

# The role of asymptotic freedom for the pseudocritical temperature in magnetized quark matter

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## Abstract.

Motivated by discrepancies observed between lattice QCD simulations and quark models regarding the behavior of the pseudo critical temperature for chiral symmetry restoration as a function of the magnetic field  $B$ , we investigate the effects of a running the quark coupling constant  $G$  with temperature  $T$  and the magnetic field  $B$  in the context of the Nambu–Jona-Lasinio model(NJL). Our point that when asymptotic freedom, an essential feature of QCD and absent in the model, is included through a running of  $G$  with  $T$  and  $B$  results from the NJL model can be brought in qualitative agreement with lattice QCD simulations.

## 1. Introduction

Recently much effort has been devoted to the understanding of the effects produced by a magnetic field in the phase diagram of Quantum Chromodynamics (QCD). The motivation is due to the fact that strong magnetic fields may be produced in non central heavy ion collisions [1, 2, 3, 4]. Other scenarios that strong magnetic fields are also important are in the phenomenology of stars, in particular in magnetars [5, 6], and in the physics of the early universe [7]. Lattice QCD simulations [8, 9] predict that at zero baryon density and zero magnetic field there is a crossover transition at a pseudo critical temperature  $T_{pc}$ . More recent lattice simulations show that this crossover persists if we include the effects of external strong magnetic fields [10, 11, 12, 13]. At zero temperature the lattice results confirm the existence of the phenomenon of magnetic catalysis (MC) [14, 15, 16], where the chiral order parameter is enhanced with the increase of the magnetic field. At finite temperature, lattice results are in agreement with effective models [17, 18, 19, 20, 21]. However, recent lattice results of Refs. [12, 13], that consider physical values of quark masses, predict an inverse magnetic catalysis (IMC), in that the pseudo critical temperature  $T_{pc}$  for the crossover decrease with  $B$ , in total disagreement with all effective model calculations in the literature [17, 18, 19, 22, 23, 24].

Many efforts have been dedicated to understand the disagreement between lattice and models in the behavior of  $T_{pc}$  with  $B$ ; see e.g. Refs. [25, 26, 34, 27]. Very sophisticated versions of quark



models were implemented, as entangled PNJL model [28, 29, 30] where the quark coupling constant of the NJL model is dependent on the Polyakov loop, but no qualitative changes have been observed. Another tentative to produce inverse catalysis was implemented in Ref. [31] where the authors fitted the lattice data assuming that pure-gauge critical temperature  $T_0$  has a dependence with  $B$ , and the model gives IMC. In the context of quark-meson (QM) models no qualitative agreement with the lattice is obtained [32], even if we include the Polyakov loop (PQM) with  $T_0 = T_0(B)$ . More refinements in QM were implemented and the model has no IMC [33].

There is an interesting suggestion [34] that the phenomenon of the IMC is the result of the back-reaction of the gluons due to the coupling of the magnetic field to the sea quarks. It seems therefore possible that the failure of models in obtaining IMC is due to the fact that the coupling constants in effective models do not depend on the magnetic field. Miransky and Shovkovy have shown [35] that the QCD coupling constant decreases for large  $B$ , an effect due to asymptotic freedom of QCD. In a recent publication we [36] have introduced an ansatz for  $G$  in way it decreases with  $B$  and  $T$ , similar to the running of the strong coupling in QCD due to asymptotic freedom. Our results show that at  $T = 0$  there is magnetic catalysis, and at  $T \neq 0$  the model realizes IMC in qualitative agreement with the recent lattice QCD simulations of Refs. [12, 13]. The idea of using a running coupling constant for the NJL coupling with the temperature is not new, it was used in Ref. [37] to understand the decoupling of the pion and recently it was used to study QCD Dyson-Schwinger equations at finite temperature and density [38, 39].

The usefulness of implementing a running coupling in the NJL model can be understood examining the relation between the quark condensate and the constituent quark mass:  $M \sim -G\langle\bar{\psi}_f\psi_f\rangle$ . For a fixed value for  $G$ , one has that  $\langle\bar{\psi}_f\psi_f\rangle$  and  $M$  (and consequently  $T_{pc}$ ) growing with  $B$ , signalling MC. We circumvent this problem assuming that the coupling constant  $G$  decreases with  $B$  and  $T$ . Our results are in qualitative agreement with the lattice; the agreement is quite impressive, given the simplicity of the model. Our results are in the direction that IMC can be result of the back reaction of the sea quarks to the magnetic field [34].

