68] $-10^{-3} \lesssim \eta_p^{(4)} \lesssim 10^{-6}$ and $-10^{-3} \lesssim \eta_\pi^{(4)} \lesssim 10^{-1}$ for $\eta_p^{(4)} > 0$ or $-10^{-3} \lesssim \eta_\pi^{(4)} \lesssim 10^{-6}$ for $\eta_p^{(4)} < 0$. Obviously the same analysis can be applied also to dimension five, CPT odd, operators leading to much stronger constraints (order $O(10^{-14})$).

5.4. SME: Neutrinos

Neutrinos physics have recently gained a prominent role in modern tests of Lorentz invariance due to the growing experimental effort devoted to their study as well as for their light mass that makes their threshold reaction very sensitive to LIV at relatively accessible energies. here we shall succinctly review the main constraints so far cast on this sector of the Standard Model.

Constraints from neutrino time of flights: In assessing the limit speed of neutrinos with respect to that of light, we have to date only a single event to rely on, the supernova SN1987a. This was a peculiar event which allowed to detect the almost simultaneous (within a few hours) arrival of electronic antineutrinos and photons. Although only few electronic antineutrinos at MeV energies was detected by the experiments KamiokaII, IMB and Baksan, it was enough to establish a constraint $(\Delta c/c)^{TOF} \lesssim 10^{-8}$ [69] or $(\Delta c/c)^{TOF} \lesssim 2 \times 10^{-9}$ [70] by looking at the difference in arrival time between antineutrinos and optical photons over a baseline distance of

1.5×10^5 ly. Further analyses of the time structure of the neutrino signal strengthened this constraint down to $\sim 10^{-10}$ [71, 72]. The scarcity of the detected neutrino did not allow the reconstruction of the full energy spectrum and of its time evolution in this sense one should probably consider constraints purely based on the difference in the arrival time with respect to photons more conservative and robust. Unfortunately adopting $\Delta c/c \lesssim 10^{-8}$, the SN constraint implies very weak constraints, $\xi_\nu^{(3)} \lesssim 10^{13}$ and $\xi_\nu^{(4)} \lesssim 10^{34}$. Note that observational constraints on $\Delta c/c$ translate in constraints on the LIV parameter via the formula [73]

$$\xi_\nu^{(n)} = \frac{2}{n-1} \left(\frac{\Delta c}{c} \Big|_{\text{Obs}}^{\text{TOF}} + \frac{m^2}{2p^2} \right) \times \left(\frac{M_{\text{Pl}}}{p} \right)^{(n-2)}. \quad (13)$$

Constraints from neutrino oscillations: At odd with the previous case, we do have a wealth of information only about neutrino oscillations which however constraints only the differences among LIV coefficients of different flavors. The best constraint to date comes from survival of atmospheric muon neutrinos observed by the former IceCube detector AMANDA-II in the energy range 100 GeV to 10 TeV [74], which searched for a generic LIV in the neutrino sector [75] and achieved $(\Delta c/c)_{ij} \leq 2.8 \times 10^{-27}$ at 90% confidence level assuming maximal mixing for some of the combinations i, j . The same constraint applies to the corresponding antiparticles as IceCube does not distinguish neutrinos from antineutrinos. IceCube is also expected to improve this constraint to $(\Delta c/c)_{ij} \leq 9 \times 10^{-28}$ in the next few years [76]. The lack of sidereal variations in the atmospheric neutrino flux also yields comparable constraints on some combinations of SME parameters [77]. Putting all together, it seems that no-flavour dependent SME can be tolerated and many studies just assume flavour independent LIV as they starting base of their analysis.