## 2. Running Coupling in the NJL Model at Finite Temperature and Magnetic Field

The  $SU(2)$  version of the NJL model can be defined by a fermionic Lagrangian density given by [40]

$$\mathcal{L}_{\text{NJL}} = \bar{\psi} (i\cancel{D} - m) \psi + G \left[ (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2 \right], \quad (1)$$

We need to introduce a sharp cut off  $\Lambda$  due the nonrenormalizability of the NJL model; the value of  $\Lambda$  is chosen to fit physical quantities. In this work we use  $\Lambda = 650$  MeV. The other parameters of the model are taken  $G = 5.022$  GeV $^{-2}$ ,  $m = 5.5$  MeV. These choices reproduce the vacuum values of the pion decay constant  $f_\pi = 93$  MeV, the pion mass  $m_\pi = 140$  MeV, and the quark condensate  $\langle\bar{\psi}_f\psi_f\rangle^{1/3} = -250$  MeV.

The evaluation of thermodynamic potential  $SU(2)$  NJL model in the mean field approximation (MFA) is well documented in the literature [41, 22] and results beyond the MFA can be found in Ref. [42]. Here we use the MFA; the gap equation is given by

$$M_f = m_f - 2G \sum_f \langle\bar{\psi}_f\psi_f\rangle, \quad (2)$$

where  $\langle\bar{\psi}_f\psi_f\rangle$  represents the quark condensate of flavor  $f$

$$\langle\bar{\psi}_f\psi_f\rangle = -\frac{N_c M_f}{2\pi^2} \left\{ \Lambda \sqrt{\Lambda^2 + M_f^2} - \frac{M_f^2}{2} \ln \left[ \frac{(\Lambda + \sqrt{\Lambda^2 + M_f^2})^2}{M_f^2} \right] \right\}$$

$$\begin{aligned}
 & - \frac{N_c M_f}{2\pi^2} |q_f| B \left\{ \ln[\Gamma(x_f)] - \frac{1}{2} \ln(2\pi) + x_f - \frac{1}{2} (2x_f - 1) \ln(x_f) \right\} \\
 & + \frac{N_c M_f}{2\pi^2} \sum_{k=0}^{\infty} \alpha_k |q_f| B \int_{-\infty}^{\infty} \frac{dp_z}{E_{p,k}(B)} \left\{ \frac{1}{e^{[E_{p,k}(B)]/T} + 1} \right\}, \quad (3)
 \end{aligned}$$

where  $E_{p,k}(B) = (p_z^2 + 2k|q_f|B + M_f^2)^{1/2}$ ,  $x_f = M_f^2/(2|q_f|B)$ , and  $\alpha_k = 2 - \delta_{0k}$ . In addition,  $|q_f|$  is the absolute value of the quark electric charge;  $|q_u| = 2e/3$ ,  $|q_d| = e/3$ , with  $e = 1/\sqrt{137}$  representing the electron charge – we use Gaussian natural units where  $1 \text{ GeV}^2 = 1.44 \times 10^{19} G$ . We consider the condition of chemical equilibrium assuming  $\mu_u = \mu_d = \mu$ . More details of the manipulations to obtain this equation can be found in Ref. [22]. It is important to note that the condensates for the flavors  $u$  and  $d$  are different due to the different electric charges of the quarks. In principle, one would have two coupled gap equations, one for each flavor:  $M_u = m_u - 2G(\langle \bar{\psi}_u \psi_u \rangle + \langle \bar{\psi}_d \psi_d \rangle)$  and  $M_d = m_d - 2G(\langle \bar{\psi}_u \psi_u \rangle + \langle \bar{\psi}_d \psi_d \rangle)$ . However, in the  $SU(2)$  version of the NJL model, when  $m_u = m_d = m$ , the different condensates contribute to  $M_u$  and  $M_d$  in a symmetric way and we can write  $M_u = M_d = M$ .

The lattice results for  $(\Sigma_u + \Sigma_d)/2$  and  $\Sigma_u - \Sigma_d$  are presented in Ref. [13]. The definition of  $\Sigma_f = \Sigma_f(B, T)$  is given by

$$\Sigma_f(B, T) = \frac{2m_f}{m_\pi^2 f_\pi^2} [\langle \bar{\psi}_f \psi_f \rangle - \langle \bar{\psi}_f \psi_f \rangle_0] + 1, \quad (4)$$

where  $\langle \bar{\psi}_f \psi_f \rangle_0$  is the quark condensate evaluated at  $T = 0$  and  $B = 0$ .

The result derived by Miransky and Shovkovy [35] for the QCD running coupling  $\alpha_s$  in the regime of sufficiently strong magnetic fields  $eB \gg \Lambda_{QCD}^2$ :

$$\frac{1}{\alpha_s} \sim b \ln \frac{eB}{\Lambda_{QCD}^2}, \quad (5)$$

where  $b = (11N_c - 2N_f)/12\pi$ . This behavior of  $\alpha_s$  with the magnetic field was our motivation in Ref. [36] to propose an NJL coupling, at  $T = 0$ , of the form

$$G(B) = \frac{G_0}{1 + \alpha \ln \left( 1 + \beta \frac{eB}{\Lambda_{QCD}^2} \right)}, \quad (6)$$

where  $G_0 = 5.022 \text{ GeV}^{-2}$  is the value of the coupling at  $B = 0$ .  $\alpha$  and  $\beta$  are free parameters that were fixed requiring a reasonable description of the lattice average  $(\Sigma_u + \Sigma_d)/2$  at  $T = 0$ . We choose this parametrization because we believe that similar physics of that of asymptotic freedom is responsible for the running of the coupling with magnetic field.

In the high temperature limit,  $\alpha_s$  also runs as the inverse of  $\ln(T/\Lambda_{QCD})$ . However, the values of  $T$  used in the lattice QCD simulations of Refs. [12, 13] are such that  $T \leq \Lambda_{QCD}$  and one cannot use a  $G$  running with  $T$  like in QCD. Since we do not know the running of  $\alpha_s$  with  $B$  and  $T$ , we propose the following ansatz

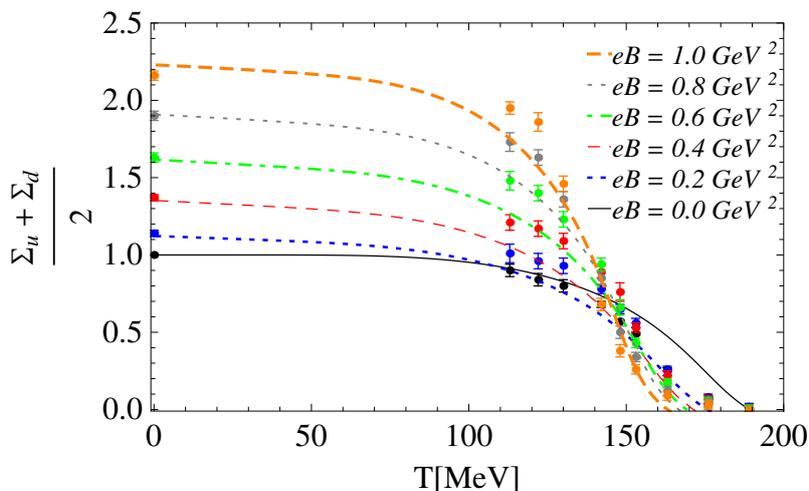
$$G(B, T) = G(B) \left( 1 - \gamma \frac{|eB|}{\Lambda_{QCD}^2} \frac{T}{\Lambda_{QCD}} \right). \quad (7)$$

The parameter  $\gamma$  is fixed when we require the lattice behavior for the temperature dependence of the lattice average  $(\Sigma_u + \Sigma_d)/2$ .

### 3. Numerical Results

We next present our numerical results, but before it is relevant to mention that Refs. [12, 13] used  $m_u = m_d = 5.5$  MeV,  $m_\pi = 135$  MeV and  $f_\pi = 86$  MeV in the multiplicative factor  $m_f/m_\pi^2 f_\pi^2$  in Eq. (4) to make  $\Sigma_f(B, T)$  dimensionless. We use this set of parameters when we compared our NJL results with the lattice simulations. In this work we consider  $\Lambda_{QCD} = 200$  MeV.

Fig. 1 displays the quark condensate average  $(\Sigma_u + \Sigma_d)/2$ . We determine the values  $\alpha = 2$  and  $\beta = 0.000327$  in Eq. (6) with a good fit of the lattice data at  $T = 0$ . In Fig. 1 we can see the phenomena of the magnetic catalysis reproduced at  $T = 0$ . The  $T$  dependence is also reasonably well reproduced (we used  $\gamma = 0.0175$  in Eq. (7)). With this set of parameters fixed we proceed in the calculation of the other quantities.



**Figure 1.** The quark condensate average as a function of the temperature. Data points are from the lattice QCD simulations of Ref. [13].