Constraints from neutrino threshold reactions: Several threshold processes have been considered in the literature, most prominently the neutrino Čerenkov emission $\nu \rightarrow \gamma \nu$, the neutrino splitting $\nu \rightarrow \nu \nu \bar{\nu}$ and the neutrino electron/positron pair production $\nu \rightarrow \nu e^- e^+$. They are all very similar, so we shall discuss here only the latter for illustrational purposes and assume no LIV modification in the electron/positron sector as we have already seen that LIV in this sector is strongly constrained. The threshold energy for arbitrary n is then

$$E_{th,(n)}^2 = \frac{4m_e^2}{\delta_{(n)}} \quad \text{with} \quad \delta_{(n)} = \xi_\nu^{(n)} \left(\frac{E_{th,(n)}}{M_{\text{Pl}}} \right)^{n-2}. \quad (14)$$

The rate of this reaction was firstly computed in [78] for $n = 2$ but can be easily generated to arbitrary n [73] (see also [79] for a more general treatment and detailed considerations). The generic energy loss time-scale then reads (dropping purely numerical factors)

$$\tau_{\nu\text{-pair}} \simeq \frac{m_Z^4 \cos^4 \theta_w}{g^4 E^5 \xi_\nu^3} \left(\frac{M_{\text{Pl}}}{E} \right)^{3(n-2)}, \quad (15)$$

where g is the weak coupling and θ_w is Weinberg's angle.

The observation of upward-going atmospheric neutrinos up to 400 TeV by the experiment IceCube implies that the free path of these particles is at least longer than the Earth radius implies a constraint $\xi_\nu^{(3)} \lesssim 40$ (taking a conservative baseline of about 6000 Km). No effective constraint can be obtained for $n = 4$ LIV, however in this case neutrino splitting (which has the further advantage to be purely dependent on neutrinos LIV) could be used on the ‘‘cosmogenic’’ neutrino flux which one expects from the decay of charged pions produced by the aforementioned GZK reaction.

It is easy to see that the neutrino splitting should modify the spectrum of the ultra high energy neutrinos by suppressing the flux at the highest energies and enhancing it at the lowest ones. In [80] it was shown that future experiments like ARIANNA [81] will achieve the required sensitivity to cast a constraint of order $\xi_\nu^{(4)} \lesssim 10^{-4}$. Note however, that the rate for neutrino splitting computed in [80] was recently recognised to be underestimated by a factor $O(E/M)^2$ [82]. An improved analysis can be found in [73] leading to an expected constraint from AUGER $\xi_\nu^{(4)} \lesssim 10^{-7}$ (note that this constraint is cast using the expected flux which won't be able to distinguish different flavours).

However, we have just seen that experimental observations of the depth of the shower maximum of UHECR interactions in the atmosphere hinted at the possible presence of nuclei heavier than protons in UHECRs [83]. In this case, pion production would be suppressed at UHE and hence the UHE neutrino flux could be much smaller than the expectation from pure proton composition.

A final comment is devoted to the so called pion decay channel $\pi^+ \rightarrow \nu_\mu \mu^+$ that was extensively explored in dealing with the recent OPERA claim [84, 85]. In this case it was found that for superluminal neutrinos and unmodified pion and muon, the process could be actually forbidden hence suppressing muonic neutrino pair production in the CERN beam. In [84] it was noticed that the detection of up to 40 GeV neutrinos at OPERA would imply a bound $\xi_\nu^{(2)} \lesssim 10^{-7}$. Also in [84, 85] a constraint $\xi_\nu^{(2)} \lesssim 10^{-13}$ was derived from atmospheric neutrinos using neutrino pair production and neutrino Čerenkov emission.

While OPERA detection of superluminal neutrinos [86] was soon recognised as flawed (see revised version of [86]), it is worth reconsider it here as a case study. Indeed, the excitement created by the OPERA's initial report has obviously subsided for the greater physics community. For the quantum gravity community that focuses on possible experimental signatures of quantum gravity, however, technical issues were raised in how to analyse these types of accelerator based experiments properly. In particular, the detailed physics of anomalous reactions, and how they reduce the intensity of a particle beam from source to detector became central to the discussion.

The most convincing theoretical objection to the initial OPERA result was produced by Cohen and Glashow [78] shortly after the OPERA report and involved just such an anomalous reaction. Cohen and Glashow used the fact that superluminal neutrinos should emit electron-positron pairs (see section 3.2) to argue that the OPERA results were not even self-consistent: any neutrino with the speed reported by OPERA should have lost most of its energy to pair production while it propagated from CERN to the detector at Gran Sasso. The maximum energy in the beam would therefore have dropped to be below some termination energy E_T , and Cohen and Glashow showed that E_T for the OPERA beam was less than the average ~ 17 GeV energy reported by OPERA.