In Fig. 2 we show the results for the difference  $\Sigma_u - \Sigma_d$ . There is a small deviation from the lattice results for some values of  $B$ , but the agreement is quite impressive, given the simplicity of the version of the NJL model we are using, with a minimum number of parameters.

The model displays a crossover at high temperatures, since we consider the physical point with nonzero current quark masses, and we can establish only a pseudo critical temperature for the partial restoration of chiral symmetry. The value of  $T_{pc}$  depends on the observable that we use to define it. In this work we use the location of the peak for the vacuum normalized quark condensates. The thermal susceptibilities is given by:

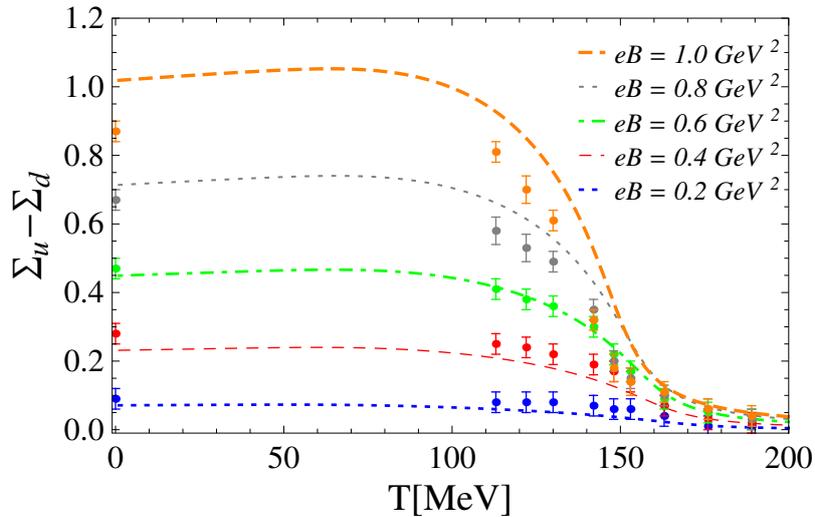
$$\chi_T = -m_\pi \frac{\partial \sigma}{\partial T}, \quad (8)$$

where  $\sigma$  is defined by

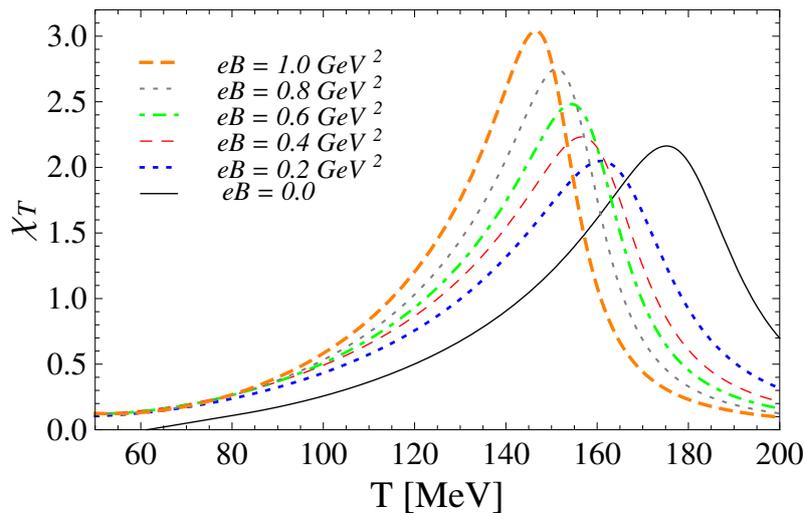
$$\sigma = \frac{\langle \bar{\psi}_u \psi_u \rangle(B, T) + \langle \bar{\psi}_d \psi_d \rangle(B, T)}{\langle \bar{\psi}_u \psi_u \rangle(B, 0) + \langle \bar{\psi}_d \psi_d \rangle(B, 0)}. \quad (9)$$

In Fig. 3 we show our results for the thermal susceptibility as a function of the temperature for different values of the magnetic field. We can see clearly that  $T_{pc}$  decrease for increasing values of the magnetic field. It is clear that the diminishing of  $G$  with  $B$  and  $T$  leads to results that are in good agreement with lattice simulations.

In Fig. 4 we show the NJL results for the pseudocritical  $T_{pc}$  temperature as a function of the magnetic field. This figure shows clearly that the  $T_{pc}$  decreases as  $B$  increases. These results are



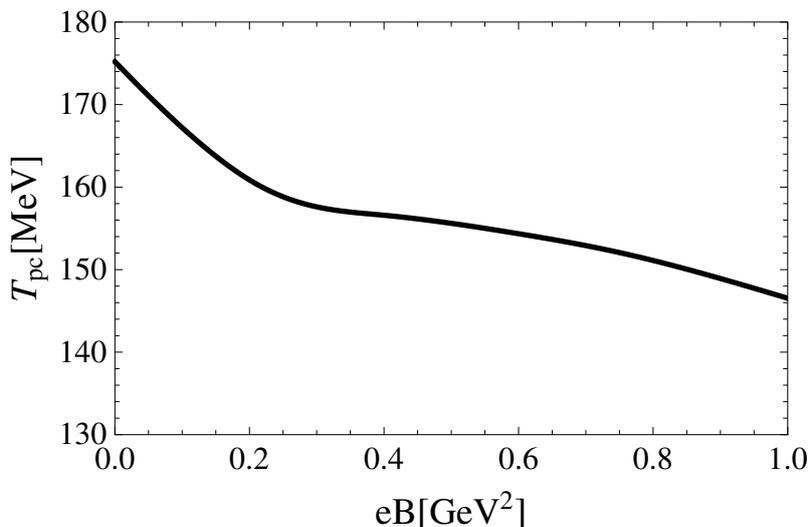
**Figure 2.** The quark condensate difference as a function of the temperature. Data points are from the lattice QCD simulations of Ref. [13].



**Figure 3.** The thermal susceptibility (normalized) as a function of the temperature for different values of the magnetic field  $B$ .

in qualitative agreement with recent lattice QCD results of Refs. [12, 13] that consider physical values of quark masses.

Fig. 4 shows our phase diagram; the lattice results are reproduced qualitatively. We can obtain a better quantitative agreement if we include more fitting parameters in our ansatz for  $G(B)$ . One very important point here is that our results avoid the undesired “turn over” effect in NJL models (with  $B$  independent couplings) where for some values of  $B$ , after a initial decrease,  $T_{pc}$  starts increase with  $B$  (this can be found in Refs. [31, 32]).



**Figure 4.** The pseudo critical temperature  $T_{pc}$  for the chiral transition of quark matter as a function of the magnetic field.

#### 4. Conclusions and Perspectives

In this work we presented results for several physical quantities associated with the partial restoration of chiral symmetry at finite temperature and in the presence of an external constant magnetic field. They were calculated within the mean field approximation in two flavor NJL model. Our aim was to understand discrepancies between effective quark models and recent lattice QCD results regarding the behavior of the pseudo critical temperature for the chiral symmetry restoration as a function of magnetic field. Motivated by the property of asymptotic freedom in QCD to introduce an ansatz that implements a running coupling with  $T$  and  $B$ . It is well known in the literature [37] that if  $G$  decreases with the temperature, general features of the chiral phase transition do not change considerably. But this is not the case in the regime of strong magnetic fields. The quark condensates  $\langle\bar{\psi}_u\psi_u\rangle$  and  $\langle\bar{\psi}_d\psi_d\rangle$  increase with  $B$ , due to the magnetic catalysis. The constituent quark mass  $M$  in the NJL model also increases with  $B$ , as it is given by the gap equation  $M_f \sim -G\langle\bar{\psi}_f\psi_f\rangle$ . In our work in Ref. [36] we have shown that the lattice result that  $T_{pc}$  decreases with  $B$  can be explained when the coupling  $G$  decreases with  $B$  and  $T$ .

There are several directions for future work. We can improve the ansatz to get a quantitative agreement with the lattice. The extension of NJL model including asymptotic freedom proposed in Ref. [36] can be used to study the behavior of other quantities such as meson-quark couplings and decay constants. We can improve the ansatz including dependence with chemical potential, working in a region that is inaccessible to the lattice. At finite density and in the presence of  $B$  we can study the phenomenology of stars [43], in particular magnetars.

The parametrization  $G(B, T)$  mimics asymptotic freedom of QCD and it can be understood as the back reaction of the sea quarks. First principles, nonperturbative calculations of the effects of an external magnetic field are provided by Dyson-Schwinger equations; for calculations at finite  $T$  and  $\mu$ , see Ref. [39]. The coupling of sea quarks with the magnetic field appear when we solve the system of integral equations for gluon and quark propagators. Such an approach was recently applied to study fermion mass generation at finite  $B$  in QED [44, 45] and QCD [26, 46, 47]. The extension of DSE studies to finite  $T$  and  $B$  should shed light on the phenomenon of IMC beyond the scope of effective models for QCD.

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