More specifically by integrating the energy loss rate from pair production (as deducible from Eq. (15)) over a distance L and by assuming that the typical energy loss length be much smaller than L one obtains

$$E^{-3n+1} - E_0^{-3n+1} = (3n-1)\xi_\nu^3 E_{\text{ref}}^{-3(n-2)} k \frac{G_F^2}{192\pi^3} L \equiv E_T^{-3n+1}, \quad (16)$$

where E is the energy on a neutrino starting with energy E_0 after propagation over the distance L and E_{ref} is the energy at which we normalize the parameter ξ_ν . The factor $k = 25/448$ was computed in [78] for the case $n = 2$, while for the general case it can be found in [79]. The ‘‘termination’’ energy E_T corresponds to the energy that a neutrino would approach after propagation over a distance L if it started with $E_0 \gg E_T$. We remark here that the termination energy E_T is a mildly varying function of n and of the energy scale E_{ref} . For a LIV extension of the standard model at order $n = 2$ (see the Coleman-Glashow model discussed briefly above) it was then shown that the Opera claim of $\xi_\nu \sim 5 \times 10^{-5}$ for $E_{\text{ref}} \sim 10$ to 30 GeV implied

$E_T \simeq 12.5$ GeV: a value of E_T obviously incompatible with the observation of neutrinos up and above 40 GeV in Opera. Similar considerations lead to an expectation of $E_T \simeq 15.0$ GeV for $n = 3$.

Note that this estimate was done without taking into account the similar mechanism of neutrino splitting. While this reaction is strictly forbidden in the $n = 2$ case, it is not so at higher orders and needs to be taken into account given that the energy loss rate of this process is comparable to the one for pair production loss (see e.g. [73, 82, 79]), and hence is not negligible for $n > 2$. In particular, this extra reaction can lead to energy losses without generating large numbers or electron-positron pairs as those searched for in an early ICARUS analysis of the Opera beam [87].

The physics of the Cohen-Glashow argument was correct, however the authors did not worry about adjusting for the finite size of the baseline. A finite baseline can be of the same order as the energy loss length of neutrinos undergoing pair production. This allows for some neutrinos to undergo only one or a few Cherenkov emissions within their time of flight. Therefore the most energetic neutrinos of the injection beam can still reach the end of the baseline with an energy larger than E_T [73]. It is then necessary, in order to cast a robust constraint on LIV by using long baseline experiments, to run a full MonteCarlo simulation of the propagation of neutrinos aimed at computing the neutrino spectrum on arrival in the presence of this energy loss process. While this was not an issue for the Cohen and Glashow result, as it was one piece of a number of experimental and theoretical concerns about OPERA [88, 87, 69, 70], if one wishes to use time of flight experiments alone to set robust constraints on neutrino LIV, the issue must be addressed.

In [73] a complete analysis for the case of OPERA in the cases $n = 2$ and $n = 3$, taking into account neutrino pair creation and neutrino splitting (for $n = 3$) has been performed. It was there found that the propagated spectrum does indeed show a pronounced bump at the expected E_T , but is also characterised by a high energy tail that extends well above E_T and has an amplitude about 10% of the amplitude of the bump. Hence, the simple calculation of E_T is not per se conclusive for casting constraints, although the reconstruction of the propagated spectra in [73] demonstrated that in the special case of OPERA the detection of neutrinos with $E > 40$ GeV would have still pointed out an incompatibility between the adopted LIV framework and the experimental observation.

6. Overview of the constraints on the Matter Sector

We can now summarize the current status of the constraints for the LIV SME (rotational invariant) in Table 1. See [12] for further discussion.

| Order | photon | e^-/e^+ | Protons | Neutrinos ^a |
|-------|---------------------|--------------------|--------------------|----------------------------|
| n=2 | N.A. | $O(10^{-16})$ | $O(10^{-20})$ (CR) | $O(10^{-8} \div 10^{-10})$ |
| n=3 | $O(10^{-16})$ (GRB) | $O(10^{-16})$ (CR) | $O(10^{-14})$ (CR) | $O(40)$ |
| n=4 | $O(10^{-8})$ (CR) | $O(10^{-8})$ (CR) | $O(10^{-6})$ (CR) | $O(10^{-7})^*$ (CR) |

Table 1. Summary of typical strengths of the available constrains on the SME at different n orders for rotational invariant, neutrino flavour independent LIV operators. GRB=gamma rays burst, CR=cosmic rays. ^a From neutrino oscillations we have constraints on the difference of LIV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. ^{*} Just expected constraint from future experiments probing cosmogenic neutrinos up to 10^{20} eV [73].

Of course at first sight this might seem a quite satisfactory state of the art, so much so that

one might ask if we haven't tests Lorentz violations enough and should now move one towards new phenomenology. As usual, the answer is not a sharp one. Let us further elaborate on this point.

7. Open issues

Let's first stick to tests of violation of Lorentz invariance in the SME. Here the main open issue is provided by the lasting uncertainty about the ultra high energy cosmic rays (UHECR) composition and hence the actual observation of the GZK cutoff (see e.g. [89]). Given that all of the best constraints on $n = 4$ (mass dimension six LIV operators) are currently based on UHECR physics it is clear that addressing such uncertainty it should be a crucial task in the next coming years. For the moment, it would be however unfair to play down the present uncertainties on UHECR and place these constraints at the same level of robustness e.g. of those cast at order $n = 3$ by using the synchrotron radiation from the Crab nebula. In this sense any UHECR-independent constraint on the dimension 6 operators of the SME would be more than welcomed.

7.1. The naturalness of Lorentz violations

Another open issue is of course the naturalness problem of Lorentz breaking in the SME: in EFT radiative corrections will generically allow the percolation of higher dimension Lorentz violating terms into the lower dimension terms due to the interactions of particles [90]. Indeed, EFT loop integrals will be naturally cut-off at the EFT breaking scale, if such scale is as well the Lorentz breaking scale present in the dispersion relation, the two will effectively cancel leading to unsuppressed, coupling dependent, contributions to the mass dimension four kinetic terms that generate the usual propagators. Hence radiative corrections will not allow a dispersion relation with only p^3/M or p^4/M^2 Lorentz breaking terms but will automatically induce extra unsuppressed LIV terms in p and p^2 which will be naturally dominant. Several mechanisms have been proposed for protecting the lowest order operators. While no proposal can be deemed as definitive it is worth noticing that in most cases these "protective mechanisms" entail extra energy scales beyond the Planck one and hence imply possible new ways to constraint LIV scenarios (for a summary see [12]).

7.2. A possible LIV explanation of the IceCube UHE neutrinos anomaly?

So far, the IceCube collaboration has identified 87_{-10}^{+14} events from neutrinos of astrophysical origin with energies above 10 TeV, (the error being due to the uncertainty in the number of astrophysical events determined by the modelled subtraction of both conventional and atmospheric neutrinos particularly at energies below 60 TeV [91]). Above 60 TeV, the IceCube data are consistent with a spectrum given by $E_\nu^2(dN_\nu/dE_\nu) \simeq 10^{-8} \text{ GeVcm}^{-2}\text{s}^{-1}$. Spectra steeper than E_ν^{-2} do not give a good fit to the existing data in the 60 TeV to 2 PeV energy range [92]. However, no neutrino induced events have been seen above ~ 2 PeV, as would be expected from extending an E_ν^{-2} spectrum beyond ~ 2 PeV. In particular, IceCube has not detected any neutrino induced events from the Glashow resonance effect at 6.3 PeV. This enhancement leads to an increased IceCube effective area for detecting the sum of the ν_e 's, *i.e.*, ν_e 's plus $\bar{\nu}_e$'s by a factor of ~ 10 [93]. It is usually expected that 1/3 of the potential 6.3 PeV neutrinos would be ν_e 's plus $\bar{\nu}_e$'s unless new physics is involved. Thus, the enhancement in the overall effective area expected is a factor of ~ 3 . Taking account of the increased effective area between 2 and 6 PeV and a decrease from an assumed neutrino energy spectrum of E_ν^{-2} , we would expect about 3 events at the Glashow resonance provided that the number of $\bar{\nu}_e$'s is equal to the number of ν_e 's. Even without considering the Glashow resonance effect, several neutrino events above 2 PeV would be expected if the E_ν^{-2} spectrum extended to higher energies. Thus,

the lack of neutrinos above 2 PeV energy and at the 6.3 PeV resonance may be indications of a cutoff in the neutrino spectrum.

In [94] the effects of LIV operators of mass dimension greater than four on the propagation and resulting energy spectrum of superluminal neutrinos of extragalactic origin was explored in particular by taking into account the possible effect of electron-positron pair production and neutrino splitting.

Interestingly, a high-energy drop off in a propagated superluminal neutrino spectrum above ~ 2 PeV can result from above mentioned processes in the \mathcal{CPT} -conserving cases. The drop off matches the observed neutrino spectra for energy dependent values of $\xi_\mu^{(4)}$ of $O(10^3)$. This drop off would be preceded by a non-negligible pile up in the neutrino spectrum which could be used as a signature of this kind of LIV phenomenology. If instead mass dimension five, \mathcal{CPT} -violating, operators dominate, it was found [94] an absence of a clear cutoff in the propagated neutrino spectrum. On the other hand, if the cutoff will be in the end explained by a natural break in the neutrino spectra of the astrophysical neutrino sources, the above number would become a constraint.

8. Lorentz breaking in the gravity sector

Of course, given that are general quantum gravity arguments the main motivation for considering departures from exact Lorentz invariance at very high energies one might wonder why the gravitational sector should not be the main one affected by this new physics. Indeed, one can introduce an analogue of the mSME for the gravitational sector, i.e. the so called Einstein–Aether theory of gravity and then try extend it in the UV.

8.1. Einstein–Aether theory

The SME is constructed by coupling matter terms to non-zero LIV tensors in vacuum. However, when dealing with extensions of this framework to gravitational phenomena it is clear that leaving this tensors non-dynamical, would break general covariance. In this respect one can proceed along two alternative approaches, the first being the introduction of a suitable dynamics for the LIV tensor, the other being the the acceptance of an explicit breaking of the four dimensional diffeomorphism invariance of GR. In what follows we shall discuss the most representative models in this respect and they relation.

If one choses to preserve general covariance by promoting the LIV tensors to dynamical fields and if restricts his/her attention to rotational invariance, then it is natural to generate LIV couplings by including in the action either a scalar or a timelike vector field that takes a vacuum expectation value. In the case of a scalar, one can use a shift symmetry ($\phi(x) \rightarrow \phi(x) + \phi_0$) to construct actions for which the derivative of the scalar takes a non-zero value [95]. In the vector case, one simply puts a potential for the vector field such that the vector acquires a vev.

We concentrate on the vector case here as it is the simplest model that allows for rotationally invariant Lorentz violation [96] and it is also the natural extension of the framework considered so far for the rotationally invariant LIV matter sector. It is the most general theory for a unit timelike vector field coupled to gravity (but not to matter), which is second order in derivatives.

Let us the express again the aether vector field by u^α and, in analogy with what we did for the SME, write the most general theory for a unit timelike vector field coupled to gravity. If we limit ourself to second order terms in derivatives (or equivalently low energies) this take the form

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_u = \frac{1}{16\pi G_x} \int d^4x \sqrt{-g} (R + \mathcal{L}_u). \quad (17)$$

where \mathcal{L}_u is given by

$$\mathcal{L}_u = -Z^{\alpha\beta}{}_{\gamma\delta} (\nabla_\alpha u^\gamma) (\nabla_\beta u^\delta) + V(u). \quad (18)$$

The tensor $Z_{\gamma\delta}^{\alpha\beta}$ is defined as [97]

$$Z_{\gamma\delta}^{\alpha\beta} = c_1 g^{\alpha\beta} g_{\gamma\delta} + c_2 \delta_{\gamma}^{\alpha} \delta_{\delta}^{\beta} + c_3 \delta_{\delta}^{\alpha} \delta_{\gamma}^{\beta} - c_4 u^{\alpha} u^{\beta} g_{\gamma\delta}, \quad (19)$$

where c_i , $i = 1, \dots, 4$ are simple coefficients of the various kinetic terms and $V(u)$ is a potential term such that it generates a non-zero vev for u^{α} . Note the indicial symmetry $Z^{\beta\alpha}_{\delta\gamma} = Z^{\alpha\beta}_{\gamma\delta}$. An additional term, $R_{ab} u^a u^b$ is a combination of the above terms when integrated by parts, and hence is not explicitly included here.

The potential in (18) is normally fixed by the requirement to remove the ghost excitation associated to the one of the vector components which will necessarily acquire a wrong sign in the kinetic term. The choice $V(u) = \lambda(u^2 + 1)$, where λ is a Lagrange multiplier, fixes the norm of u^{α} and removes the ghost excitation (c.f. the discussions in [98] and [99]). It is this choice of the potential that it is normally associated to the so called ‘‘Einstein–Aether theory’’ [96] which we shall consider here.

8.1.1. Constraints on Einstein–Aether gravity: For what regards the constraints on Einstein–Aether gravity, they can be divided in those on the aether kinetic terms and those on the aether-matter couplings. Given the latter can be reduced to mSME constraints, we shall here discuss specifically only the first kind (one can find a discussion of the second kind in e.g. [100]).

Before of doing so, we can note however that the couplings between aether and the standard model field content while being the same as those discussed before in this review, they do have new features as the aether field has now dynamics, so there can be position dependent violations of Lorentz symmetry. Interestingly, some of the couplings to matter that are unobservable for a single fermion field can have relevant effects when the aether varies. For example, the $-au_{\mu} \bar{\psi} \gamma^{\mu} \psi$ term in the mSME could be removed by making a phase change for the fermion. However, once u^{α} is dynamical and varies with position, only a single component of the term can actually be removed by a phase change [101]. This leads to a new type of term which requires gravitational/position dependent tests in the matter sector [101].

In order to constrain instead aether kinetic terms one can adopt two different approaches. One consists of course in adopting, as normally done any modified gravity theory, a PPN analysis that allows to compare the theory with observations e.g. solar system constraints. Alternatively, one can use the fact that the theory predicts extra degrees of freedom in the gravity sector (there are naively four, but the unit constraint removes one) and that these excitations strongly couple to the metric via the unit constraint (for a more detailed discussion see [97, 100]).

Constraints from PPN analysis: All the PPN parameters vanish except for α_1, α_2 which describe preferred frame effects. α_1 and α_2 were calculated in [102]

$$\alpha_1 = \frac{-8(c_3^2 + c_1 c_4)}{2c_1 - c_1^2 + c_3^2} \quad (20)$$

$$\alpha_2 = \frac{\alpha_1}{2} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{(c_1 + c_2 + c_3)(2 - c_1 - c_4)}. \quad (21)$$

Current constraints are $\alpha_1 < 10^{-4}$ and $\alpha_2 < 4 \times 10^{-7}$ [103] and so from a PPN analysis alone there is still a large 2-d region of parameter space that remains consistent with available tests of GR. Furthermore, recently the orbital evolution of binary pulsars was studied in this framework and was found to induce a much more rapid decay of the binarys orbital period due to the emission of dipolar radiation and modifies quadruple emission [104, 105]. This can be used to cast strong constraint on $(c_+, c_-) \approx O(10^{-2}, 10^{-3})$ where $c_{\pm} = c_1 \pm c_3$ which, in the limit of small $\alpha_{1,2}$, are the only remain free LIV parameters of the theory.

Constraints from gravity-aether wave modes: The combined aether-metric modes consist of the two usual transverse traceless graviton modes, a vector mode, and a scalar mode [106]. The speeds of each of the modes can differ from the speed of light. Hence if the speeds are less than unity, high energy cosmic rays will emit vacuum gravitational Čerenkov radiation [107]. If we denote the speeds of the spin-2, spin-1 and spin-0 modes by s_2, s_1, s_0 then we have [106]

$$s_2^2 = (1 - c_1 - c_3)^{-1} \quad (22)$$

$$s_1^2 = \frac{2c_1 - c_1^2 + c_3^2}{2(c_1 + c_4)(1 - c_1 - c_3)} \quad (23)$$

$$s_0^2 = \frac{(c_1 + c_2 + c_3)(2 - c_1 - c_4)}{(c_1 + c_4)(1 - c_1 - c_3)(2 + 3c_2 + c_1 + c_3)}. \quad (24)$$

The requirement that all these speeds are greater than unity therefore puts constraints on a combination of the c_i coefficients. However, even after imposing all of the above constraints there is still a large region of parameter space allowed. Indeed, the PPN and gravitational Čerenkov constraints are all satisfied provided quite loose conditions on the model coefficients are satisfied [97, 100]. Hence, the gravitational sector is only minimally constrained compared to aether-matter couplings.

8.2. Hořava–Lifshitz gravity

The underlying idea of the Hořava–Lifshitz (HL) gravity (see *e.g.* [108] for a review) is to achieve power-counting renormalizability by modifying the graviton propagator in the ultraviolet by adding to the action terms containing higher order spatial derivatives of the metric, but not higher order time derivatives, so to preserve unitarity. This procedure naturally leads to a space-time foliation into spacelike surfaces, labeled by the t coordinate and with x^i being the coordinates on each surface. The resulting theory is then invariant only under the reduced set of diffeomorphisms that leave this foliation intact, $t \rightarrow \tilde{t}(t)$ and $x^i \rightarrow \tilde{x}^i(t, x^i)$.

It was shown that power counting renormalizability requires the action to include terms with at least 6 spatial derivatives in 4 dimensions [109, 19]. Of course, all lower order operators compatible with the symmetry of the theory are expected to be generated by radiative corrections, so the most general action takes the form [110]

$$S_{HL} = \frac{M_{\text{Pl}}^2}{2} \int dt d^3x N \sqrt{h} \left(L_2 + \frac{1}{M_\star^2} L_4 + \frac{1}{M_\star^4} L_6 \right), \quad (25)$$

where h is the determinant of the induced metric h_{ij} on the spacelike hypersurfaces,

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi {}^{(3)}R + \eta a_i a^i, \quad (26)$$

where K is the trace of the extrinsic curvature K_{ij} , ${}^{(3)}R$ is the Ricci scalar of h_{ij} , N is the lapse function, and $a_i = \partial_i \ln N$. L_4 and L_6 denote a collection of 4th and 6th order operators respectively and M_\star is the scale that suppresses these operators which does not coincide *a priori* with M_{Pl} .

It is perhaps tempting to call M_\star the Lorentz breaking scale, but the theory exhibits Lorentz violations (LIV) at all scales, as L_2 already contains LIV operators. These Infrared (IR) Lorentz violations are controlled by three dimensionless parameters that take the values $\lambda = 1$, $\xi = 1$ and $\eta = 0$ in General Relativity (GR). While, ξ can be set to 1 by a suitable coordinate rescaling, it is presently unclear if the running of the remaining two parameters will converge on the GR values in the IR. Nonetheless, it seems that the theory could still be viable and consistent for suitable choices of the dimensionless parameters λ , η which admits the GR values as extremal limits of the allowed range [111].

Action (25) does present, however, the unappealing feature to contain (in L_4 and L_6) a very large number, $O(10^2)$, of operators and independent coupling parameters. In remedy of this situation, restrictions to the theory have been proposed which would limit the proliferation of independent couplings. We shall not deal with such restrictions here (but see e.g [108, 112] for a concise review) as they will not be determinant for the phenomenological discussion on HL that we shall present later on in this review.

8.3. Constraints on Hořava gravity

Coming to constraints on HL gravity theory, it was shown in [113] (but see also [114]) that the Einstein–Aether theory is equivalent to the the infrared limit of HL gravity if the aether is assumed to be hypersurface orthogonal before the variation. Then it should be obvious that given the relation in the IR between hypersurface orthogonal Einstein–Aether and Hořava–Lifshitz gravity, the previously presented constraints on Einstein–Aether gravity can be related to constraints for the latter theory (see e.g. [104, 105]).

Looking then at the UV complete the theory from the point of view of QG phenomenology, it then interesting to know the available constraints on the Lorentz breaking scale of the theory M_\star . Remarkably, this scale happens to be bounded both below and above. Indeed, for the theory to preserve power counting renormalizability and be at the same time compatible with current observations (microgravity experiments and solar system tests) one has to require $O(1) \text{ meV} < M_\star < 10^{16} \text{ GeV}$ [111, 108, 41], a quite broad opportunity window for the theory.

We have seen however, that radiative corrections will always allow for LIV operators to percolate from a SM sector to the other, gravity being no exception. In this sense it can be generically expected for the above theory to induce Lorentz breaking operators in the matter sector at all orders. Let us assume here that no LIV is present in the matter sector at tree level and that again some protective mechanism will prevent the percolation of the Lorentz breaking terms to the lowest order (mass dimension 3 and 4) operators of the matter sector. Also, one can assume that the CPT and Parity (P) invariance of the gravitational action is preserved in the matter sector. This assumption forbids helicity dependent terms and allows only even power of the momentum in the matter dispersion relation. Hence matter and anti-matter are expected to share the same dispersion relation which within this framework one can then expect to be

$$E^2 = m^2 + p^2 + \eta \frac{p^4}{M_{\text{LIV}}^2} + O\left(\frac{p^6}{M_{\text{LIV}}^4}\right). \quad (27)$$

From a logic point of view, there are now two options: (a) $M_\star = M_{\text{LIV}}$, i.e. M_\star is a universal scale; (b) $M_\star \ll M_{\text{LIV}}$ (as we have seen in this review current phenomenological constraints already rule out the case $M_{\text{LIV}} \ll M_\star < 10^{16} \text{ GeV}$). Clearly, option (b) requires some mechanism which suppresses the percolation of gravity LIVs into the matter sector (see e.g. [115]). Here, reporting the work done in [41], we shall conclude the necessity for such mechanisms.

For doing so one can adopt option (a) and demonstrate that, in this case, matter LIV constraints imply $M_\star = M_{\text{LIV}} > 10^{16} \text{ GeV}$, thus closing the available window for M_\star . Of course this could be easily achieved using the constraints on QED for modified dispersion relation of order $n = 4$ using UHECR. However, we have seen that these constrains are somewhat questionable nowadays while further evidence about the nature of the highest energy particles is awaited. However, a more robust observation is that of the synchrotron radiation from the Crab Nebula, which for Lorentz breaking scales of order 10^{16} GeV can be sensitive to modified dispersion relation of order $n = 4$! In [41] the possible LIV induced modifications to the standard Fermi mechanism (which is thought to be responsible for the formation of the spectrum of energetic electrons in the CN) were considered and the synchrotron spectrum of the CN was recomputed taking into account all the new, LIV induced, phenomena. This was done for $n = 4$ and the specific modified dispersion relation Eq. (27).

The free parameters of the model (electron/positron density and spectrum and magnetic field strength) were fixed in order to reproduce the low energy part of the spectrum, which is not affected by LIV. This allows to reconstruct how LIV affects the higher energy part of the spectrum ($E \gtrsim 100$ keV) and to use a χ^2 statistics to measure when deviations from the observed spectrum due to LIV become unacceptably large. By considering the offset from the minimum of the reduced χ^2 exclusion limits at 90%, 95% and 99% Confidence Level (CL), according to [116] it was shown in [41] that mass scales $M_{\text{LIV}} \lesssim 2 \times 10^{16}$ GeV are excluded at 95% CL.

9. Perspectives

Perhaps a final comment is due to the present state of the art of the field. Lorentz breaking phenomenology has been a remarkable success, a community effort which has built (in a bit over a decade) a wealth of knowledge, methods and constraints that are now at our disposal for efficiently testing candidate theories of QG once their low energy limit is known. However, this state of affair is not and cannot be the end of the story as EFT phenomenology is not yet a true quantum gravity phenomenology. There are probably many more phenomenological consequences of QG beyond and apart LIV and maybe some of them will go beyond the realm of local EFT. For example, causal sets models seems to entail non-locality as a counterweight to the requirement of a Lorentz invariant discretisation of spacetime [117]. The development of more specific phenomenological studies tailored on specific quantum gravity scenarios is probably what we should be heading to in the years to come. Hopefully, the training provided by Lorentz breaking effective field theories would help us facing this sort of future challenges.

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